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Bachelor Thesis

Comparison of Neoclassical Theories of Asset Pricing

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Chapter 1

Introduction

"Price is what you pay; value is what you get"
Benjamin Graham [n.d.][1].

How do we price an asset - especially how do we price a risky asset? This is the domain of asset pricing theory, which is a central theme in financial economics. It has attracted the attention of many of the most renowned researchers in finance and a vast number of papers on asset pricing have been written. They all have one central question in common: how do individuals allocate scarce resources through a price system based on the valuation of risky assets? [Copeland, Weston, and Shastri, 2005]

1.1 Motivation

This thesis aims to compare the most important asset pricing theories in the Neoclassical Paradigm. Most of our understanding of modern finance has been build on four interconnected but independent theories: first the State-Preference Approach, second the Modern Portfolio Theory and the Capital Asset Pricing Model, third the Arbitrage Pricing Theory and fourth Option Pricing Theory. Those milestones in financial economics allow us to price risky assets. Since comparisons of the theories are seldom made on a satisfying level in finance textbooks this thesis aims to bridge this gap.

1.2 Purpose

The main purpose is to allow the reader to develop a coherent understanding of the main pillars of modern finance and bring them together in a greater context. This will allow to

[1] As cited in Buffett [2008].

relate all theories from different introductory finance courses. The theories under consideration will all be developed under the usual neoclassical assumptions of perfect markets. This means that asset markets are assumed to be perfectly competitive and frictionless. Hence, there are no taxes, indivisibilities, restrictions on borrowing, lending and short-selling or any other market imperfections [Dybvig and Ross, 2003, Copeland et al., 2005].

Furthermore, the efficient market model is assumed to hold and to be known to the reader. More specifically, one can state market equilibrium conditions in terms of expected returns and prices "fully reflect" all available information [Fama, 1970].

1.3 Structure

First, this thesis analyzes the State Preference Theory, SPT henceforth, pioneered by Arrow [1964] and Debreu [1959]. The framework allows for a very simple representation and understanding of financial markets and the derivation of security prices. In the SPT the objects of choice are characterized by contingent consumption claims over alternative future states of the world. The main results of the theory will be discussed afterwards. The chapter will serve as a good basis for the understanding of the arbitrage principle discussed subsequently.

Parallel to this approach Markowitz [1952] has pioneered the Mean Variance Approach which defines the objects of choice in terms of mean (desirable) and variance (undesirable) properties of asset returns. Thus, investor's preference curves are assumed to be defined in terms of the mean and variance of returns. From this starting point the whole theoretical construct known as Modern Portfolio Theory, MPT henceforth, has been developed. One can imagine the implications of the theory when considering that despite its age the theory is still termed "modern" by economists nowadays. The Capital Asset Pricing Model by Sharpe [1963] and Lintner [1965], CAPM henceforth, as the MPT's crown jewel, is taught in virtually every introductory finance course. The CAPM allows defining and pricing "risk" for single assets and portfolios. A descriptive interpretation will follow the derivation thereby presenting the great intuitive value of the CAPM.

The third approach can be termed No-Arbitrage Approach. Arbitrage is a key concept in finance and will be examined elaborately in this chapter. It was used in the SPT framework to some extent, but not nearly as exhaustively as it was used by Ross in the 1970s to develop the important Arbitrage Pricing Theory. Thus, the concept of Arbitrage will be first defined informally and formally to gain a better understanding. Second, the theories that can be derived from this principle will be explained. An excursus to the Option Pricing Theory will also be made to underline the importance of No-Arbitrage.

Following this, a comparative analysis of the theories will be given. The emphasis will lie on the CAPM and APT due to their popularity. First, the main assumptions will be compared as those are the foundations of the models. After that the author will point out the pedagogy and intuitive value of the models to justify why the CAPM is the traditional workhorse in introductory corporate finance texts. Moreover, the implications of the models will be analyzed to allow the reader to grasp the essence of Neoclassical Finance. The empirical content will be discussed in the following section to show how well the models explain real world financial markets. A conclusion will then follow to summarize the main findings in the Neoclassical Paradigm.

Chapter 2

State Preference Approach

"The state preference approach [...] resolves the assets or securities into distributions of dated contingent claims to income defined over the set of all possible 'states of the world'" [Hirshleifer, 1966, p. 252].

The State Preference Theory (SPT) impresses through its simplicity and provides a very basic intuition and "feel" of how financial markets function. The approach defines the objects of choice to be contingent claims in alternative future states of the world - securities will be represented as vectors of state contingent claims. The theory will be kept very simple and will only serve to get a better understanding of financial markets.

This approach has been pioneered by Arrow [1964] and Debreu [1959]. It has undergone some modifications and generalizations by Hirshleifer [1964, 1965, 1966] and Myers [1968]. This chapter mostly follows the explanations by Copeland et al. [2005] and Zimmermann [1998], as those are very intuitive descriptions of the theory.

2.1 Basic State Preference Framework

The SPT framework features two points in time: t_0 as today and t_1 as tomorrow. Trading and portfolio optimization only occur in t_0. The uncertainty in this framework is characterized through various mutually exclusive and exhaustive future states that can occur at time t_1 from the finite set $\Omega = \{w_1, \cdots, w_S\}$ with cardinality S. The investor might know the different probabilities of the states, but he does not know which one is going to occur. Securities can therefore be seen as a set of possible payoffs each occurring in a mutually exclusive state of nature. Mathematically speaking, they can be represented as a vector[1] of state contingent claims or as a random variable.

[1] Henceforth, vectors are denoted in underlined letters.

Represented as vectors, securities assign a payoff to every possible state ω_s:

$$\underline{a}_j = \begin{pmatrix} a_j(\omega_1) \\ \vdots \\ a_j(\omega_S) \end{pmatrix}.$$

At time t_0 it is not known which state will occur, but the individuals know each possible payoff. The set $A = \{a_1, \cdots, a_J\}$ represents the securities and has cardinality J. At time t_0 the prices of the existing securities are given by the vector

$$\underline{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_J \end{pmatrix}$$

where each p_j is the price of a security $\underline{a}_j$. An important concept to be introduced are the Arrow-Debreu-Securities, ADS henceforth, $(\underline{e}_1, \underline{e}_2, \cdots, \underline{e}_s)$. Those securities yield a payoff of one monetary unit in a certain state s and zero otherwise:

$$\underline{e}_s = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \leftarrow \text{state s.}$$

This concept allows for the intuitive decomposition of every payoff into a linear combination of ADS. One can now further examine the array of possible payoff structures. To do so one can condense the elements introduced so far in a $S \times J$ payoff matrix D, that can be seen as one of the simplest representations of a financial market. Each row represents a state and each column represents a security:

$$D = \begin{pmatrix} a_1(\omega_1) & a_2(\omega_1) & \cdots & a_J(\omega_1) \\ a_1(\omega_2) & a_2(\omega_2) & \cdots & a_J(\omega_2) \\ \vdots & \vdots & & \vdots \\ a_1(\omega_S) & a_2(\omega_S) & \cdots & a_J(\omega_S) \end{pmatrix}.$$

Furthermore, a portfolio $\underline{x}$ is defined as a linear combination of securities of the following form, where each x_j denotes the number of each security held:

$$\underline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_j \end{pmatrix}.$$

2.1.1 Complete Markets

One crucial assumption in the Arrow-Debreu world is the completeness[2] of the market. The market is said to be complete if every payoff structure is achievable, i.e. if the asset's returns span the s states. Formally completeness is achieved if every ADS $\underline{e}_s$ can be constructed through a portfolio $\underline{x}_s$. That means

$$D\underline{x}_s = \underline{e}_s \ , \ s = 1, \cdots, S.$$

Linear algebra tells us that the linear equation system can be solved as rank (D) = S. An interesting insight one can already gain here is that one could eliminate all risk by choosing a portfolio that will yield equal payoffs in every state of nature. This will be of interest in a later chapter.

2.1.2 No-Arbitrage Condition

One further important assumption which has to be made is the No-Arbitrage profit condition. Interestingly, this condition was stated as a "by-product" in the original works e.g. a necessary condition in relation to the *single-price law of markets*, as in Hirshleifer [1966], or excluded through assumptions about the prices of ADS, as in Arrow [1964]. At that time nobody thought of Arbitrage itself as a powerful tool for the valuation of assets and a basis for sophisticated asset pricing theories on its own. In modern textbooks which start with the introduction of an Arrow-Debreu-like financial market the Arbitrage argument is followed more elaborately and conclusions are made that go far beyond the original works of SPT. Those results will be derived in Chapter 4 and one will see to what extend the harmless assumption of No-Arbitrage can help us understand and develop financial market models.

For the purposes of this chapter the following notion is sufficient: a capital market equilibrium requires that market prices are set so that supply equals demand for each individual security. This means that any two securities or portfolios with the same cash flows at t_1 must have the same price. This is the *single-price law of markets*. [Copeland et al., 2005]

2.2 Derivation of Security Prices

Having established a simple complete-markets model one can now derive the relationship between the given market prices of securities and the prices of ADS by simply solving a linear equation system. Any new stream of cash flows is valued based on the prices of the original j securities. This becomes more obvious if the so-called *state price vector* $\underline{v}$ is introduced:

[2]The thesis will assume complete capital markets from now on. Incomplete capital markets have severe implications on pricing as the price vector will not be defined uniquely anymore. The reader is referred to Zimmermann [1998] for a more detailed discussion.

$$\underline{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_S \end{pmatrix}$$

where

$$\underline{p} = D^T \underline{v}.$$

The interpretation is straightforward: each v_s is the price of the ADS $\underline{e}_s$ at time t_0. The state prices are inferred from existing security prices. This state price vector allows us to value any stream of uncertain cash flows by multiplying each state contingent claim by its state price. The next section will look at the determinants of existing asset prices. But before that the author introduces a very intuitive example found in Copeland et al. [2005] to illustrate the concept of the state-price vector:

Imagine a fruit market where a stand sells two baskets of fruits composed of bananas and apples. The prices of the baskets can be interpreted as the given security prices in the SPT. Each basket represents a security. The amounts of fruit are the "state contingent" payoffs.

	Bananas	Apples	Prices
Basket 1	10	20	$ 8
Basket 2	30	10	$ 9

Table 2.1: Fruit baskets: prices and amounts. (Source: Copeland et al. [2005, p. 79])

Simple algebra allows us to determine the prices of one banana and one apple easily. Let the fruit prices be v_A and v_B, respectively, and the quantities $Q_{1A,B}$ and $Q_{2A,B}$. The given basket prices are p_1 and p_2. Solving the linear equation system

$$p_1 = v_A Q_{1A} + v_B Q_{1B}$$
$$p_2 = v_A Q_{2A} + v_B Q_{2B}$$

gives us the prices of a single banana and single apple. Those can easily be interpreted as the state prices in this analogy. Hence, the vector

$$\underline{v} = \begin{pmatrix} v_a \\ v_b \end{pmatrix} = \begin{pmatrix} \$\,0,30 \\ \$\,0,20 \end{pmatrix}$$

is the state price vector. One is able to price every imaginable fruit basket consisting of bananas and apples. This is what the state price vector can do in the real world: price every imaginable future cash flow.

2.3 Determinants of Security Prices

In this section the determinants of security prices will be derived to learn more about asset pricing. As in most types of analysis in finance the starting point is the optimal portfolio decision of a representative individual. The first order conditions (FOC) of those problems have led to numerous results in finance, with asset pricing relationships being among the most important, as listed in Dybvig and Ross [2003]. More precisely, the portfolio choice problem characterizes the certain investment today for the uncertain benefit in the future. The author will examine this decision with regard to ADS, since the real securities can be constructed from ADS. The analysis is facilitated by using ADS.

The expected utility can be written as $\sum \pi_s U(Q_s)$, where Q_s simply denotes the number of pure securities $\underline{e}_s$ in a given state s. π_s is the probability of state s and $U(\cdot)$ is a concave and differentiable utility function. This is equivalent to stating that the expected utility equals $\sum \pi_s U(W_s)$ where W_s denotes the end of period wealth in the state s. Hence, $\underline{W}$ denotes all possible values of future wealth and can be interpreted as a random variable.

The analysis considers a representative individual who tries to choose the optimal portfolio of ADS in order to achieve the utility-maximizing investment. The individual has initial wealth of W_0 that can be used for consumption C_0 now and the investment in ADS for future consumption. The expected utility $\hat{U}(\underline{W})$ of the end of period wealth can be written as:

$$\hat{U}(\underline{W}) = \sum_{s=1}^{S} \pi_s U(W_s) = \sum_{s=1}^{S} \pi_s U(Q_s).$$

The objective function to be maximized is the utility of today's consumption and expected utility of tomorrow's consumption:

$$\max_{C_0, Q} \left[U(C_0) + \hat{U}(\underline{Q}) \right]$$

subject to

$$B = C_0 + \sum_{s=1}^{S} Q_s v_s$$

where B is some given budgetary constraint. Writing the sum as a product of vectors the following Lagrangian function is obtained:

$$L(C_0, \underline{Q}, \lambda) = U(C_0) + \hat{U}(\underline{Q}) - \lambda(C_0 + \underline{Q}^T \underline{v} - B).$$

From the first order conditions one gets helpful insights into the decision making of individuals. The first order conditions of the problem are as follows:

$$\frac{\partial L}{\partial C_0} = U'(C_0) - \lambda = 0 \tag{2.1}$$

$$\frac{\partial L}{\partial \underline{Q}} = \hat{U}'(\underline{Q}) - \lambda \underline{v} = \hat{U}'(\underline{W}) - \lambda \underline{v} = 0 \tag{2.2}$$

$$\frac{\partial L}{\partial \lambda} = C_0 + \underline{Q}^T \underline{v} - B = 0 \tag{2.3}$$

where the $U'(\underline{W})$ in the second FOC (2.2) can be written as:

$$U'(\underline{W}) \equiv \begin{pmatrix} \frac{\partial \hat{U}}{\partial W_1} \\ \frac{\partial \hat{U}}{\partial W_2} \\ \vdots \\ \frac{\partial \hat{U}}{\partial W_s} \end{pmatrix} = \begin{pmatrix} \pi_1 U'(W_1) \\ \pi_2 U'(W_2) \\ \vdots \\ \pi_s U'(W_s) \end{pmatrix}. \tag{2.4}$$

Rearranging the equations one can obtain important results. Plugging (2.1) into (2.2) and using (2.4) one obtains:

$$U'(C_0) = \frac{\partial \hat{U}}{\partial W_s} \frac{1}{v_s} = \pi_s U'(W_s) \frac{1}{v_s}.$$

This already allows a very intuitive insight: the marginal utility of today's consumption must equal the *expected, price-adjusted marginal utility* of wealth in every state of nature. Additional information can be inferred when considering that this relationship must hold for any arbitrary state z:

$$U'(C_0) = \frac{\pi_s U'(W_s)}{v_s} = \frac{\pi_z U'(W_z)}{v_z}$$

This simply means that the *expected, price adjusted marginal utility* of wealth must be the same across all states. Another reformulation yields the important insight that prices must reflect the marginal rates of substitution of wealth in different states:

$$\frac{\pi_s U'(W_s)}{\pi_z U'(W_z)} = \frac{v_s}{v_z}. \tag{2.5}$$

One can see the similarities between general equilibrium models with trade in microeconomics. In equilibrium the prices indicate the marginal rates of substitution between goods - in our case between state contingent claims. In other words, this is the usual result from neoclassical economics: the gradient of the utility function is proportional to prices [Dybvig and Ross, 2003].

So in general, the price of a security j is determined by multiplying its cash flows[3] by their respective state price

$$p_j = \sum_{s=1}^{S} v_s a_{sj}. \tag{2.6}$$

where each v_s can be expressed as

$$v_s = \frac{\pi_s U'(W_s)}{U'(C_0)}.$$

Sandmann [2001] derives this relationship with regard to the marginal utility of consumption now and consumption in future state s instead of wealth in future state s. Assuming that the individual will consume all wealth in state s, this is the same.

It seems difficult to make this relationship operational in practice since it is impossible to measure marginal utility with such precision.[4] But one can derive very important implications from the pricing equation and the general price relationship in (2.5). Since marginal utility is decreasing (reflecting risk aversion), one can infer something about the prices in relation to aggregate wealth levels. States with low levels of aggregate consumption will lead to a higher price for the correspondent ADS. Thus, "insurance" for states with low aggregate consumption is relatively expensive. [Bossaerts, 2002]

Introducing the concept of the *state price density* one can explain more formally, why some ADS are "more expensive" than others. Dividing the state prices by their respective probabilities of the state occurring one obtains the probability-adjusted willingness to pay. Or as Dybvig and Ross [2003] describe it: "it is a measure of priced relative scarcity in state of nature s" [p. 608]. They reflect the marginal utility of consumption and are high in states with low aggregate wealth since "the marginal utility of consumption is proportional to the relative scarcity". Defining the state price density as

$$\rho_s \equiv \frac{v_s}{\pi_s}$$

the asset pricing equation (2.6) can be written as an expected value:

$$\begin{aligned} p_j &= \sum_{s=1}^{S} \pi_s \rho_s a_{sj} \\ &= E[\tilde{\rho}\tilde{a}_j] \end{aligned}$$

where the tilde "˜" denotes a random variable. Dividing the expression above by p_j will yield a more convenient representation for further calculations. By common economic

[3] For notational purposes the abbreviated form, $a_{sj} := a_j(\omega_s)$ as used in Sandmann [2001], is adopted.

[4] Agreeing on a set of possible states and their outcomes is as well an obstacle as will be discussed in a later chapter.

reasoning the quotient $\tilde{a}_j/p_j$ denotes one plus the (uncertain) expected return on security j written as $(1+\tilde{R}_j)$ [Zimmermann, 1998, p. 39]. Furthermore, $(1+\tilde{R}_j)$ will be denoted as $\tilde{X}_j$. Thus, one can write:

$$1 = E\left[\tilde{\rho}\frac{\tilde{a}_j}{p_j}\right] \equiv E[\tilde{\rho}(1+\tilde{R}_j)] \equiv E[\tilde{\rho}\tilde{X}_j].$$

Denoting the return on a riskless asset by $X = (1+R_0)$ it can be proven[5] that the following relationship holds for the expected return on security j:

$$E[\tilde{X}_j] = X - XCov[\tilde{\rho}, \tilde{X}_j].$$

This equation allows us to state our previous intuition about state prices and their relation to aggregate wealth formally. First, it shows that the expected return of a security depends on its covariance with the state price density. The more negative the covariance the higher the return. The interpretation is very valuable for our understanding of asset prices: a negative covariance means that asset payoffs are high when the state price density is low (hence, the willingness to pay is low) and vice versa. Having assumed risk averse investors those are the states of the world where aggregate wealth is high. So assets with payoffs that correlate with the aggregate wealth have higher expected returns, because they offer no insurance against economic risk. Bearing this risk is rewarded with more return. This equation and reasoning is very similar to the CAPM as will be shown in the next chapter.[6]

The insight from this is, that securities with a high proportion of *non-diversifiable* risk, i.e. with payoffs that reflect the end of period wealth closely, will have higher expected rates of return. The securities that do not share that economy risk will have lower rates of expected return since they do not involve a lot of risk bearing in terms of aggregate wealth levels. The following chapters will show how to arrive at the same result in a more formal way.

2.4 Risk Neutral Valuation

This section will quickly introduce an important concept known as "Risk Neutral Valuation" that will take on added importance in Chapter 4. As noted, one can construct a security that yields a payoff of one monetary unit in each state, thus, making it risk free. Such a security would be a pure discount bond trading at a risk free interest rate discount.

[5] The proof can be found in Zimmermann [1998, p. 41 - 42]. It has been omitted for the lack of space, but in essence it makes use of the usual expectations and covariance laws from statistics.

[6] The SPT framework allows to actually derive a state-price beta model which is a special version of the CAPM as in Duffie [2001, p. 11 - 12]. Another possible development is the Consumption CAPM as in Dybvig and Ross [2003, p. 621 - 622]. With further assumptions such joint normal distribution of asset returns the result can be extended to develop the CAPM.

Thus, the sum of the state prices should equal the price of the riskless investment:

$$\sum_{k=1}^{S} v_k = \frac{1}{1+R_0}. \tag{2.7}$$

This insight allows to transform the state prices into a common discount factor known as risk neutral probabilities. Following Müller [2009] define $\underline{\psi}$ as

$$\underline{\psi} = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_S \end{pmatrix} \equiv (1+R_0) \begin{pmatrix} v_1 \\ \vdots \\ v_S \end{pmatrix}$$

where

$$\psi_s >> 0 \quad \text{and} \quad \sum_{s=1}^{S} \psi_s = 1.$$

The $\underline{\psi}$ vector can be interpreted as a vector of probabilities since they are all between zero and one and sum to one. Those probabilities are called risk neutral probabilities - of course they are not the the "real probabilities", but using them simplifies mathematical finance since one can use the rich mathematical toolkit known from statistics. The value of a cash flow under risk neutral valuation is its expected value under risk neutral probabilities discounted at the risk free rate:

$$p_j = \frac{1}{1+R_0} \sum_{s=1}^{S} a_{sj} \psi_s = \frac{1}{1+R_0} E^{\psi}[\tilde{a}_j].$$

Valuing with risk neutral probabilities is different from the "traditional" approach. In the traditional approach the asset j is valued by taking the expected value of the cash flows under statistical probabilities denoted $E^P[\tilde{a}_j]$ and discounting it with a risk adjusted rate of return denoted R_j. Thus, the risk adjustment takes place in the denominator.[7] Under risk neutral probabilities the risk adjustment takes place in the numerator when taking the expected value $E^{\psi}[\tilde{a}_j]$. To illustrate this mathematically:

Pricing:		traditional		risk neutral
p_j	$=$	$\frac{E^P[a_j]}{(1+R_j)}$	or	$\frac{E^{\psi}[a_j]}{(1+R_0)}$

In a risk neutral world the investors assume that securities grow at the risk free rate since the risk adjustment is done by taking the expected value. The equivalence of those two approaches is very important in modern quantitative finance. Risk neutral probabilities will be encountered in the No-Arbitrage chapter again, especially in the Option Pricing Theory.

[7] Of course the traditional pricing equations can be reformulated for the risk adjustment to take place in the numerator, but that is not common.

2.5 Summary

The State Preference Theory provides an elegant and general framework for the analysis of financial markets and yields a pricing rule for securities. This so-called *state price vector* can be inferred from existing security prices in a complete capital market and can value any new security introduced into the market. Common economic reasoning has been encountered: prices reflect the scarcity among states. In addition, it was shown that securities with payoffs that resemble the end of period aggregate wealth, i.e. are positively related to the economy, have higher expected rates of return than securities that do not show this dependency on the economy as a whole.

Many authors have developed this approach with more elaborate reasoning on the No-Arbitrage assumption. However, this was not originally included in the theory and will be discussed in the No-Arbitrage chapter. For now, the SPT provides us with a useful addition to a financial economist's toolkit and with a basic understanding of financial markets and prices. It is in fact very general and one can derive the MPT and APT by adding assumptions to this framework as will be shown in the following chapters. The following model will be more specific about the concept of non-diversifiable risk.

Chapter 3

Mean Variance Approach

"We next consider the rule that the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing" [Markowitz, 1952, p. 77].

The following approach to asset pricing has become one of the centerpieces in financial economics. "It represents an almost perfect blend of elegance and simplicity" as Grauer [2003, p. xiii] puts it. The Mean Variance Approach describes the objects of choice in terms of mean and variance properties of their returns. The investor's preferences are assumed to be defined in terms of the mean and variance of the asset returns. Asset returns are assumed to be random variables and furthermore, investors are only interested in financial aspects of the portfolios, and nothing else. Markowitz pioneered this approach in 1952.

Based on Markowitz's works Sharpe [1963, 1964] and Lintner [1965][1] developed the famous Capital Asset Pricing Model (CAPM). Using simple mathematics Sharpe derived a model that still counts as one of the most fundamental achievements in financial economics. Although empirical testing might not fully validate the theory, it is taught and used widely because of the "insight it offers and because the results are accurate enough for a wide range of applications" [Bodie, Kane, and Marcus, 2008, p. 293].

The chapter will start with Markowitz's ideas on portfolio selection. The formal development of the theory closely follows Spremann [2008a] and Copeland et al. [2005]. References to the original papers will also be made.

[1] Treynor [1961] developed the CAPM simultaneously but his work was never published and Sharpe and Lintner remained the most common citations.

3.1 Markowitz - Portfolio Selection

Markowitz [1952, 1959] was the first to describe portfolio optimization as the mean variance choice theoretic approach. He positioned single assets in a risk-return diagram to analyze the selection of *efficient* portfolios. He also developed algorithms for the systematic selection of all efficient portfolios thereby deriving the *efficient frontier*. With elaborations by Tobin [1958] this has led to the central concepts of the *market portfolio* and *Capital Market Line* (CML). This section closely follows Spremann [2008a].

3.1.1 Central Building Blocks

The development of the MPT by Markowitz is based on three central building blocks to be discussed briefly. More implicit assumptions will be pointed out in Chapter 5.

1. **Single Period Model**
   There are two points in time t_0 and t_1 as in the SPT framework. At t_0 the investor determines his asset allocation and does not change it until t_1. As a portfolio choice problem, the MPT is about the initial composition of the portfolio.

2. **Parameters of the Discrete Returns**
   The preferences are defined over the uncertain asset return in terms of its mean and variance. The mean is the desired property and should be maximized while the variance is undesired and should be minimized. Moreover, the returns of assets exhibit the portfolio properties. The portfolio mean return is the weighted average of single asset means as follows:
   $$\mu_p = \sum_{k=1}^{n} x_k \cdot \mu_k \; , \tag{3.1}$$
   where μ_k is the mean and x_k the portfolio weight of an asset. The variance can be computed as follows:
   $$\sigma_p^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} x_j \cdot x_k \cdot \sigma_j \cdot \sigma_k \cdot \rho_{j,k} \; , \tag{3.2}$$
   where σ_n is the standard deviation of an asset's return and $\rho_{j,k}$ is the correlation coefficient of assets j and k.

3. **Normal Distribution**
   The parameters of the asset returns are assumed to be normally distributed which leads to the following simplifications:

   - Parameters of portfolio returns are normally distributed, too.

- The first two parameters of the distribution are sufficient for a complete description of the Investment Opportunity Set (IOS).

With this in mind one can proceed to the implications of those assumptions. To do this the Risk Return Diagram will be introduced.

3.1.2 Risk Return Diagram

As already mentioned only the first two moments of the distribution of asset returns are relevant - a graphical representation suggests itself. This facilitates the analysis and yields interesting insights.

3.1.2.1 Efficient Portfolios

Figure 3.1 below displays that certain portfolios or assets are *dominated* by others in mean variance terms. Mathematically speaking, a portfolio B with mean μ_b dominates a portfolio C with mean μ_c if $\mu_b \geq \mu_c$ and $\sigma_b < \sigma_c$ hold. This allows a definition of *efficient* portfolios. In Markowitz's [1959] words: "If a portfolio is 'efficient', it is impossible to obtain a greater average return without incurring greater standard deviation; it is impossible to obtain smaller standard deviation without giving up return on average" [p. 22]. In Figure 3.1 asset B clearly dominates asset C.

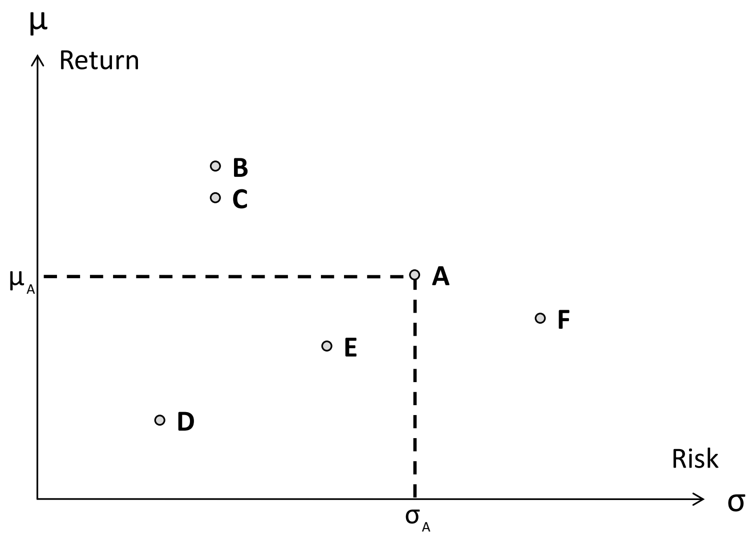

Figure 3.1: Risk return diagram. (Source: own adaptation from Spremann [2008a, p. 178])

3.1.2.2 Minimum Variance Portfolio and Efficient Frontier

Now the *diversification effect* can be examined. When positioning two arbitrary assets in the risk return diagram, one can identify the corresponding portfolios that can be generated with those two assets. Assuming a correlation that is not perfect, all possible risk return combinations can be plotted to obtain a *hyperbola* of possible investment opportunities. Interestingly, one can achieve smaller standard deviation than the weighted average of the standard deviations of the single assets would suggest.[2] This can be seen in the figure below representing the possible investment opportunities with assets A and B on the dark line.

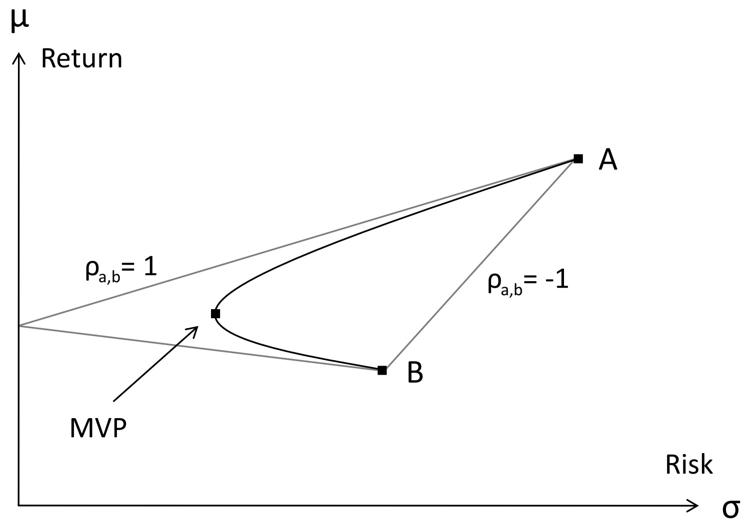

Figure 3.2: Efficient Set and Minimum Variance Portfolio. The cases of perfectly negative and positive correlations bound the hyperbola in a triangle. (Source: own adaptation from Spremann [2008a, p. 179] and Copeland et al. [2005, p. 118])

The upper bound of the hyperbola represents all the efficient portfolios while the lower bound represents the non-efficient portfolios. Those two types are separated by the *Minimum Variance Portfolio* (MVP). It features the smallest standard deviation achievable. The upper bound of the hyperbola is also called the *efficient frontier* and has the same shape for a set of n risky assets.[3] In Markowitz's theory an investor will now choose a portfolio *on* the efficient frontier. What return this portfolio will have depends on the investor's individual preferences. He will choose the portfolio at the point of tangency between his indifference curve and the efficient frontier, thereby maximizing the mean and minimizing the variance.

[2] The cases of perfectly negative and positive correlation bound this hyperbola in a triangle. For a proof and further explanation see Copeland et al. [2005, p. 117 - 119].

[3] See Merton [1972] for derivation and proof.

3.1.3 Capital Market Line

The investment opportunity set in Markowitz's theory only contains risky assets, thus, leading to an hyperbolic efficient frontier. Tobin [1958] considered an investment opportunity set with a riskless asset. This leads to the fundamental result of the *Capital Market Line* (CML). The CML is tangent to the efficient frontier. The *market portfolio* in the point of tangency plays a significant role as will be shown.

3.1.3.1 Efficient Frontier with a riskless asset

To develop the efficient frontier with a riskless asset, consider an investor holding a portfolio P of n risky assets with return R_p and parameters μ_p and σ_p. The individual then forms a portfolio Q consisting of a combination of the portfolio P and a riskless asset with relative amounts w and $1 - w$, respectively. The return of the riskless asset is R_0 and it has a standard deviation of zero.[4] Referring to Spremann [2008a] one can show that once a riskless investment is introduced all portfolios Q generated by an investment in any arbitrary combination of n risky assets and the risk-free asset lie on a straight line in the risk return diagram. It starts at the riskless asset positioned at return R_0 on the ordinate and continues through the risky portfolio of assets.[5] This yields many interesting insights and opens the field for intuitive thought as Spremann [2008a] suggests:

First, one can see that there is a linear relationship between risk and return. Second, one would want that relationship to be as favorable as possible, i.e. the line as *steep* as possible. Third, the line allows the investor to build new portfolios that clearly dominate the efficient frontier. Following those insights, one will now choose the portfolio of risky assets to be the tangent portfolio between the straight line and the old efficient frontier. This reasoning is depicted in Figure 3.3. The portfolio that is positioned at the point where the tangent starting at the risk-free rate touches the efficient frontier allows for the most favorable trade off between risk and return.

[4] It will be assumed that one can hold negative amounts of the riskless asset, i.e. borrow at the risk-free rate.

[5] The proof is analogous to the derivation of the CML in the next two paragraphs and is postponed to the next footnote.

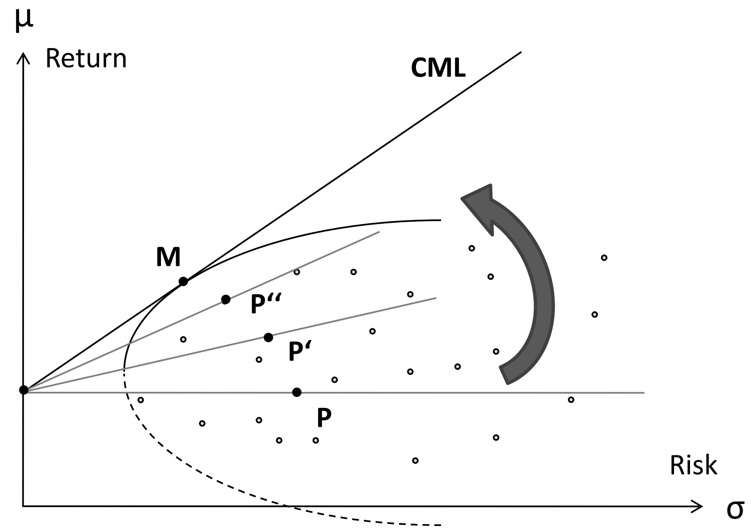

Figure 3.3: Derivation of the Capital Market Line. (Source: Own adaptation from Spremann [2008a, p. 220])

This line starts at return R_0 and continues with equation[6]:

$$\mu_q = R_0 + \frac{\mu_m - R_0}{\sigma_m} \cdot \sigma_q.$$

This line has become known as the *Capital Market Line* (CML) and the *tangent portfolio* is the famous *market portfolio M*. All portfolios except for the market portfolio on the efficient frontier are now dominated by portfolios on the CML. The CML is the *new* efficient frontier.

According to Spremann [2008a] these insights have revolutionized and standardized asset management world wide as one will see in the upcoming section.

[6]Proof: using the portfolio properties introduced in (3.1) and (3.2) one knows:

$$\mu_q(w) = w \cdot \mu_m + (1-w) \cdot R_0 = R_0 + w \cdot (\mu_m - R_0) \quad (3.3)$$

$$\sigma_q(w) = w \cdot \sigma_m \quad (3.4)$$

Rearranging (3.4) to

$$w = \sigma_q / \sigma_m$$

and plugging this into (3.3) one gets

$$\mu_q = R_0 + \frac{\mu_m - R_0}{\sigma_m} \cdot \sigma_q.$$

3.1.3.2 Tobin Separation

Any combination of the market portfolio and the riskless asset dominates the portfolios on the efficient frontier. Thus, assuming that all investors have *homogeneous expectations*[7] regarding the parameters of the return, the market portfolio will be identical for everybody. Hence, every investor will choose a portfolio on the CML. Where exactly it will lie depends on the individual's preferences. The crucial point is that every investor will hold a combination of the market portfolio and the riskless asset.

This was a revolution in terms of portfolio management as it allowed for much greater standardization. While every optimization in portfolio choice before Tobin would yield a different portfolio for different individuals one now only needed to know the market portfolio.[8]

3.1.3.3 The Price of Risk

The Tobin Separation has important implications for the price of risk. As everyone decides to hold a portfolio on the CML, the marginal rate of substitution between return and risk is the same for all individuals and equals the slope of the CML which is:

$$\frac{\mu_m - R_0}{\sigma_m}. \tag{3.5}$$

Knowing the *price* of risk one would like to be able to *measure* the risk. The standard deviation as a measure of risk has already been used in the derivation of Markowitz's theory - it is an adequate measure of risk when considering portfolios of assets. However, measuring the risk of individual assets that do not lie on the CML is not possible with the present method of measuring risk.

Consider the following example: Modigliani and Pogue [1974], as cited in Copeland et al. [2005], have collected data for 306 months measuring the return and standard deviation of a single asset (Bayside Smoke) and a 100-stock portfolio.

	Bayside Smoke	100-stock portfolio
Average Return per year	5,4 %	10,9 %
Standard Deviation	7,26 %	4,45 %

Table 3.1: Risk and return for Bayside Smoke and a 100-stock portfolio. (Source: Modigliani and Pogue [1974] as cited in Copeland et al. [2005])

[7] See Chapter 5 for a discussion of how realistic this assumption is.

[8] The analytical derivation of the market portfolio itself does not add anything to the intuition and pedagogy of the theory. The reader is referred to Spremann [2008a, p. 243 et seq.] for the calculation of the market portfolio.

One sees that Bayside Smoke has higher standard deviation, but lower return. This is counterintuitive to what was developed so far. On the other hand, the diversification effect was already mentioned which is why one can think of the more relevant measure of risk for single assets: the contribution to the overall risk of the portfolio. As the number of assets in a portfolio is increased its variance decreases and approaches the average covariance. Another reason why variance or standard deviation cannot be the right measure is understood very easily. Drawing a horizontal line at some given return in the risk return diagram one will find many assets with the same return but different standard deviations. Thus, they all have different "risk" but the same return - this seems unappealing. Hence, the following subchapter will develop the CAPM that will show us what the appropriate measure of risk for single assets is. [Copeland et al., 2005]

3.1.4 Summary

Markowitz's works were so fundamental that it is difficult to imagine what the world of finance was like before his contributions to the field. His research and ideas allowed to speak of portfolio risk in a quantifiable fashion and the two concepts of risk and return are basic vocabulary in finance classes nowadays. Before Markowitz, the common belief was that an investor should maximize expected return - this would lead to investors only holding a portfolio of one stock with the highest expected return. Markowitz's concept was such a novelty in 1952 that even Milton Friedman did not want to acknowledge that it was a valid microeconomic theory of choice when Markowitz was defending his thesis [Markowitz, 1991, p. 476]. In the end, the key insight of Markowitz was that when adding an asset to a well-diversified portfolio the increase in risk is due to the crucial covariance effect rather than due to the addition of variance [Varian, 1993, p. 161]. The author will develop further conclusions from this concept in the next section.

3.2 Capital Asset Pricing Model

The CAPM was mainly developed by Sharpe [1963, 1964] and shows how the return depends linearly on the *Beta*. The model will be mathematically derived from the results in the last section and discussed, as it is one of the most important neoclassical tools for valuing assets. The following sections will mainly follow Spremann [2008a] and Copeland et al. [2005], except for the mathematical derivation of the CAPM itself which follows the original paper by Sharpe [1964].

3.2.1 Main Idea

Considering a single asset k in the risk return diagram one knows its mean and variance, but still cannot explain the rate of return the market rewards for holding that asset. When examining this single asset k in the market portfolio one sees a correlation in returns between the asset and the market. The correlation will be positive but not perfect since there are many other assets, i.e. $0 < \rho_{k,m} < 1$. This holds for all single assets. So movements in market returns translate to a certain extent into movements in single asset returns.[9] The market has to be some *common factor* that drives single asset returns. In terms of risk, one can speak of 'common' risk inherent to all single assets. Following Spremann [2008a] the author will show that this is the so-called *systematic risk*, that cannot be diversified away, compared to the *unsystematic risk*, that can be diversified away. A measure will be derived for the single asset risk called *Beta*. It measures the extent to which single asset returns are correlated with the market's return.

3.2.2 Efficiency of the Market Portfolio

For the CAPM to hold the market portfolio needs to be mean variance efficient.[10] For now the market portfolio is expected to be efficient. Speaking intuitively: the theory has assumed rational investors that are only concerned about mean and variance. Additionally, all investors use the same information available in the market - a "market opinion" emerges [Spremann, 2008a, p. 224]. Hence, the individual's marginal rates of substitution between return and risk will equal the market price for risk. Since the market is simply the sum of all efficient individual holdings, the market portfolio itself will be mean variance efficient.

3.2.3 Derivation of the CAPM

The CAPM can be derived from the definition and construction of the market portfolio and therefore - mathematically speaking - it is a valid model. The derivation discussed here follows the approach by Copeland et al. [2005] which in turn is closely based on the original works by Sharpe [1964].

In equilibrium, all assets in the market portfolio will be held in proportion to their market value weights. All prices must adjust until there is no excess demand for any asset. Hence, in equilibrium every portfolio weight of each asset must be:

$$w_i = \frac{\text{market value of the individual asset}}{\text{market value of all assets}}.$$

[9] This does not imply a causal relationship. The existing correlation is just pointed out.

[10] That might actually entail an additional assumption as will be shown in a later chapter: that all assets are perfectly divisible and liquid. This means that assets like human capital and private real estate are accounted for in the market portfolio.

Consider a portfolio P consisting of $a\%$ of the risky asset R and $(1-a)\%$ of the the market portfolio M. Using the portfolio properties introduced in (3.1) and (3.2) it has the following mean and standard deviation:

$$\mu_p = a\mu_r + (1-a)\mu_m$$

$$\sigma_p = \sqrt{a^2\sigma_r^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{r,m}}.$$

A key fact for later findings is that the market portfolio already contains the risky asset R according to its market value weight since it was assumed to be efficient. To use this fact the author examines the change in the mean and standard deviation of the portfolio with respect to the change of the amount of risky asset R in terms of a:

$$\frac{\partial \mu_p}{\partial a} = \mu_r - \mu_m \tag{3.6}$$

and

$$\begin{aligned} \frac{\partial \sigma_p}{\partial a} = & \frac{1}{2\sqrt{a^2\sigma_r^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{r,m}}} \\ & \times[2a\sigma_r^2 - 2\sigma_m^2 + 2a\sigma_m^2 + 2\sigma_{r,m} - 4a\sigma_{r,m}]. \end{aligned} \tag{3.7}$$

Sharpe [1964] realized that in equilibrium asset R is already included in the market portfolio according to its market value weight. The percentage a can consequently be interpreted as the excess demand for the asset. However, in equilibrium the excess demand for any asset must equal zero. Calculating the above equations (3.6) and (3.7) where $a = 0$, one gets:

$$\begin{aligned} \left.\frac{\partial \mu_p}{\partial a}\right|_{a=0} &= \mu_r - \mu_m \\ \left.\frac{\partial \sigma_p}{\partial a}\right|_{a=0} &= \frac{1}{2\sigma_m^2}(-2\sigma_m^2 + 2\sigma_{r,m}) \\ &= \frac{\sigma_{r,m} - \sigma_m^2}{\sigma_m}. \end{aligned}$$

Thus, the slope of the rate of substitution between return and risk at point M is:

$$\left.\frac{\partial \mu_p/\partial a}{\partial \sigma_p/\partial a}\right|_{a=0} = \frac{\mu_r - \mu_m}{(\sigma_{r,m} - \sigma_m^2)/\sigma_m}. \tag{3.8}$$

The CML derived earlier was chosen to be tangent to the efficient frontier at the market portfolio. Hence, the rate of substitution between risk and return above (3.8) has to equal the slope of the CML. So equaling (3.8) and (3.5) it can be written:

$$\frac{\mu_m - R_0}{\sigma_m} = \frac{\mu_r - \mu_m}{(\sigma_{r,m} - \sigma_m^2)/\sigma_m}.$$

Now solving for μ_r the famous CAPM relationship is obtained:

$$\mu_r = R_0 + \beta_r \cdot [\mu_m - R_0]$$

$$\beta_r = \frac{\sigma_{r,m}}{\sigma_m^2}.$$

This equation reveals that for any single asset the expected return equals the risk-free rate plus β - times the market risk premium. The β, verbally Beta, is explained in the next section.

3.2.4 Properties of the CAPM

The CAPM is a valid mathematical model if it has been derived from an efficient and correctly calculated market portfolio [Spremann, 2008a, p. 333]. This market portfolio plays a pivotal role in the CAPM. The following sections explore three important properties in more detail as discussed in Copeland et al. [2005].

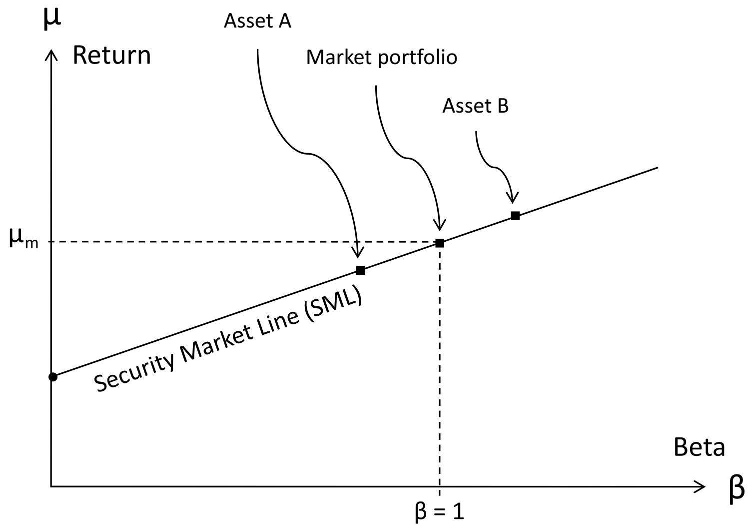

Figure 3.4: Illustration of the Security Market Line. (Source: own adaptation from Spremann [2008a, p. 295])

3.2.4.1 Security Market Line

The basic CAPM equation can be displayed graphically in a Beta Return Diagram that is similar to the Markowitz Risk Return Diagram. Figure 3.4 shows that *every single asset and portfolios of assets can be positioned on the so-called Security Market Line (SML).* Thus, one can correctly measure the risk of a single asset.

3.2.4.2 Systematic Risk

The CAPM allows the decomposition of risk into two parts. One part is the *unsystematic risk* inherent to the asset. The other part is the *systematic risk*, which is a measure of how the asset returns covary with the economy as a whole - measured by the Beta.

$$\text{total risk} = \text{systematic risk} + \text{unsystematic risk}$$

Systematic risk cannot be diversified away. Hence, the investor is only rewarded for bearing systematic risk, but he will not be rewarded for bearing unsystematic risk since this risk can be diversified at no cost. In consequence, the explanation for the risk premium on single assets is the *Beta*: it measures the systematic risk inherent to the single asset.

3.2.4.3 Linear Additivity of Risk

Another important property of the CAPM is that Betas are linearly additive when portfolios are formed. Hence, the Beta of a portfolio is simply the weighted average of the single asset Betas in the portfolio.

3.2.5 Risk-adjusted Rate of Return Valuation Formula

So far only expected returns have been examined and not explicitly asset pricing. But the application of the CAPM to asset pricing is obvious. A risk adjusted rate of return formula, that is used to discount expected future cash flows, can be obtained. Let $E^P[C]$ be the expected end of period cash flow C and P_0 its price today. Then

$$P_0 = \frac{E^P[C]}{1 + R_0 + \beta_c[\mu_m - R_0]}$$

is the general formulation of the asset pricing formula as shown in Copeland et al. [2005].

3.2.6 Summary

Starting with Markowitz on Portfolio Selection this chapter derived fundamental results. Sharpe's discovery of the CAPM was revolutionary for financial economics and shows how the risk premium can be determined for single assets. As Varian [1993] states "it is a prime example of how to take a theory of individual optimizing behavior and aggregate it to determine equilibrium pricing relationships" [p. 165]. The risk premium on assets can be explained with the CAPM. The relevant measure is the portion of total variance that is correlated with the whole economy - the Beta. Any uncorrelated risk or unsystematic risk - idiosyncratic to the asset - is not rewarded and can be diversified away at no cost.

Despite its age the approach is still summarized under the term "Modern" Portfolio Theory. Empirical tests are actually ambiguous about the validity of the model so there is always "room for refinements" as Spremann [2008a, p. 333] notes. But due to its intuition it still serves as a basic tool taught in class and used in practice. The empirical content will be discussed in a later chapter.

Chapter 4

No-Arbitrage Approach

"The study of the implications of No-Arbitrage is the meat and potatoes of modern finance"
[Ross, 2005, p. 2].

Finally, arbitrage-based models will be examined. The results in this section are also termed preference-free results as in Dybvig and Ross [2003] to underline that hardly any restrictions are posed on preferences (utility functions) of investors except for the "ubiquitous human characteristic that one prefers more to less" [Ross, 2005, p. 1].

The author will first briefly define Arbitrage and then explore its full implications and meaning in finance. The arbitrage principle and its related results have "unified the understanding of asset pricing and the theory of derivatives" [Ross, 2005, p. 1].

4.1 Arbitrage: A Definition

First, Arbitrage will be defined in words to and then formally for a better understanding. According to Sandmann [2001, p. 15] an Arbitrage opportunity exists if it is possible

- to form a portfolio at no cost that has a non-negative payoff in each state and a positive payoff in at least one state or
- to form a portfolio with a negative price (i.e. through short selling the investor has a cash inflow) that has non-negative payoffs in each state.

The following formal definition of Arbitrage closely follows Ross [2005, p. 3 - 5] and takes place in the SPT framework. As a short repetition: there is a set of possible states $\Omega = \{w_1, \cdots, w_S\}$, a vector $\underline{p}$ which represents today's security prices and the payoff matrix D of securities across states. Next let

$$\underline{\eta} = \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix}$$

denote an arbitrage portfolio of available securities. The price of such portfolio will be accordingly:

$$\underline{p}^T \underline{\eta} = \sum_n p_n \eta_n .$$

The thesis will refer to the combination $\underline{\eta}$ as an arbitrage portfolio if it has no positive cost:

$$\underline{p}^T \underline{\eta} \leq 0 \, .$$

The payoff of such arbitrage portfolio is determined by:

$$D\underline{\eta}.$$

Hence, one can define arbitrage mathematically in the same manner as it was done verbally before. An arbitrage opportunity exists if the following holds:

$$\begin{aligned} \underline{p}^T \underline{\eta} &\leq 0 \\ &\text{and} \\ D\underline{\eta} &> 0. \end{aligned}$$

To further simplify the mathematic representation of No-Arbitrage one can define a stacked matrix A as follows:

$$A = \begin{bmatrix} - & \underline{p}^T \\ & D \end{bmatrix}.$$

Using this setting Ross [1977] has defined Arbitrage as a portfolio, $\underline{\eta}$, such that:

$$A\underline{\eta} > 0.$$

The formal definition of the No-Arbitrage (NA) condition is then as follows:

$$NA \Leftrightarrow \left\{ \underline{\eta} | A\underline{\eta} > 0 \right\} = \emptyset. \tag{4.1}$$

Put simply, all this mathematical notation means that there is no portfolio and accordingly no way of buying and selling the traded assets in the market in a manner that would *generate riskless profit without any cash outlay at time* t_0. [Ross, 2005, p. 3 - 5]

4.2 Arbitrage Pricing Theory

The Arbitrage Pricing Theory was developed by Ross [1976] under a set of very weak assumptions. The pricing equation follows from the absence of arbitrage and homogeneous investor beliefs about the linear return generating process. Returns are believed to be governed by a linear k-factor model. There are no restrictions on investor's utility functions except for the appealing assumption that one prefers more to less. A third assumption is that the number of assets has to be "sufficiently large"[1] as Ross [1977, p. 195] himself puts it. The resulting relationship is an example for approximate arbitrage pricing whereas option pricing for example is referred to as exact arbitrage pricing.

4.2.1 Intuition

The intuition of the APT, as described in Cochrane [2005], comes from the observation that stock prices tend to move together in groups. The first big component of stock returns is the market. Further certain groups of stocks move together such as pharmaceutical stocks, financial institution stocks, growth stocks and so on. And lastly each stock has its own idiosyncratic movement independent of the other influences. Those idiosyncratic price movements should not be priced as they can be diversified in portfolios. The covariance of stock movements with the common components or "factors" should only be priced. It will be shown that the expected return can be shown to depend linearly on the exposure of assets to each of those factors.

4.2.2 Formal Development of the APT

The formal development of the APT in this chapter stems mostly from Roll and Ross [1980] and the original works by Ross [1977, 1976]. For further understanding the author has also included many remarks from Copeland et al. [2005] and Spremann [2005]. The APT begins with assumptions about the return-generating process. The linear return generating process looks like a multi-factor CAPM. But the essential point is, Ross [1977] argues, that this model "of and by itself constitutes a far more satisfactory basis for a capital market theory without the additional baggage of mean variance theory" [p. 195]. The random return on some asset i is believed to be governed by a k-factor model of the following form:

$$\begin{aligned} \tilde{r}_i &= E_i + b_{i1}\tilde{\delta}_1 + \cdots + b_{ik}\tilde{\delta}_k + \tilde{\epsilon}_i, \\ i &= 1, \cdots, n. \end{aligned} \tag{4.2}$$

where

- E_i is the expected return on the i^{th} asset

[1] To permit the law of large numbers to work.

- $\tilde{\delta}_k$ denotes a mean zero k^{th} common factor to the returns of all assets under consideration
- b_{ik} states the exposure or factor loading of the asset $i's$ returns to the movements of the common factor $\tilde{\delta}_k$
- $\tilde{\epsilon}_i$ is a random zero mean noise term. It represents the unsystematic risk or risk idiosyncratic to the single asset that is unrelated to other assets.[2]

To develop the APT an individual holding a portfolio will be examined. The individual wants to rearrange the portfolio in investment proportions by adding a self-financing portfolio, i.e. the investment proportions w_i must sum to zero:

$$\sum_{i=1}^{n} w_i = 0$$

or equivalently

$$\underline{w}^T \underline{1} = 0.$$

This means that sales of assets in the self-financing portfolio must balance the purchases of assets. Now, one can examine the additional return obtained from altering the portfolio to determine whether the individual should chose to alter the portfolio or not. Adding the self-financing portfolio entails the following additional return:

$$\sum_{i=1}^{n} w_i \tilde{r}_i = \sum_{i=1}^{n} w_i E_i + \sum_{i=1}^{n} w_i b_{i1} \tilde{\delta}_1 + \cdots + \sum_{i=1}^{n} w_i b_{ik} \tilde{\delta}_k + \sum_{i=1}^{n} w_i \tilde{\epsilon}_i. \tag{4.3}$$

If the portfolio is engineered to use no wealth and have no risk then it must also earn no return on average. To eliminate unsystematic risk the portfolio has to be well diversified. In order to so n is chosen to be large and the investment proportions to be small. Consequently, each asset has to be approximately $1/n$ in portfolio weight ensuring it will be well-diversified. Thus, one can neglect the last term $\sum_{i=1}^{n} w_i \tilde{\epsilon}_i$ in (4.3).[3] Additionally, $\underline{w}$ will be constructed to have no systematic risk exposure. This means that for each k one obtains:

$$\sum_{i=1}^{n} w_i b_{ik} = 0.$$

Those specifications lead to the following change in return of such an self-financing portfolio:

$$\sum_{i=1}^{n} w_i \tilde{r}_i = \sum_{i=1}^{n} w_i E_i.$$

[2]This also means that $E(\tilde{\epsilon}_i|\tilde{\delta}_k) = 0$ and that $\tilde{\epsilon}_i$ is independent of $\tilde{\epsilon}_j$ for all i and j.

[3]By the law of large numbers the term approaches zero as n grows.

The portfolio has now been engineered to have neither systematic nor unsystematic risk. The obvious consequence is that if the individual has been satisfied with his old portfolio, then $\sum_{i=1}^{n} w_i E_i = 0$ must be the return on any shift in investment weights at no cost. Otherwise the individual could generate more return without incurring additional risk - arbitrage would be possible.

The conditions so far are really statements in linear algebra. Recapitulating and restating the findings in vector notation as in Copeland et al. [2005]: any vector orthogonal to the constant vector, i.e.

$$\underline{w}^T \underline{1} = 0,$$

and to each of the b_{ik} coefficients, i.e.

$$\underline{w}^T \underline{b}_k = \sum_i w_i b_{ik} = 0 \text{ for each k},$$

must also be orthogonal to the expected return vector, i.e.,

$$\sum_{i=1}^{n} w_i E_i = \underline{w}^T \underline{E} = 0.$$

The purely algebraic consequence of those statements is that the expected return vector must be a linear combination of a constant and the coefficient vectors. In other words, as Roll and Ross [1980] note, there exists a set of $k+1$ positive numbers, $\lambda_0, \lambda_1, \cdots, \lambda_k$, that allow to properly explain the expected return on assets as follows:

$$E_i = \lambda_0 + \lambda_1 b_{i1} + \cdots + \lambda_k b_{ik} \text{ , for all } i. \tag{4.4}$$

The exposure of a riskless asset to each factor is zero, $b_{ok} = 0$, hence, the return E_0 is:

$$E_0 = \lambda_0 = R_0.$$

Thus, one can restate (4.4) more generally in an excess return form as:

$$E_i - R_0 = \lambda_1 b_{i1} + \cdots + \lambda_k b_{ik}.$$

Though (4.4) is the central pricing relationship used in empirical testing, an interpretation of the factor risk premia λ_k suggests itself. If portfolios with unit systematic risk on some factor k and no risk on other factors are formed, each λ_k can be interpreted as

$$\lambda_k = E^k - R_0$$

which is the market risk premium for assets with exposure to only systematic factor risk k. As a consequence one can rewrite (4.4) as

$$E_i = R_0 + [E^1 - R_0]b_{i1} + ... + [E^k - R_0]b_{ik}. \tag{4.5}$$

Equation (4.5) can in fact be interpreted as a linear regression which leads to the b_{ik} to be defined in the same manner as in the previously discussed CAPM model. Each b_{ik} is then defined as the covariance between the return of asset i and a linear transformation of the the factor k divided by the variance of the linear transformation of factor k [Copeland et al., 2005]. Hence, it can be seen that an asset's return once again depends on the co-movements of its returns with the systematic risks as a whole. In the APT the market portfolio plays no role and it remains unspecified what those systematic risk factors are. But the CAPM can in fact be seen as a special case of the APT with a normal distribution of returns and only one return generating factor (the market portfolio). This will be discussed in depth in the next Chapter.

The following section will shortly explore another application of the arbitrage principle in financial economics. This time *exact* arbitrage pricing will lead to fundamental results in derivative pricing and risk neutral valuation will be encountered again.

4.3 Excursus: Option Pricing

In a first approximation one can *bound option prices* using arbitrage arguments or arrive at the important result that *an American call option has the same value as an European call option*. Although those results are of great importance this part can be skipped to arrive at even more fundamental insights in option pricing. As a guide for further reading those results are developed in Merton [1973], Cox and Ross [1976].

Now, "an extremely *relative* pricing approach" will be adopted as Cochrane [2005, p. 313] puts it. Thus, the prices of other securities as stocks and bonds will be taken as given. This theory is not exactly an asset pricing theory as it only partially describes the financial market which is why this section can be seen as an excursus. But it helps to underline the importance of the SPT framework as a pedagogical tool and highlights the concept of risk neutral probabilities. Lastly, it shows the importance of the No-Arbitrage condition.

4.3.1 Binomial Trees

Just a few decades ago no one had a clear understanding of how to value an option. The intuition was clear, but no formula or numerical procedure existed. In 1973 Black and Scholes provided the well-known closed-form solution to option pricing. However, its derivation requires advanced financial mathematics (e.g. stochastic differential equations). Hence, the thesis will present the more intuitive binomial approach that was developed by Cox, Ross, and Rubinstein [1979]. Understanding the binomial tree will also allow us to grasp the relationship between the positive linear pricing rule found in the Arrow-Debreu world and the risk-neutral valuation in this chapter.

The derivation will be kept very brief and closely follow the descriptions in Hull [2008] and Spremann [2005]. The starting point is the simple assumption that a stock whose price is S_0 at time t_0 can either move up to S_0u or down to S_0d at time t_1 with $d < 1$ and $u > 1$. Similarly, an option with current price f has two possible payoffs f_u and f_d, respectively, at time t_1. The time period between t_0 and t_1 is denoted as T. Next, consider a portfolio consisting of a long position in Δ shares and a short position in one option. One can calculate the value of Δ that yields a riskless portfolio meaning that it will have two identical payoffs in both states.

In case of an up movement one has the payoff:

$$S_0u\Delta - f_u.$$

In case of a down movement one has the payoff:

$$S_0d\Delta - f_d.$$

Equating those two payoffs yields:

$$S_0u\Delta - f_u = S_0d\Delta - f_d$$

or

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d}. \tag{4.6}$$

As mentioned, the portfolio will be riskless and under the assumption of No-Arbitrage one can safely assume it will earn the risk-free interest rate R_0. The present value of the portfolio is either of the payoffs at t_1 discounted at the risk-free rate

$$(S_0u\Delta - f_u)e^{-R_0T}.$$

As the present value should equal the cost of setting up the portfolio which is

$$S_0\Delta - f,$$

one can equate those two values

$$S_0\Delta - f = (S_0u\Delta - f_u)e^{-R_0T}$$

and solve for f:

$$f = S_0\Delta(1 - ue^{-R_0T}) + f_ue^{-R_0T}.$$

Now, substituting from (4.6) for Δ and simplifying, one obtains:

$$f = e^{-R_0 T}[pf_u + (1-p)f_d] \tag{4.7}$$

where

$$p = \frac{e^{R_0 T} - d}{u - d}.$$

No more assumptions than the one of No-Arbitrage are needed to obtain a result that allows to price options in a one-step binomial tree. It is the expected value under *risk neutral probabilities* discounted at the risk-free rate. In the pricing equation (4.7) the stock's expected return is irrelevant for the option price. This is counterintuitive, but the option is valued in relative terms and the real probabilities of up or down movements are already implicated in the stock price. Admittedly, this one step binomial tree is a rather drastic simplification, but the next section will show how to make the model more realistic. [Hull, 2008, p. 237 - 241]

4.3.2 Increasing the Number of Steps

To make the model more realistic one can include more time steps and therefore more possible option values at the end of the tree. Without going into the details of the derivation one can show that for a more complex multi-period model the option price still remains the *expected payoff under risk neutral probabilities* discounted at the risk-free rate. Assuming n to be the number of up movements in the stock price $(n = 0, 1, 2, \cdots, T)$, T to be the total number of periods and K to be the exercise price of the option one obtains:

$$f = e^{-R_0 T}\left(\sum_{n=0}^{T} \frac{T!}{(T-n)!n!} p^n (1-p)^{T-n} \mathrm{MAX}[0, u^n d^{T-n} S_T - K]\right).$$

The result that the option price is the expected payoff using risk neutral probabilities discounted at the risk-free rate still holds. Fixing the time period and dividing it into more and more binomial steps would in the limit yield a continuous stochastic process. One can derive the famous Black-Scholes Formula from the binomial model, but this derivation would go beyond the scope of this paper and clearly the insight stays the same: the option price is the expected payoff under risk neutral probabilities. [Spremann, 2005, p. 362 - 369]

4.4 Summary

The results in this chapter have shown that the simple Arbitrage Principle can derive fundamental results and serves as a basis for asset and option pricing. The APT pricing equation is very similar to a multi factor CAPM and Roll and Ross [1980] confirm that the "APT agrees perfectly with what appears to be the intuition behind the CAPM" [p. 1074]. But the derivation needed a set of much weaker assumptions than the CAPM as will be discussed in the next chapter.

The section on Option Pricing resulted in risk neutral pricing by relying on exact arbitrage arguments. The financial market model from the SPT framework is especially helpful in understanding options and their payoffs as they can easily be interpreted as vectors of state contingent payoffs. As a consequence, a vector of state contingent payoffs is found at the end of every binomial tree. This is also why many books on option pricing start with a basic SPT framework to later introduce more complex concepts.

One can actually relate the SPT and Option Pricing very closely, but this will be omitted as it goes beyond the intention and scope of this thesis. Derivative pricing is a very quantitative science. Several authors have shown important properties of the risk neutral probabilities obtained and the relationship with true Arrow-Debreu state prices. For example Breeden and Litzenberger [1978] have eventually shown the conditions under which option-based state prices are equivalent to the true Arrow-Debreu state prices.

Chapter 5

Comparison

"The lack of any clear-cut understanding of which theories may prevail is discouraging"
[Grauer, 2003, p. *li*].

5.1 Introduction

As Cochrane [2005] notes asset pricing theory shares the tension between *normative* and *positive* theories present in the rest of economics. "Do the models describe how the world *does* work or how it *should* work" he asks [p. xiii]. Consequently, there is no ultimate truth in finance. There are several models that economists think explain the world pretty well, but which one will prevail is not clear yet. There are literally hundreds of papers[1] on asset pricing and no clear consensus among researchers.

This section aims to compare the different models developed earlier and tries to look at the differences between them. A clear distinction between the models is sometimes difficult when considering the extensions and refinements, especially when considering the CAPM and APT. Many authors have developed the models from very different assumptions. Therefore the thesis will focus on the original assumptions made by the "fathers" of the theories.

The SPT is the most general theory and serves as a solid framework for the understanding of financial markets. Both, the CAPM and APT, can be developed from the SPT framework when adding assumptions on the preferences (or distribution of returns) or by elaborating more on the No-Arbitrage condition when assuming a linear k-factor return generating process. Thus, the next sections mainly compare the CAPM and APT.

[1] As an example Robert Korajczyk from the Kellogg School of Management has put together a list of references on the APT and multi factor models which contains 351 papers and is not exhaustive. (http://www.kellogg.northwestern.edu/faculty/korajczy/htm/aptlist.htm retrieved on 30.07.2010). Grauer [2003] has reviewed 155 of the most influential papers on asset pricing and has not come to a clear cut conclusion which theory works best.

5.2 Assumptions

In general, financial economists tend to classify the CAPM or APT based on their main assumptions or way of formal development. Thus, the APT is often said to be an arbitrage- or beliefs-based model[2] yielding "preference-free results" as in Dybvig and Ross [2003, p. 612] while the CAPM is classified as an utility-, optimization-, tastes- or preference-based[3] asset pricing model depending on the source. Or more general, the CAPM and Portfolio Theory are often termed Mean Variance Approach/Framework/Model.[4] In the case of the SPT, as Sharpe [1991, p. 491] admits, there is no such clear cut classification since some results are utility based, but most are arbitrage based. The security prices can be inferred by optimizing individual portfolio choice or one can also price the assets in an arbitrage free and complete market without considering utility functions. Hence, the SPT framework "is one of the most general frameworks available" for asset pricing as Breeden and Litzenberger [1978, p. 621] note. Those two ways are also the starting points for the CAPM and APT, respectively. In the spirit of Arrow [1964] and Debreu [1959] the SPT can be termed as a general equilibrium model, though.

No-Arbitrage

The APT, as the name suggests, relies heavily on the No-Arbitrage principle. Many famous economists have repeatedly emphasized the importance and consequence of the No-Arbitrage condition making it one of the most accepted assumptions in financial markets. For example Ingersoll [1987] admits "the absence of arbitrage is one of the most convincing and, therefore, farthest-reaching arguments made in financial economics" [p. xiii]. Ross [2005] asserts that "the study of the implications of No-Arbitrage is the meat and potatoes of modern finance" [p. 2]. And Varian [1987] suggests, that "it serves as one of the most basic unifying principles of the study of financial markets" [p. 56].

So this simple assumption leads to a promising asset pricing model and has also served to derive many other fundamental results in finance. Considering the quotes above and the competition in financial markets, No-Arbitrage is a weak assumption and easily acceptable. More precisely, as Kardaras [2010] suggests in the Encyclopedia of Quantitative Finance: "It is difficult to imagine a normative condition that is more widely accepted and unquestionable in the minds of anyone involved in the field of quantitative finance other than the absence of arbitrage opportunities in a financial market" [p. 74].

The arbitrage principle has led to many more results like the Fundamental Theorem of Finance, the famous Black Scholes Formula or Option Pricing Theory in general. Arbitrage

[2]See for example Sharpe [1991] or Grauer [2003].
[3]See for example Dybvig and Ross [2003], Grauer [2003], Pennacchi [2008] or Sharpe [1991].
[4]See for example Hirshleifer [1966], Grinblatt and Titman [1987] or Jensen [1972].

related results serve as the pillars of modern mathematical finance. In this thesis the author has only discussed the arbitrage principle in relation to the APT and OPT, but it should be emphasized what far-reaching implications this simple assumption has.

Moreover, as Ross and Dybvig [1987] argue, the Efficient Market Hypothesis is "clearly consistent with the intuition of the absence of arbitrage". If a price does not fully reflect available information one either short sells the overpriced asset or buys the underpriced asset - even if this is only an "approximate arbitrage possibility" since it might require a cash outlay. [Ross and Dybvig, 1987, p. 194]

In fact, a competing paradigm evolved around the dissatisfaction with the arbitrage principle. The "Limits to Arbitrage" as described in Shleifer and Vishny [1997] serve as one of the two founding pillars of Behavioral Finance - a competing paradigm. It has evolved around the dissatisfaction of the lacking explanatory power of Neoclassical Models when it comes to anomalies. Those deficiencies of Neoclassical Models are discussed in the Empirical Content section.

Preferences and Beliefs

The CAPM poses restrictions on investor's preferences as those are assumed to be defined with respect to the mean and variance of asset returns. The model then results from the optimal portfolio choice problem of the individual investor. This assumption is "extreme" as Fama and French [2004, p. 37] put it. They note that investors might also care about other risks such as labor income risks as also noted in Cochrane [1999a] and Spremann [2008a]. More complicated models offer remedies as will be discussed in the Empirical Content Section.

For the Mean-Variance optimizing behavior to be consistent with expected utility theory either of the following two assumptions needs to be made: a quadratic utility function or a joint normal distribution of returns.

The justification for assuming quadratic utility is that only the first two moments of the distribution matter when determining the expected utility. This can be seen when the utility function is expanded in a Taylor series around the mean. All derivatives of higher order than two are zero then [Pennacchi, 2008]. Consequently, no moments of the distribution higher than second order are needed. But quadratic utility functions have an increasing absolute risk aversion (ARA) which is a very unrealistic investment behavior. Additionally, the assumption would actually not rule out arbitrage in a financial market: "investors with quadratic utility will not take unlimited arbitrage positions, for this would increase their wealth past the point where marginal utility becomes negative" [Ingersoll, 1987, p. 99]. Thus, most economists avoid that assumption in portfolio theory.

To circumvent this one can just assume a general monotonically increasing, concave utility function, but make an additional assumption on returns. We need probability distributions that make expected utility depend only on the first two moments of the distribution. Theoretically there are many two-parameter distributions where the higher order moments can be expressed as functions of the first two moments. But it was also assumed that the portfolio distributions exhibit the desirable properties of being fully described by the first two moments. Thus, the sums of the random returns in portfolios need to be distributed normally again. This can be achieved by assuming a joint normal distribution of asset returns, which is the second crucial assumption in the CAPM. But in the light of newer findings this assumption also seems unrealistic since returns have been shown to often exhibit skewness or "fat tails" as Spremann [2008b] notes.

The APT on the other hand makes no assumptions on investor preferences except for that "one prefers more to less" Ross [2005, p. 2]. This is a condition that is easily accepted and poses hardly any restrictions which is why arbitrage-based models are often termed *"preference-free"* as in Dybvig and Ross [2003, p. 9]. Furthermore, no assumptions on the return distributions are made which is another point that makes the APT more general than the CAPM.

However, the APT makes assumptions on the return generating process and, therefore, imposes homogeneous beliefs of investors. The investors are assumed to agree on a linear factor model of asset returns. In this model asset returns are driven by a few common factors. It is similar to a multi factor CAPM, but gives no specifications as to what those factors might be. There is no specified market portfolio that plays a role. Hence, as shown in Connor [1995], there are three possible types of multi factor models: macroeconomic, fundamental and statistical factor models.

The CAPM also makes the assumption of homogeneous investor beliefs (also termed "homogeneous expectations" as in Spremann [2008a] for example). They are assumed to have the same beliefs on means, variances and covariances of asset returns.

The assumption of homogeneous beliefs in both models is objectionable and often comes under attack. It is a crucial assumption since the models discussed in this thesis are built on a representative-agent-based framework. There have been extensions of the models with heterogeneous beliefs. Lintner [1969] for example discusses a CAPM with diverse judgments of investors. This does not necessarily alter the model, but implies that the market portfolio might not be mean variance efficient. However, those extensions might not be necessary as Spremann [2008a, p. 224] notes: homogeneous beliefs are a reasonable assumption since financial data is public. Hence, all investors use the same information available in the market and a "market opinion" emerges. Consequently, all investors form the same expectations (beliefs).

No-Arbitrage and Complete Markets

Two important assumptions in the SPT framework are the completeness of markets and the No-Arbitrage condition as for example stated in Pennacchi [2008, p. 98]. Though it has to be noted, that those are normative grounds and it is diffcult to objectively state what the "most important" assumptions in the framework are. Some authors explicitly name the axioms of expected utility theory as further assumptions (as in Sandmann [2001]), but some also leave that out and take them for granted (as in Zimmermann [1998]). In general, a differentiable, concave utility function, representative agents with homogeneous beliefs about the possible states of the world and the general one-period financial market model described earlier needs to be assumed.

Further Assumptions

Besides the "main" assumptions above there are some further assumptions made in the models that are worth being mentioned. For example a troubling implicit assumption of the CAPM model is that all assets are supposed to be perfectly divisible and liquid to be able to capture all economy related risk in the market portfolio as noted in Jensen [1972, p. 359]. This assumption ensures that individuals take into account their whole portfolio when optimizing their investment. Their real estate or labor income risk for example should be taken into account when optimizing the portfolio. Thus, today's research tries to take into account more macroeconomic risk factors to capture all the risks that the CAPM market portfolio might actually omit due to the violation of the implicit assumption. The International Library of Critical Writings in Financial Economics has dedicated a whole volume on the theme of Financial Markets and the Real Economy emphasizing this important development. This development is completely in line with what the intuition won from the previous models has been as the quote by Cochrane [2006] in Richard Roll's foreword to the series suggests:

> *"'Good' assets pay off well in bad times when investors are hungry. Since investors all want them, they get lower average returns and command higher prices in equilibrium. High average return assets are forced to pay those returns or suffer low prices because they are so 'bad' - because they pay off badly precisely when investors are most hungry." [Cochrane, 2006, p. xi]*[5]

Moreover, in the CAPM the investors are assumed to be allowed to borrow or lend unlimited amounts at the risk free rate with no restrictions on short sales of assets. Those assumptions are actually already included in the "neoclassical setting" since it assumes

[5] As cited by Richard Roll in the Foreword.

frictionless and perfect markets. Together all the assumptions might seem very strict, making the model world unrealistic. Thus, many authors have tried to relax some assumptions to make the model more robust:

- Brennan [1971] has derived a CAPM for diverging lending and borrowing rates.
- Black [1972] has derived a CAPM with no riskless asset. Instead he used zero beta portfolios.
- Merton [1973] has derived a CAPM where trading takes place in continuous time and asset returns are distributed lognormally.
- Lintner [1969] has discussed a version of the model with heterogeneous expectations about future returns. While it does not critically alter the theoretical model, the market portfolio will not always be efficient. This makes the CAPM non-testable.

But in the end, as Dybvig and Ross [2003] have put it: "finance has work to be done and seeks specific models with strong assumptions and definite implications that can be tested and implemented in practice " [p. 606]. Thus, financial economists need some minimum set of assumptions or they end up with models that are useless for practical purposes which will be discussed in a later section.

5.3 Absolute and Relative Pricing

In this thesis examples of absolute and relative pricing have been given. The CAPM, as a prime example of a general equilibrium model, stands for an absolute pricing relationship: assets are priced according to their exposure to fundamental risk factors (in the CAPM only the market portfolio is relevant). Relative pricing models price assets in relation to the prices of other assets. The prime example of relative pricing is Option Pricing where the Option are priced in relation to their underlyings. Those theories are two polar extremes.

The APT also prices assets in dependence of their exposure to fundamental risk factors, hence, is an absolute pricing model. But it has the advantage that one only needs to consider a subset of assets in the investment opportunity set to infer the pricing relationship. The APT is also referred to as an equilibrium pricing relationship by many authors such as Copeland et al. [2005], but Ross [1976, p. 343] himself asserts that it "does not only hold in equilibrium, but in almost all sorts of profound disequilibria".

Both ways of pricing assets have been encountered in the SPT framework. The state price vector can be derived from existing security prices, thus, assets are priced relative to existing prices. Going further into the theory and looking at the determinants of security prices one could get a absolute pricing relationship as the dependence of asset prices on economy risk was shown. But the SPT framework was never developed so far nor has it been practically employed in asset pricing in an absolute way. As Jensen [1972] noted: "it provides an elegant framework for investigating theoretical issues, but it is unfortunately difficult to give it empirical content" [p. 358].

5.4 Pedagogy

The CAPM can be seen as a special case of the APT with only one return generating factor (namely the market portfolio) and a joint normal distribution of returns. Accordingly, the APT seems to be much more robust, general and less restrictive. Yet, the CAPM is a key concept taught in most introductory and MBA finance courses. The reason is that the derivation of the CAPM highlights many of the key ideas in finance and rewards the student with an insightful understanding of risk. The student of the MPT has to go through the concepts of risk and return, the study of efficient portfolios, the decomposition of risk into systematic and unsystematic or idiosyncratic risk and efficient sets. Ross himself, the founder and fervent supporter of the APT, admits that the study of the CAPM is "of great intuitive value" [Ross et al., 2005, p. 310]. The development of the APT on the other hand uses concepts from linear algebra and does not provide the student with the intuition as to what the risk factors are.

Even the multi-factor CAPMs have more intuitive value: A well known multi-factor version of the CAPM is the Intertemporal CAPM by Merton [1973]. Merton extended the model to multiple periods and identified more risk factors that investors take into account such as labor-income risk. This adds to the explanatory power of the CAPM since stocks with desirable hedging characteristics are demanded more, i.e. their price bid up. Thus, leading to a lower return than the CAPM would suggest. The major difference in pedagogy between a multi-factor CAPM and the APT is that the CAPM specifies what additional factors to account for in the pricing relationship. In the APT on the contrary, Ross et al. [2005] state this in their own textbook, the selection of factors includes "convenience and common sense" [p. 311].

The SPT offers a lot of insight in the beginning and allows for very simple and abstract representations of financial markets as a payoff matrix. Especially in connection to derivative pricing the vector notation simplifies the understanding of options greatly. The state price densities calculated offer the "standard" microeconomic insight that marginal rates of substitution between consumption now and across the different states have to balance. In the end the SPT also offers the insight that security returns correlate with the aggregate level of wealth. This reasoning is not obtained by explaining non-diversifiable risk, but rather through emphasizing marginal utility of wealth across different wealth levels. When aggregate wealth is low each additional monetary unit has a high marginal utility. Hence, state prices for those states are high, offering low rates of return, making insurance against economy related risks expensive. That is similar to the intuition from the CAPM where a low beta would suggest low economy related risks and a low rate of return (i.e. high price). This intuition will be discussed in the next section more thoroughly.

In conclusion one can say that the great importance of the key concepts in the MPT make it "one of the most useful and enduring bits of economics developed in the last 50 years" [Cochrane, 1999b, p. 60]. It has endured in textbooks until now despite its shortcomings in empirical tests. Certainly, this is in part due to its *great pedagogical value*.

5.5 Intuition

The general intuition, that all models share, is that expected returns are proportional to the covariance with aggregate risk. In the SPT framework that is the aggregate wealth level, in the CAPM this is the market portfolio and the APT does not specify what the common fundamental risk sources are. But it does show that the co-movements of returns and fundamental risk factors are the only risk priced and that idiosyncratic risk is negligible.

In the CAPM the idiosyncratic risk is the part of the asset variance that is not correlated with the market portfolio (economy) and can be diversified away at no cost. The model "presupposes that one single representative market portfolio can capture all the risk exposure" [Bodie et al., 2008, p. 348]. Hence, not the variance, but the covariance is the relevant measure of risk since it measures how much a single asset contributes to the risk of well-diversified portfolios. This is the risk that cannot be diversified away and investors need a reward to bear that risk. The CAPM leaves no doubt about how the systematic risk is measured and proves this with an elegant derivation.

In the APT the idiosyncratic risk - expressed as the error term in the return generating process - can also be easily diversified away. It is not of great significance economically and should not be priced. Roll and Ross [1980] confirm that the "APT agrees perfectly with what appears to be the intuition behind the CAPM" [p. 1074]. But as mentioned the APT leaves a loose end and does not specify what the common risk factors are. Still, Roll and Ross [1980] argue further that the real "grey eminence" of the CAPM is the single factor model which results from the "dichotomy between diversifiable and non-diversifiable risk". Thus, the APT expands the ideas behind the CAPM and might actually provide the student with "a deeper but perhaps not fully formulated intuition" as Grauer [2003, p. xx] argues. In essence, this has also be shown in this thesis: though the factors are not specified the APT itself provides a good framework for intuitive thought about what the common factors might be. The justification for factors is left out by the theory.

The SPT does not present that intuition in such an accessible way. Theoretically interpreting the state price densities one can derive the same results - only economy related risks are reflected in the state prices.[6] But in this framework this insight remains highly theoretical and cannot be put into practice.

Additionally, all models share the insight that the compensation for risk should be linear. Another relationship other than linear would allow for arbitrage. In Dybvig and Ross [2003, p. 634] this intuition is explained: the example assumes there is a single factor and two assets have different risk exposures to the factor. The excess return must be proportional to the risk exposure. If there was a larger risk compensation per unit risk on the riskier asset, then some portfolio of a risk-free asset and the higher risk asset will have the same risk exposure as the lower risk asset. But due to the assumed non-linearity it has a higher expected return. So buying the portfolio and short-selling the low-risk asset would yield an arbitrage profit in the absence of idiosyncratic risk.

5.6 Implications

In this section the author wants to go a step further and look at the implications of the models. This entails an interpretation of the assumptions as well as deducing the main statements of the models.

The SPT framework, as it originated in general equilibrium theory, does not offer very specific implications. One can infer very general principles, though. This is why it never came to prominent use for asset pricing in practice.[7] The prices of ADS for example are not easily observable or more precisely "the absence of a natural, agreed upon, and manageably small set of state definitions puts severe obstacles in the way of examining data about observable security behavior in terms of underlying choices for sequences of time state claims" [Hirshleifer, 1970, p. 277].

Its main implication, though, that asset prices depend on the covariance of the assets payoff with the economy is at the very heart of Neoclassical Finance and has been in essence the main lesson learned from all models. But to measure those risks further assumptions need to be made to get more specific statements.

From the CAPM one can infer that the market portfolio is mean variance efficient - it has to be for the pricing relationship to hold. That is especially troubling when trying to empirically test the CAPM as Roll [1977] has pointed out. This point will be discussed further in the section on the Empirical Content of the models. Furthermore, all individuals hold mean variance efficient portfolios, or more precisely, a combination of the market portfolio and the riskless asset. But for this to hold everybody would have to follow the portfolio theory by Markowitz. Many authors such as Cocca et al. [2008] have shown that

[6] For more elaboration on the diversifiable versus non-diversifiable risk issue see Copeland et al. [2005, p. 83]

[7] Though it was already mentioned that the framework was extended and serves as a basis for derivative pricing.

individuals clearly do not hold mean variance efficient portfolios. In the recent cited study it was shown that the individual Swiss investor only holds three different stocks on average [p. 8].

The APT has many advantages here as there is no need to specify a market portfolio nor does it need to be mean variance efficient. Furthermore, by allowing more factors to generate return it offers a more comprehensive way to account for risk in the pricing equation. As was shown the market portfolio might not capture all economy related risk. Thus, the APT is an "empiricist's dream" since it allows to freely specify the risk factors [Fama, 1991, p. 1594].

Another implication from the CAPM is that whenever the market is in disequilibrium all agents need to optimize their individuals holdings again in order for the market to equilibrate. For the APT to work only a few arbitrageurs are needed that will push the market to the right prices. In the exact definition of arbitrage the arbitrageur does not need to take any risks and will take on unlimited amounts of arbitrage positions to maximize his profit. Consequently, in the MPT everyone has to take action. In the APT only a few individuals need to act.

Both models share the idea of the importance of diversification. They show that it allows to avoid idiosyncratic risks at no cost and that no reward is given for bearing those unsystematic risks. In the APT diversification allows the error term to tend to zero. In the CAPM the total variance of asset returns is not crucial, but only the portion it adds to the variance of a portfolio. The rest is eliminated by diversification.

5.7 Empirical Content

Up front the author has to admit that it is a very difficult task to summarize the literature on testing and empirical results of the asset pricing models. Grauer [2003], who has prepared the "Asset Pricing Theory and Tests" collection for The International Library of Critical Writings in Financial Economics and, therefore, reviewed 155 of the most influential articles on asset pricing and tests, himself admits that "the lack of any clear-cut understanding of which theories may prevail is discouraging" [p. li]. Or as Richard Roll puts it in the foreword to Grauer's collection: "Perhaps the only non-controversial opinion about asset pricing today is that it remains poorly understood" [p. xi]. For that reason only the main tests and shortcomings of Neoclassical Models will be discussed.

Empirical testing revealed patterns in average stock returns that cannot be explained with the CAPM or APT. Those patterns are called anomalies and have been emerging in research papers constantly since the late 1970s. Since then there have been many studies revealing anomalies. Well known anomalies include size-effects as discovered by Banz [1981], the January effect laid out in Keim [1983], overreaction as in Bondt and Thaler [1985, 1990], the outperformance of value stocks to growth stocks as shown in Fama and French [1998], slow responses and momentum of stock prices as first reported in Ball and Brown [1968] and later confirmed by Chan et al. [1996] and Rendleman et al. [1982], excess volatility as reported by Shiller [1981, 2003a] and stock market bubbles in general.

But it was not until 1992 that those results received a lot of attention since they never really fully rejected the CAPM. Fama and French [1992] developed a three factor model that explained returns with the market index, book to market ratios and firm size. As Grauer [2003] notes they "shot straight at the heart of the CAPM" [p. xxviii]. They used the largest database so far and a well thought out test design that separates the firm-size-effects from the beta-effects. Their results actually *rejected* the CAPM [Spremann, 2008a, p. 341].

This test had widespread implications in the academic finance world and many different reactions. Some declared the beta is dead and some believed in investor irrationality. According to Grauer [2003] four different schools of thought have emerged. The first believes that the CAPM has been rejected spuriously due to data mining[8], survivorship bias[9] or poor proxies for the market portfolio[10]. The next school believes that three factor ICAPM or APT pioneered by Fama and French [1992] simply does not reduce to the CAPM and is the right pricing model. The third group argues with missing factors in the models, namely taxes and liquidity.[11] The fourth school of thought adheres to the concept of investor irrationality that prevents the CAPM relationship to hold.[12]

In addition to the ambiguous results of the tests, some scientists remark that the CAPM is *not testable* anyway. Roll's [1977] critique is the most famous. He asserts that testing is not possible unless the exact market portfolio is observable. But instead most tests use proxies for the market portfolio and never observe the true portfolio. Hence, the Betas which are a function of the portfolio from which they are calculated are never the true Betas.

Tests of the APT have been as ambiguous, too. According to different sources there are several approaches to specifying factors and testing the theory. Connor [1995] differentiates between three possible factor models: macroeconomic, fundamental and statistical factor models. Grauer [2003] identifies two main approaches to testing: statistical and theoretical. The theoretical tests can be divided into three groups again. First macroeconomic variable models as mentioned above. Second, the three factor model by Fama and French. And a third method that deals with "firm descriptors that may help in predicting changing measures of risk" [p. xlvii]. It is associated with the risk management firm Barra.

When considering the diversity of possible factors in the multi factor models they might

[8] Lo and MacKinlay [1990] and Black [1993] as cited in Grauer [2003].

[9] Kothari et al. [1995] as cited in Grauer [2003].

[10] Roll and Ross [1994], Kandel and Stambaugh [1995] or Grauer [1999] as cited in Grauer [2003].

[11] Klein [2001], Amihud and Mendelson [1986], Brennan and Subrahmanyam [1996], Brennan et al. [1998], Amihud [2002] and Pastor and Stambaugh [2003] as cited in Grauer [2003].

[12] Lakonishok et al. [1994], Haugen [1996], MacKinlay [1995] and Daniel and Titman [1997] as cited in Grauer [2003].

actually explain the excess returns better than the original single factor CAPM. So those models seem more general since they allow for e.g. firm size and value effects that could be prized risk factors. Still, many anomalies mentioned above have no obvious explanation with our present models.

In general, one has to distinguish between anomalies that are inconsistent with the Neoclassical Models, but can be explained through refinement and anomalies that are inconsistent with the whole paradigm of Neoclassical Finance. A thorough treatment of that issue is beyond the scope of this thesis and the reader is referred to Barberis and Thaler [2003] for a comprehensive survey of that topic. In general it can be noted that there is a "gripping, ongoing debate" on whether models are being erroneously rejected because of data mining, newly discovered risk premiums that do not fit our current understanding of risk, poor proxies for the market portfolio or survivorship bias [Grauer, 2003, p. xxi].[13]

While Grauer's classification of different schools of thought is very detailed one can see a more general division between groups of scientists as noted in Ross et al. [2005] and Fama and French [2004]. Based on the interpretations of empirical evidence basically three camps of scientists have formed over the past few decades - those who argue in favor of efficient markets and stick to Neoclassical Finance, those who belief in the inefficiency of markets and adhere to a new school of thought termed *Behavioral Finance*. And then there are the scientists that are not convinced by either approach. As a warning one has to say there is little agreement among scientist or as Ross et al. [2005] note "only in this area do grown-up finance professors come close to fisticuffs over an idea" [p. 370].

5.8 Applications

The CAPM and APT are very versatile in their applications in practice. The SPT in its original form is not found in practice to a great extent. But many books on derivative pricing and mathematical finance start with an SPT framework as a good introduction to option pricing theory and financial markets as it is an ingenious theoretical framework.

Uses for the CAPM and APT include the cost of capital estimation, capital budgeting decisions, portfolio performance evaluation/risk adjusted performance measurement and capital structure decisions. Graham and Harvey [2001] have shown that the CAPM is used by almost three quarters of financial managers in US corporations and Brounen, de Jong, and Koedijk [2004] have confirmed this result for Europe - the CAPM is the most widely used tool in capital budgeting and cost of capital estimation. But as many recent corporate finance texts admit the APT is becoming more common lately (e.g. Ross et al. [2005], Copeland et al. [2005] and Brealey et al. [2008]).

In portfolio performance measurement Lehmann and Modest [1987] have provided a

[13] This discussion can also be found in Bodie et al. [2008] or Schwert [2003].

comparison of APT and CAPM based performance measures and show that the results differ. As noted in the previous section it remains unclear to what model should be generally used as empirical tests are ambiguous. It is clear that multi factor models are in wide use in empirical finance and also many risk management firms employ multi factor models e.g. the risk management firm Barra.[14]

In general the CAPM remains the traditional approach taught in Corporate Finance texts due to its intuitive value and easy handling.

5.9 Summary: Lessons Learned

Though all models share the same intuition it was shown that they differ in many aspects. The popular CAPM and APT start from very different main assumptions:

CAPM Main Assumptions

- Investor preferences are defined over the mean and variance of asset returns and investors are expected utility of end of period wealth maximizers.
- Investors have homogeneous beliefs on means, variances and covariances of asset returns which are normally distributed.
- All assets are perfectly divisible and perfectly liquid, i.e. all assets are marketable.

APT Main Assumptions

- The No-Arbitrage condition holds.
- Investors prefer more to less.
- Investors have homogeneous beliefs on the linear k-factor return generating process.

From this point the comparison has shown that there is much more behind the intuition of diversifiable and non-diversifiable risk than often noticed. The notion of "non-diversifiable risk" is the centerpiece of the asset pricing models. This is the only risk that the investor is rewarded for. That risk is a recurring theme through all theories: In the SPT investors would try to insure themselves in a way to ensure equal marginal utilities of wealth across different states. The non-diversifiable risk of different aggregate wealth levels played a pivotal role here. In the CAPM this non-diversifiable risk is the covariance with the market portfolio and

[14]Now MSCI Barra: http://www.mscibarra.com/products/analytics/models/.

in the APT it is the exposure to some unspecified fundamental risk factors. All theories try to explain the amount of non-diversifiable risk inherent to a single asset and try to somehow define that concept. Though the intuition is perfectly clear in the SPT, most will agree that the CAPM provides the most elegant framework for explaining the systematic risk.

Ultimately, the APT can be seen as more robust and general as the CAPM for several reasons as Copeland et al. [2005] note: First, it makes no assumptions on the distribution asset returns. Second, no strong assumptions on preferences are needed except for that one prefers more to less. Third, the pricing equation can be obtained for any subset of assets. There is no market portfolio that needs to be measured. Hence, the whole investment opportunity set does not need to be taken into account. Lastly, the APT allows the returns of assets to depend on more factors than just the market portfolio. This is convenient when discovering new risk premiums that are not included in the CAPM market portfolio.

Chapter 6

Conclusion

"Science progresses funeral by funeral" [Mankiw, 2006, p. 38].[1]

The comparison of our simple single period context models has shown what rich theoretical development the different asset pricing theories share. All asset pricing models shared the idea that an investor is only rewarded for bearing systematic risk as opposed to unsystematic risk that can be diversified away at no cost. While the SPT and CAPM explicitly point out with what kind of aggregate risk the expected returns correlate, the APT made no predictions about what the risk factors are.

The SPT allows for a general understanding of the determinants of asset prices. It was shown that the state price densities are consistent with marginal utility across states, thus, making asset prices dependent on economy related risks i.e. the aggregate wealth level. While this framework is an invaluable addition to a financial economists toolkit it has hardly ever been made operational for practical purposes. Hirshleifer [1964, 1965, 1966] and Myers [1968] were among the first and only ones to apply the original SPT to corporate finance problems. But due to its descriptive representation of financial markets the SPT has proven to be a solid basis for courses on derivative pricing. Many textbooks on the mathematics of arbitrage and mathematical finance start with an SPT model to introduce the concepts.

The MPT with its flagship the CAPM offers the most intuitive and insightful understanding of risk of the models and is a great pedagogical tool. As Fama [1991, p. 1593] puts it: "before it became a standard part of MBA investments courses, market professionals had only a vague understanding of risk and diversification." Rubinstein [2002, p. 1044] even puts it more metaphorically:

> *"Near the end of his reign in 14 AD, the Roman emperor Augustus could boast that he had found Rome a city of brick and left it a city of marble. Markowitz can boast*

[1] Max Planck and Paul Samuelson are often credited with that quote. It is unclear where it originated.

> *that he found the field of finance awash in the imprecision of English and left it with the scientific precision and insight made possible only by mathematics."*

It was not until the development of Markowitz theory and its extensions by Sharpe and Lintner that one was able to speak of risk in a quantifiable fashion and make an unambiguous statement about the risk-return relationship. Because of the insight and intuition about risk and return it offers, the CAPM has been the centerpiece of financial economics for decades and remains to be the traditional workhorse of financial managers. It has been criticized for its strong assumptions and ultimately rejected in empirical testing. But its main implications remain intact, and there is "room for refinements" as Spremann [2008a] puts it.

The APT as the last asset pricing model was based on very weak assumptions, but yielded a very similar insight to the other models: idiosyncratic risks can be diversified easily and should not be priced. Only the common factors that move security returns should be priced according to the exposure to those factors. The APT has no answer to what those factors are and leaves room for intuitive thought or statistical methods such as factor analysis. Thus, it lacks an unambiguous statement about the risk-return relationship as the CAPM. This has advantages and drawbacks. On the one hand Fama [1991, p. 1594] argues that the APT is an "empiricist's dream" as it allows for more risk factors. On the other hand Connor and Korajczyk [2006, p. 137] admit that "the APT would be a better model if we could relate the factors more closely to identifiable sources of economic risk".

One can claim that the generality of the APT comes from another well-known economic principle: "People face trade-offs" as Gregory Mankiw[2] tends to emphasize. Thus, one can either build models (e.g. the CAPM) with strong (arguably unrealistic) assumptions, but very strong implications and "instructions" on how to measure risk or one can employ a set of weak assumptions (e.g. the APT) with less specific implications and "instructions" on where to look for risk factors.

Lastly, a remark on the Empirical Content should be made. Many authors such as Shleifer [2000] or Shiller [2003a] argue that it is time for a paradigm shift towards Behavioral Finance. But research is progressing and many authors such as Spremann [2008a], Cochrane [1999a, 2005] or Ross [2005] note that the Neoclassical Models are open for refinements. More importantly, researchers tend to embrace the influences of the real economy on financial markets. Although "science progresses funeral by funeral" this thesis has shown that it is too early for an obituary of Neoclassical Models.

[2] http://gregmankiw.blogspot.com/2008/12/principle-1-people-face-tradeoffs.html retrieved on 10.08.2010

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Declaration of authorship

St. Gallen, August 22, 2010

I hereby declare,

- that I have written this thesis without any help from others and without the use of documents and aids other than those stated above,
- that I have mentioned all used sources and that I have cited them correctly according to established academic citation rules.

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