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Risk Measurement in Electricity Markets

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Content

1. Introduction

Electricity contracts differ substantially from financial contracts making traditional derivatives inapplicable. The main difference lies in the inability to store electricity causing the production to cover demand instantaneously. Therefore, electricity prices often jump to a multiple of their current value only to come back to normal level within a few hours. Spot price volatility is driven by demand whereas prices in the long run are rather affected by the physical ability of technology and generation capacity. A one factor price model like the geometric Brownian motion is insufficient to capture the mean-reverting behaviour of the electricity prices. The model needs to be extended by an additional stochastic factor to reflect electricity price movements realistically.

The evolution of sophisticated models and numerical techniques had a lasting effect on the risk perception of companies active in these markets. Facing the problem of managing their risk exposure, market participants seek to offset their risk by hedging and rebalancing their positions. The approach is referred to as the ‘Greeks’ or sensitivity analysis whereby each risk factor is assigned to a *'Greek Letter'*. An alternative risk management technique which summarizes the total risk in a single number is known as VaR. Despite its popularity, the VaR concept should be handled with caution when it is applied to electricity markets.

This work aims at providing further insights into risk management in electricity markets in general and into sensitivity analysis in particular. The goal of this paper is a systematic analysis and comparison of the ‘Greeks’ under the assumption of different price dynamics. Moreover, it tries to demonstrate the limits of traditional risk management methods such as VaR and their modification. In the last part, model test are carried out in order prove the accuracy of the used software tool for option valuation.

2. Electricity Contract Modelling

Each power plant can be described as a leasing contract, which gives the holder the right to the plant's output at predetermined period in the future[1]. All rights and liabilities are embedded in electricity contracts allowing for valuation through replication of a no-arbitrage portfolio[2]. The software tool BIT@EPI.VPP enables the valuation of a range of electricity contract types, including *futures*, *option on futures* and the so-called *swing options* or *Virtual Power Plants*[3].

BIT@EPI.VPP prices electricity contracts by transferring them into an analytical structure using multistage stochastic linear programming as a measure[4]. The model is built on a filtrated probability space induced by an appropriate underlying electricity spot price process. This methodology is superior to comparable measures such as Monte Carlo Simulation insofar as it optimizes the decision process and provides the best exercise strategy as part of the result[5].

The value of an electricity contract not only depends on the specific characteristics such as price, volume and timing of delivery but also on the evolution of the underlying price. While the price forward curve represents an average scenario and thus the mean, stochastic processes simulate the probabilistic behaviour of the underlying price through time. *Figure 1* summarizes the relevant input parameters used for valuation of electricity contracts with BIT@EPI.VPP.

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[1] Eydeland and Wolyniec (2003), p. 58.

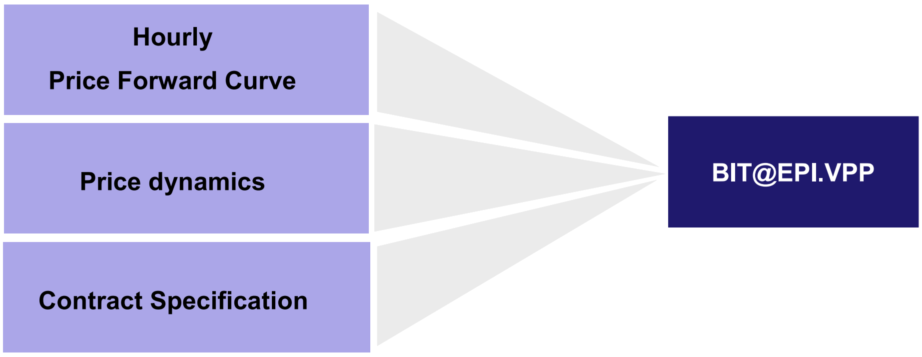
[2] Güssow (2006), p. 65.

[3] The name 'Virtual Power Plants' stems from the fact that they can be seen as a virtual storage, giving a holder the right to operate at a predetermined period of time. Doege (2006), S. 78. For an accurate technical description see Davison, Thompson and Rasmussen (2004).

[4] Blöchinger, Haarbrücker, Kuhn (2006), p. 6.

[5] Güssow (2006), p. 65.

Figure 1. Input Parameters Electricity Contract Valuation with BIT@EPI.VPP



2.1. Hourly Price Forward Curve

When dealing with the valuation of electricity contracts, one should be aware of the fundamental differences between electricity and most other traded products. In particular, the non-storabilty makes the application of finance and storable commodity market models unfeasible. Unlike for products, which can be physically stored, the cost of carry relationship between spot and forward prices is invalid[6]. One has to consider the economic characteristics of demand and supply for electrical energy in the short run as well as the issues that must be addressed in the long run, namely the physical capability of a regional market to generate electricity[7]. *Borovkova and Geman* conclude that the characteristics of spot and future electricity markets are so dissimilar that the relationship breaks down. This fact is supported by the Nordpool historical data where correlation between spot and futures prices ranges from 0,65 to -0.15[8]. Pilipovic names this phenomenon characteristically the "split personality" of electricity prices indicating that the relationship between spot and forward prices in electricity markets is impossible to capture with only one risk factor[9].

In the short run, the price elasticity of the demand for electricity is relatively small[10]. For industrial customers electricity is indispensable in manufacturing and they are willing to pay a premium, the so-called convenience yield, in order to keep the production process running. In addition, the demand behaviour of residential consumers exhibit strong hourly, daily and monthly seasonalities[11]. Seasonality stems from regular demand fluctuations often caused

[6] Hull (2006), p. 60.

[7] Henney and Keers (1998), p. 39.

[8] Borovkova and Geman (2006), p.3.

[9] Pilipovic (1998), p. 4.

[10] Kirschen (2003), p. 520.

[11] Burgert, Bernhard, Müller and Schindlmayr (2006), p. 2.

by weather related factors. Fluctuations in demand go together with supply side constraints such as capacity limits in production and transformation grid[12].

Limits in production capacity combined with low elasticity of demand in the short run result in severe price spikes and jumps caused by shortage conditions. In the long run, however, seasonalities tend to diminish due to adjusted economic conditions and improved technology. Users can act more flexible by shifting their technology to other forms of energy while the suppliers can expand the physical potential of electricity generation. The long term price level therefore hardly depends on current economic market conditions but on the physical potential to enhance supply and increase demand sensitivity[13].

The electricity forward curve reflects the expectation about electricity prices for different maturities. The corresponding forward contracts have a huge advantage over spot electricity because they are traded and storable, thus allowing for a risk-neutral valuation. Generally, electricity forward contracts are traded on an hourly basis giving credit to different hourly demand patterns and price levels. For convenience, market participants often agree on standard qualities: 'Cheap' generation capacity is referred to as *off-peak quality* while high price level hours characterize *peak quality*. *Base quality*, however, guarantees a constant hourly supply over the whole delivery period.

2.2. Forward Price Dynamics

Market participants base their decisions on forward price information that changes over time in an uncertain way. The price forward curve bears only information about the expected price level without making any further assumption about the stochastic evolution of the curve. In order to capture the forward price dynamics adequately we must incorporate stochastic dynamics into the model. The standard model for financial markets describes the dynamics of the underlying through the geometric Brownian motion assuming the variance to grow over time. However, electricity prices follow a mean reversion process where the spot prices move back towards their long-term level. The 2-Factor mean reversion model introduced by Pilipovic provides a more realistic description of the electricity price dynamics incorporating a stochastic long-term level as the key extension to conventional mean reversion models[14].

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[12] Clewlow and Strickland (2000), p. 38.

[13] Pilipovic (1998), p. 4.

[14] Pilipovic (1998), p. 36.

2.2.1. 1-Factor Model: Geometric Brownian Motion

The model capturing stochastic price behaviour commonly used in financial markets is known as geometric Brownian motion (GBM). The process is described by only one factor, dx, symbolizing the change in price in terms of dz[15]:

$$dx(t) = \alpha x dt + \sigma x dz(t) \text{ or}$$

$$x(t+dt) \ = x(t) + \alpha dt + \sigma dz(t)$$

According to the above equation, the price process has a constant drift rate of *α* and a variance rate of *σ*. The probability distribution of the price in any particular future time *(t+dt)* only depends of the current price *x*. This feature is known as the Markov property[16]. Furthermore, the GBM implies that the stochastic term *dz(t)* is normally distributes with a mean of zero and a variance of *dt*[17].

$$dz_t \sim \mathrm{N}(0, dt)$$

In fact, the model implies that the variance of the return distribution depends on the time span *dt* over which returns are calculated. GBM implies a growing variance over time whereas the price development in electricity markets suggests that prices fluctuate around a long term mean[18]. The applicability of GBM for modelling energy price processes is therefore questionable.

2.2.2. 2-Factor Model: Mean-Reversion

The GBM model can be extended to a more realistic description of the forward price dynamics by including a second factor into the model and giving credit to the ‘split personality’ characteristics of electricity prices. The existing literature has well documented that an important characteristics of electricity prices is mean reversion where short term prices fluctuate around a long term level[19]. While forward prices with short maturities are strongly correlated and can be captured by only one factor, prices with long maturities need an additional factor to describe their evolvement[20]. A model which can simulate the electricity price process in a more realistic way is the 2-factor Pilipovic model (2F)[21]:

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[15] Geman (2005), p. 60.

[16] Hull (2006), p. 263.

[17] Pilipovic (1998), p. 102.

[18] Geman (2005), p. 65.

[19] Pilipovic (1998), Clewlow and Strikland (2000).

[20] Pilipovic (1998), p. 115.

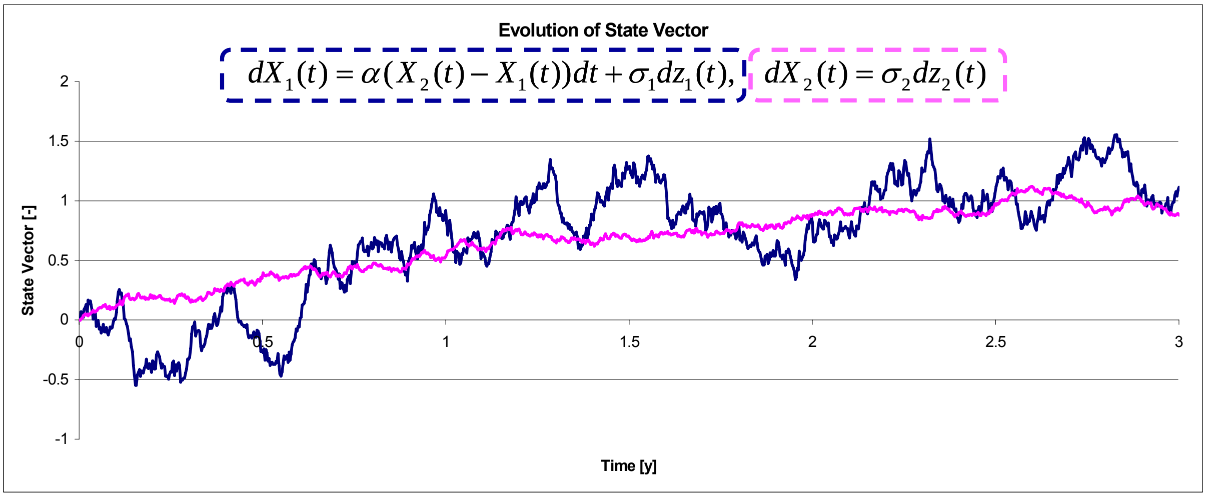
[21] Pilipovic (1998), p. 64.

$$dX_1 = \alpha(X_2 - X_1)dt + \sigma_1 dz_1(t)$$

$$dX_2 = \sigma_2 dz_2(t)$$

The first factor X_1 describes the short term deviation from the long term price level X_2 following a mean reverting process with the drift of *α*. They key extension of the model is the assumption of a stochastic rather than constant long term level with the volatility σ_2[22]. *Figure 2* illustrates the price dynamics of the 2F model.

Figure 2. Price Dynamics of the 2F Model



Frauendorfer (2006)

The forward price *F(t,T)* at time t is given by the following equation:

$$F(t,T) = F(0,T)\frac{E_t(\exp(X_1(T)))}{E_0(\exp(X_1(T)))},$$

$$E_0(F(t,T)) = F(0,T),$$

$$0 \le t \le T$$

where *F(0,T)* represents the deterministic forward price as determined by the price forward curve. The stochastic process is implemented through the first factor X_1 into the model. As forward prices converge to the spot prices at maturity the lack of arbitrage is guaranteed.

$$S(t) = F(t,t) = F(0,t)\frac{(\exp(x_1(T)))}{E_0(\exp(x_1(T)))}$$

Note that the GBM model represents a special case of the 2F model where the mean reversion parameter *α* equals zero and the volatility is equal to the short term volatility.

[22] Clewlow and Strickland (2000), S. 19.

2.3. Contract Specification

The software package BIT@EPI.VPP allows for valuation of three different contract types: *futures*, *option on futures* and *swing options* or the so-called *Virtual Power (VPP)*. These energy contracts differ substantially in the level of flexibility they offer to their holder. In the following, each contract type will be considered in detail and it will be shown how flexibility influences the profit and loss (P&L) distribution in power markets.

2.3.1. Future

A *future* is a standardized contract between two parties, which obligates the holder to buy or to sell the underlying asset at a predetermined price during a specified future time period[23]. The contract standardization of electricity futures enables both, the reduction of transaction costs and the trade on a continuous basis, making electricity futures comparable with traditional financial instruments. Another advantage lies within the elimination of counterparty default risk due to a guarantee of delivery and to the financial compensation by the power exchange[24].

What makes electricity futures different from traditional financial futures is the termination of the contract. The physically settled future obligates the seller to deliver electricity at constant power over a period of time rather than at a point in time. The traditional convergence criterion, which assumes that the future price converges to the spot price at maturity, is only valid on average[25].

$$S(t) = F(t,t)$$

The holder of the future contract is exposed to the deviation of the settled future price from the spot prices within the delivery period. This specific risk exposure in case of unfavourable evolution of the spot price within the delivery period is known as the basis risk of an electricity future. *Eydeland* and *Wolyniec* point out that the lack of convergence may result in significant basis risk[26].

2.3.2. Option on Future

An options on future is priced with BIT@EPI.VPP as a European option because it can only be exercised at maturity. The value of an option, as compared to a future, stems from the flexibility it provides to the holder with the right rather than the duty to exercise the option.

[23] Hull (2006), S. 6.

[24] Eydeland and Wolyniec (2003), S. 19.

[25] Eydeland and Wolyniec (2003), S. 21.

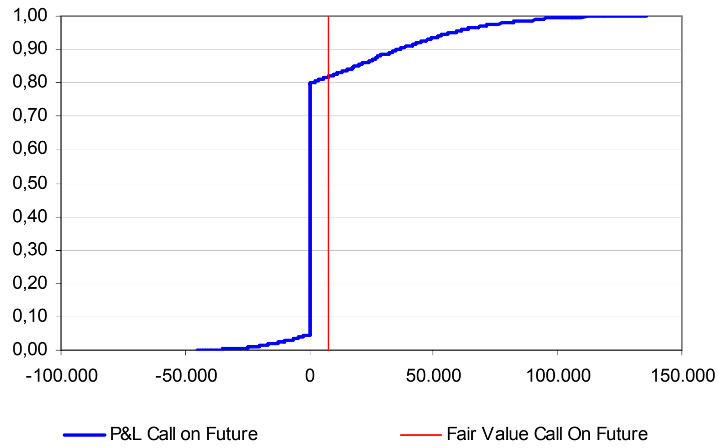
[26] Eydeland and Wolyniec (2003), S. 31.

The holder of a call option benefits if the price increases above the strike price while at the same time he possesses a hedge against the downside exposure.

What makes spot electricity options problematic is the inability of the underlying to be stored causing the spot price to depend on a particular time and geographical location[27]. By contrast, an option on future provides additional flexibility and benefits from the liquidity of its underlying asset. As mentioned above, the standardisation of the future contract enables the holder to trade in the power exchange, to settle not only physically but also financially and hence to factor out the problem of storage. Furthermore, the liquidity of future contracts assures the future price to be known whereas the spot price is often not accessible[28].

The basis risk of the underlying future has an impact on the P&L distribution of the option, which can be observed in the left part of the distribution in *Figure 3*. This part reflects the probability of generating a loss when an option is exercised despite the downside hedge provided by a call option. Consequently, the right part of the distribution represents the profit area whereas the step within the P&L distribution measures the probability of an option not being exercised.

Figure 3. P&L Distribution of a Call on Future



2.3.3. Swing Option or Virtual Power Plant (VPP)

The contract types described so far are basically used to hedge against an unfavourable development in the electricity spot price. Unlike in other commodity markets the supplier of electricity faces a "swing" demand risk[29]. This uncertainty is known as *volume* risk and can be tracked back to the physical incapacity of power plant facilities in the short run. *Swing*

[27] Hampton (1999), S. 41.

[28] Hull (2006), S. 327.

[29] Nagarajan (1999), p. 248.

options or so-called *Virtual Power Plants* address the need of the market participants to have flexibility in both timing and volume of energy delivered.[30]. Jaillet, Ronn and Tompaidis sum up the *raison d'être* of swing options as following: *„They address the need to hedge in a market subject to frequent, but not pervasive, price- and demand-spiking behaviour that is typically followed by reversion to normal levels.“*[31]

Swing options are tailor-made contracts that are composed of two parts[32]: The first part is similar to a base-load future contract that fixes the delivered amount of electricity over a specified period in the future. This part addresses the buyer's need to hedge against the price risk. The second part gives the option holder the right to swing his demand freely within predefined thresholds, allowing to buy more or less of the predetermined amount. Thus, the swinging part of the contract includes an insurance against stochastic demand fluctuations.

Due to the limited right to swing within predetermined volume constraints, the contract becomes path-dependent allowing for the fact, that future demand restrictively depends on the past energy consumption. Swing options are superior to both American and Asian options.[33]. While European options provide no flexibility with respect to volume, American Options can offer more flexibility than needed making the consumer overpay the protection[34]. Thus, a *swing option* offers sufficient protection at the lowest price.

2.3.4. Comparison of the three contract types

The value of each contract reflects the flexibility offered to the holder. The comparison of the three contract types provides a deeper understanding of the financial instruments introduced above. As an example, we consider the following contract: The valuation date is October 1st 2006; the delivery period lasts from January 1st to December 31st 2007. In order to make a comparison, the total output is set to 3.132 MWh for the contracts 1-3 whereas additional volumetric flexibility is embedded in contract 4 by allowing the amount to swing between 3.000 and 3.264 MWh. The option right can only be exercised in peak hours from 8.00 am to 20.00 pm. Thus, the number of exercise hours within the delivery period totals 3.312 and implicitly sets the upper power limit for the contract 1 and 2 to 1 MW. When it comes to swing option contracts, namely contract 3 and 4, we introduce volume flexibility through power constraints, which allows to swing between 0 and 2 MW.

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[30] Dahlgren (2005), p.27.

[31] Jaillet, Ronn and Tompaidis (2004), p. 909.

[32] Jaillet, Ronn and Tompaidis (2004), p. 909.

[33] Dahlgren (2005), p. 28.

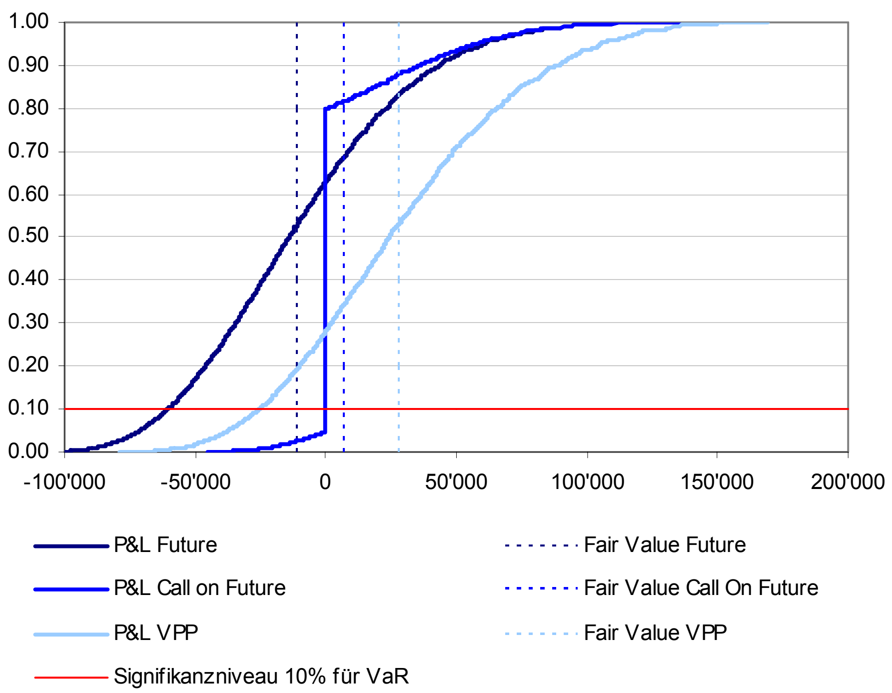
[34] Jaillet, Ronn and Tompaidis (2004), p. 909.

The strike price is set at 82.88 Euro and the underlying price evolution is based on the 2F model introduced by Pilipovic with the parameter values σ1=30%, σ2=10% and α=5, which are assumed to be risk neutral estimation from the market. We compare the four contracts with respect to their fair value and Value-at-Risk (VaR) at 10% level of significance as presented in *Table 1* and *Figure 4*.

Table 1: Value and risk profile of the contracts 1-4

Contract	**1. Future**	**2. Call on Future**	**3. VPP**	**4. VPP**
Volume	3'132 MWh	3'132 MWh	3'132 MWh	3'000-3264 MWh
Price	-10.693,61	7.501,30	26.229,17	28.274,06
VaR (10%)	-60.383,11	1,83	-28.313,58	-25.489,52

Figure 4: Comparison of the P&L-distribution of contracts 1, 2 and 4.



The expected loss of the future as represented by contract 1 equals -10'693 Euro due to the above-average delivery price. With a probability of 90% the loss will not exceed 60'383 Euro. A call on future for the same amount of electricity in contract 2 provides a hedge against the downside price risk. The option holder has the right to let the option expire without exercising it which is done in 75% of the cases as measured by the step in the distribution function. Thus, the fair value equals 7'501 Euro with a corresponding VaR at 2 Euro.

In contract 3 we enhanced additional timing flexibility by introducing lower and upper power limits ranging between 0 and 2 MW. Literally, this means that at some point within the delivery period the consumer can double his demand whereas on other occasions he can suspend the delivery. The total amount continues to be fixed at 3.132 MW. The fair value of this contract equals 26.229 Euro and exceeds the value of the call on future by 18.728 Euro. One has to bear in mind that a further increase in timing flexibility would result in an even stronger impact on the fair value of the contract. The VaR at 10% level of significance equals -28.314 Euro reflecting the different exercise probabilities of contract 2 and 3. While the call on future expires with a probability of 75%, Virtual Power Plant has a future as a basis part and thus is always exercised.

In a second step, we introduced volumetric flexibility by allowing the total output to swing between 3.000 and 3.264 MWh as represented in contract 4. The additional flexibility has a positive impact on both the fair value and the risk of the VPP: We observe an increase in fair value to 28'274 Euro and a decline in VaR to -25'489.52 Euro. However, we expect the impact to be more severe along with further flexibility enhancement.

To sum up, the future only provides an insufficient insurance with respect to price and no insurance with respect to volumetric risk. A call on future causes an overprotection of the risk exposure. The contract seems ho have excellent hedging abilities because of no downside risk but one has to bear in mind that the option is only exercised with the low probability of 25%. The introduction of timing flexibility results in an increase in the fair value of the option. This effect is further amplified by volumetric flexibility, which at the same time affects the risk profile in a favourable way.

3. The Greeks

In finance literature, the rate of change of the option price with respect to change of a risk factor is commonly referred to as the 'Greeks'. Each Greek letter measures a different dimension to the risk in an option position. The Greeks can be split into first and second order: First order Greeks are defined as the rate of change of the option price with respect to the price of the underlying *(delta)*, to the volatility of the underlying *(vega)* and to the passage of time *(theta)*. *Rho*, the sensitivity with respect to interest rates won't be covered in the course of this work and without loss of generality are assumed to be zero. *Gamma* is the second partial derivative of the option price with respect to the price of the underlying and corresponds to the rate of change of the option's delta with respect to asset price.

The Greeks offer an alternative approach to manage the risk exposure of an open position by creating a hedging scheme through the immunisation of risk. This is achieved by neutralizing a position in such a way that it is not affected by small changes in the underlying risk factor. A more descriptive term commonly used in practice is referred to as sensitivity. The change in the contract value resulting from a change of the risk parameter in question enables us to undertake a sensitivity analysis and to plot the 'Greek' patterns.

In order to evaluate Greeks we perform a sensitivity analysis by setting up a model according to the conceptual considerations presented in the previous section. The first step is to determine a standard contract where the risk factor in question will vary keeping everything else constant. For our numerical calculation, we choose a base contract (all hours) with the power of 1MW: The contract type is a call on future with a six months delivery period ranging from 1. July 2007 to 31. December 2007. The valuation date is 1. April 2007. *Table 2* provides an overview over the parameters of the standard contract.

Table 2. Details of the Standard Contract

Contract details	
Contract specification	Call on Future
Delivery Period	1. July 2007 - 31. December 2007
Valuation Date	1. April 2007
Power	1 MW
Quality	Base
Model Parameter GBM	$\sigma = 0.2$
Model Parameter 2F	$\sigma_1 = 0.3$, $\sigma_2 = 0.1$, $\alpha = 5$

The valuation is based on the GBM and 2F model introduced in chapter 2.2. The volatility coefficients corresponding to the short and long term fluctuations are set to $\sigma1=0.3$ and $\sigma2=0.1$, respectively. The mean-reversion parameter α is set to 5. These coefficients meet the risk neutral estimation from the market. With regard to benchmarking the obtained results, we undertake an additional valuation based on the GBM with a constant volatility of σ=0,2. Moreover, our model is based on a flat price forward curve so the option can be classified in the categories *in the money*, *at the money* and *out of the money*. This convenience does not affect the explanatory power of our results at any time.

3.1. Sensitivity with Respect to Price: Delta and Gamma

The first derivative with respect to the price of the underlying is known as delta, the second derivative is commonly referred to as gamma. The delta of an option is defined as the rate of change of the option price with respect to the price of the underlying asset[35]. Delta is closely related to the Black-Scholes pricing formula based on the assumption that a delta-neutral portfolio should earn the risk-free rate[36]. One should bear in mind that a position is delta neutral in a small period of time only. The steady adjustment of the position in order to eliminate the price risk is known as rebalancing or dynamic hedging[37]. Graphically, delta represents the slope of the curve that puts the option price and the underlying asset price into relationship. The conventional definition of delta as the first derivative of the option price with respect to the price of the underlying asset is only feasible for closed form solutions. In our case, it is appropriate to use a discrete form of delta with delta representing a change in option price stemming from a finitely small shift in the price of the underlying. We obtain an even better approximation by taking down- und up-moves of the price likewise into account[38]. Thus, the approximation for delta will be defined by the following equation:

$$ApproxDelta = \frac{1}{2}\left(\frac{\Delta V}{\Delta F^{-}} + \frac{\Delta V}{\Delta F^{+}}\right)$$

Gamma measures the effectiveness respectively the sensitivity of a delta hedge as the underlying price moves away from the hedged level. A gamma of zero implies that the delta hedge stays effective although the underlying price exhibits fluctuations. The rebalancing of the position then remains redundant. The gamma hedge is commonly used for options due to their non-linear payoff structure. A simple delta hedge does not fully offset the movement of the option price due to a shift in the price of the underlying. Therefore, we need an option in order to rebalance the position[39]. Analogue to delta, we define the approximation of gamma in its discrete form.

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[35] Hull (2006), p. 302.

[36] Hull (2006), p. 303.

[37] Hull (2006), p. 303.

[38] Taleb (1997), p. 118.

[39] Steiner, Wenninger and Willinsky (2002), p. 71.

$$ApproxGamma = \frac{1}{2}\left(\frac{\Delta D}{\Delta F^{-}} + \frac{\Delta D}{\Delta F^{+}}\right)$$

In order to continue our analysis with numerical examples, we price our standard contract with BIT@EPI.VPP and set the strike price equal to 55 euro. The finitely small change in the underlying price is simulated through a parallel shift of the flat price forward curve in 5 euro steps. All numerical results are presented in the annex.

3.1.1. Price Sensitivity for Different Maturities

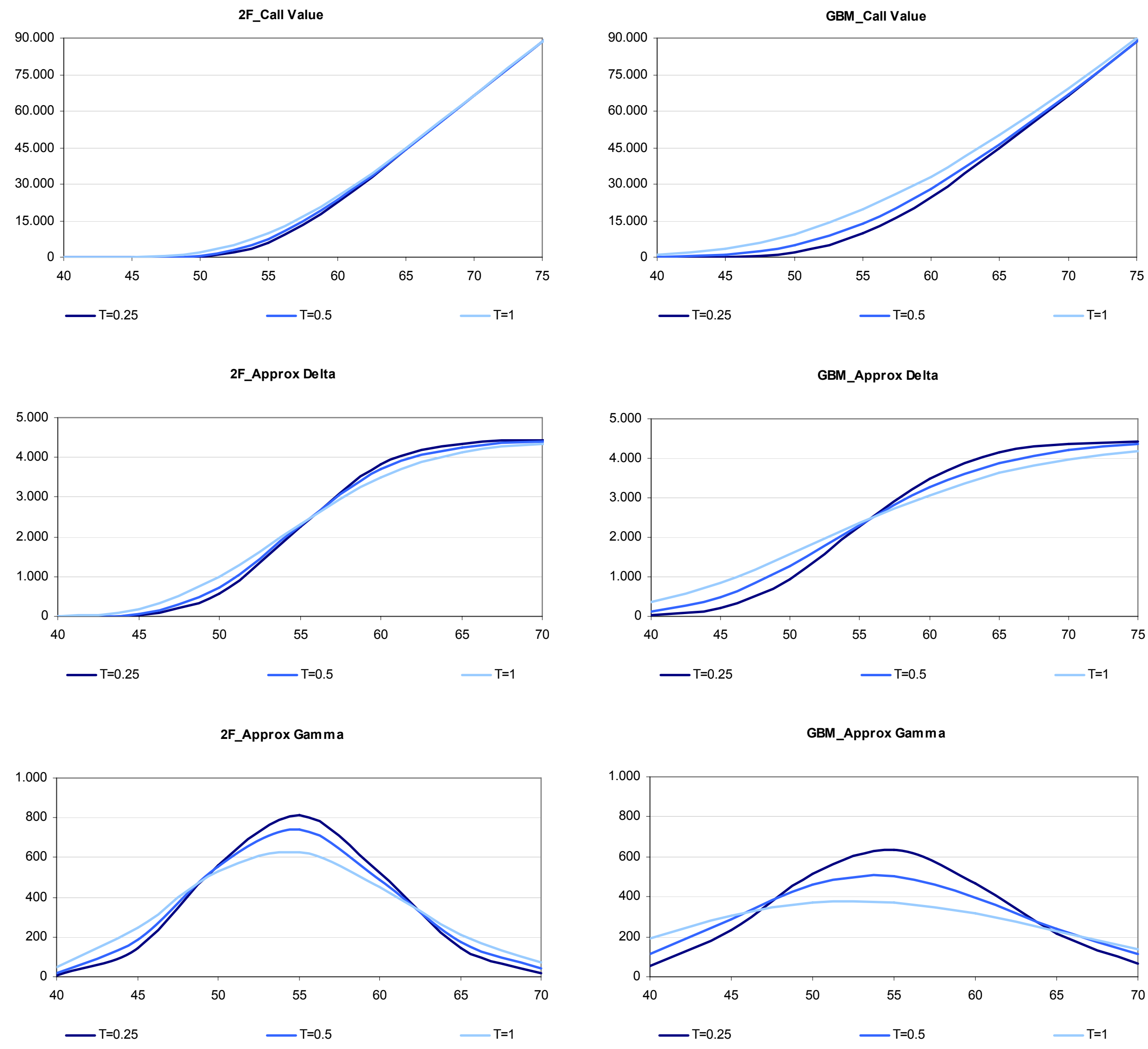
The value of an option before maturity consists of the intrinsic value and the time value. The intrinsic value is the maximum of zero and the value from immediate exercise. The time value, however, gives credit to the possibility of future movements of the underlying price in a favourable direction. With proceeding maturity, the value of an option converges towards its intrinsic value due to a decrease in time value. Therefore, it is necessary to consider the time value when examining the sensitivity with respect to the price of the underlying.

For our analysis, we price the standard contract at three different points in time: 1. April 2007 with a holding period of 3 months (T=0,25), 1. January 2007 with a maturity of 6 months and 1. July 2006 with a maturity of one year (T=1) respectively. The forward price underlying the option is assumed to vary between 40 and 75 euro, the strike price is set at 55 euro. *Figure 5* plots the payoff structure and the associated delta and gamma profiles for the standard contract based on 2F and GBM. The numerical results are documented in the annex.

Figure 5 shows that the time value is maximum at the money. For deep out of the money and deep in the money options, the time value plays an inferior role as the remaining time to maturity has a minor influence on the option value. Further, we can observe that the time value is grater for the option based on GBM than for the option based on 2F. At the money, the price decay equals 5.769 euro (GBM) respectively 2.634 euro (2F) when the maturity decreases by 6 months and 9.762 euro (GBM) respectively 4.218 euro (2F) for a maturity decrease of 9 moths.

Figure 5 also plots the associated delta profiles based on 2F and GBM. When the option is out of the money, the delta of the option converges against zero as the forward price decreases reflecting the sinking probability of the option of ending in the money at expiration. Analogue, when the forward price is high relatively to the strike price, the delta of the option increases and converges against the boundary value of 4.416 euro. As the standard contract provides the total volume of 4.416 MWh, we observe a one-to-one relationship between the option value and the underlying price when the option moves deeper in the money giving credit for the increasing possibility of remaining in the money at maturity. Further, we can observe that the delta profile becomes steeper as the option approaches expiration reflecting the growing sensitivity to small changes in the forward price at the money. Similar to the option value, the maturity plays a greater role when the option pricing is based on GBM. The delta profile for 2F is steeper at the money and becomes flatter as the forward price moves away from the strike price making the delta profile converge faster against the boundary values.

Figure 5: Value, Delta Profile and Gamma Profile for a Call on Future based on 2F and GBM for Different Maturities



When we consider gamma as shown in *Figure 5*, we can observe a bell-shaped profile with a maximum at the money. As we have seen before, delta was most sensitive with respect to the forward price at the money while the profile remained relatively stable on the endings. A gamma close to zero implies that the delta hedge is effective and need not to be rebalanced while a large gamma reflects the need for a steady adjustment in case of small shifts of the underlying price. Further, we can proof time dependence for gamma. When the forward price is close to the strike price, gamma increases when the option nears expiration. The opposite is the case for from the money options for gamma reaches its maximum for long maturities. When comparing the gamma profiles based on different price dynamics, we observe a higher gamma based on 2F at the money and a lower gamma when the option is far from the money. As mentioned before, the sensitivity of the option value based on mean reversion is higher at the money and lower far from the money than for an option value based on GBM.

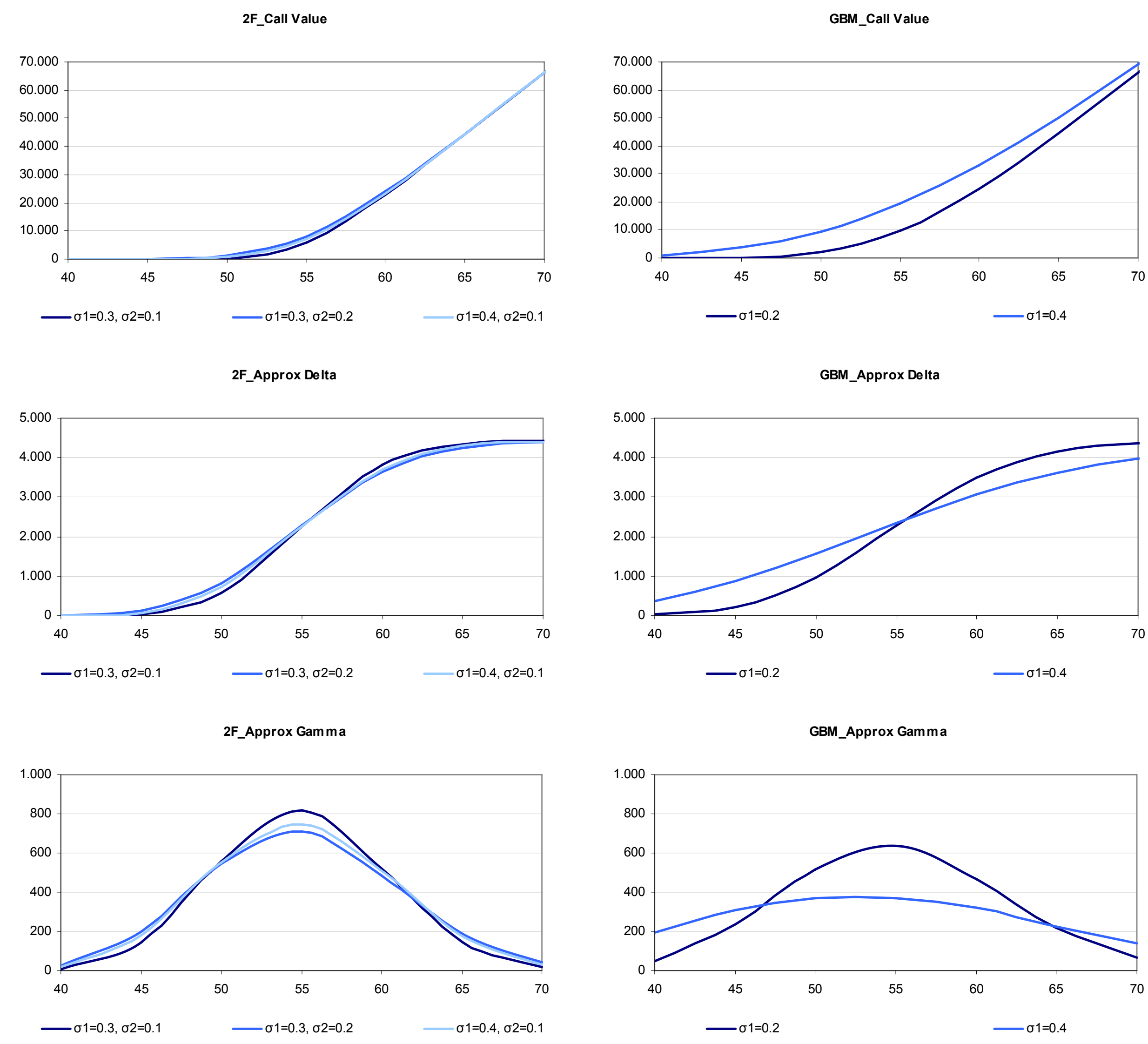
3.1.2. Price Sensitivity for Different Volatility Levels

The implied volatility of the price dynamics has a strong impact on the delta and gamma profiles of an option. The volatility of the underlying asset measures the degree of uncertainty about the future price evolution. Due to the convex payoff structure, the effect of a price increase does not offset the effect of a price decrease. The owner of a call option has an unlimited upside potential, however only a limited downside risk. Therefore, the value of an option benefits when volatility increases. To analyze this relationship, we price the standard contract based on 2F and GBM for different volatilities. *Figure 6* shows the change in price, the delta profile and the gamma profile of a call on future when volatility increases. The numerical results are presented in the annex.

Figure 6 confirms a positive effect of a volatility increase on the option value. However, the effect is smaller, when valuation is based on 2F rather than on GBM. Assuming the dynamics of 2F, we can further observe that an increase in long term volatility has a stronger effect on option value than the effect stemming from an increase in short term volatility. This observation is valid for a mean reversion parameter of 5 only as it weightings the long term volatility stronger than the short term volatility.

Next, *Figure 6* shows, that the delta profile becomes flatter as volatility increases indicating that the probability of finishing in the money becomes less sensitive to small changes in the underlying price. Consequently, gamma declines at the money. The effect is stronger for GBM than for 2F. Under the assumption of 2F, however, the impact stemming from the mean reversion coefficient α can be analogue transferred to the delta and the gamma profile. An increase in long term volatility reduces the sensitivity of an option with respect to price stronger than an increase in short term volatility.

Figure 6: Value, Delta Profile and Gamma Profile for a Call on Future based on 2F and GBM for Different Volatilities



3.2. Sensitivity with Respect to Time Decay: Theta

The value of an option before expiration consists of the intrinsic value and the time value. Theta describes the loss in time value when the option nears expiration[40]. In other words, theta is the difference between today's and future value of an option. In the following, we approximate theta in its discrete form as the rate of change of the value of the portfolio with respect to time decay.

[40] Taleb (1997), p. 167.

$$ApproxTheta = \frac{1}{2}\left(\frac{\Delta V}{\Delta T^{-}} + \frac{\Delta V}{\Delta T^{+}}\right)$$

There is a huge difference between theta and delta. While there is uncertainty about the future forward price, the passage of time is sure. As theta bears no risk, it is needless to hedge against it.

In financial literature, we find different explanations for theta. According to *Hull,* the buyer of an option prefers longer maturities because the strike price is not paid until expiration[41]. The interests accumulated during the holding period equal the time value of the option indicating that an option with a longer maturity is worth more than the same option with a shorter maturity. In our case, this explanation is secondary because we assume the interest rate to be zero.

Another explanation stems from the stochastic modelling of the forward prices where the price variance depends on the time to maturity. To demonstrate this, consider a call option before maturity. Even if the forward price is below the strike price, the numerical value can be positive due to the probability of finishing in the money. GBM assumes a linear relationship between time to maturity and the variance of the underlying price whereas the 2F model implies a constant price variance along the passage of time. Therefore, we expect a higher time value for an option assuming GBM. Furthermore, *Taleb* claims the following relationship: *"If the option is priced at the right volatility (assuming interest rates are 0), time decay will be expected to be 0."*[42]. This statement allows for the conclusion that a price dynamics model approximates the volatility best if it results in a low theta. We conclude that the ‘true’ parameters will lead to a lower theta.

In practice, it has been well documented that an important property of volatility is mean reversion.[43]. Thus, we assume that the 2F based valuation delivers better approximation of the ‘true’ volatility than the GBM dynamics where volatility is assumed to be constant. The goal of the following analysis is to verify these results by illustrating the theta profiles subject to volatility. Therefore, we price the standard contract with respect to time to expiration for different volatility levels. *Figure 7* shows the patterns for the variation of option price and theta with time to maturity for in the money, at the money and out of the money call on futures.

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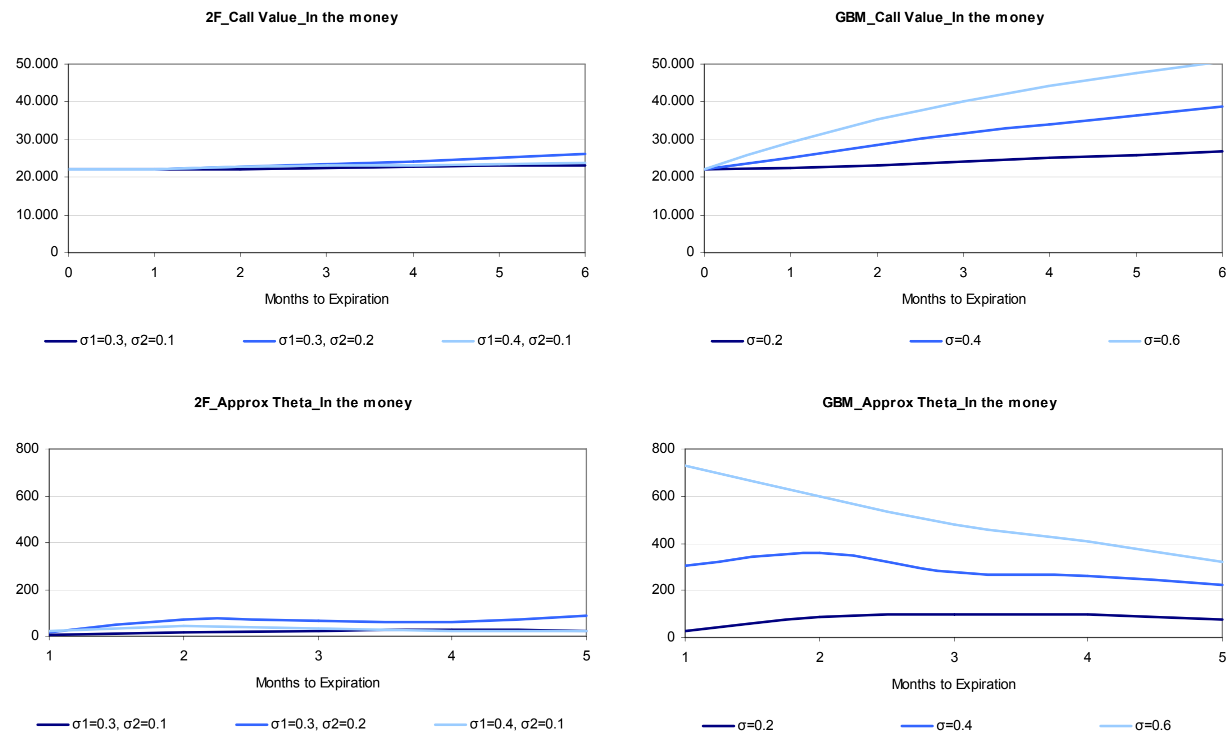
[41] Hull (2006), p. 216.

[42] Taleb (1997), p. 167.

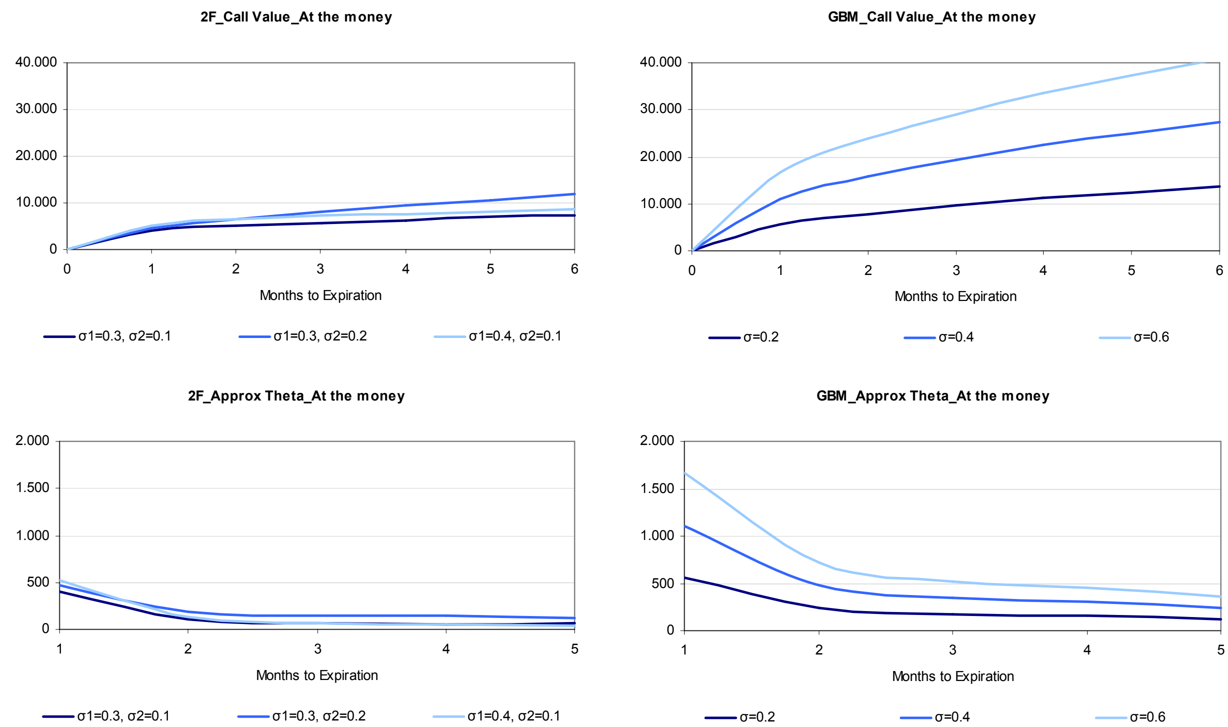
[43] Malz (2001), p. 7.

Figure 7: Value and Theta Profile for a Call on Future based on 2F and GBM for Different Volatilities

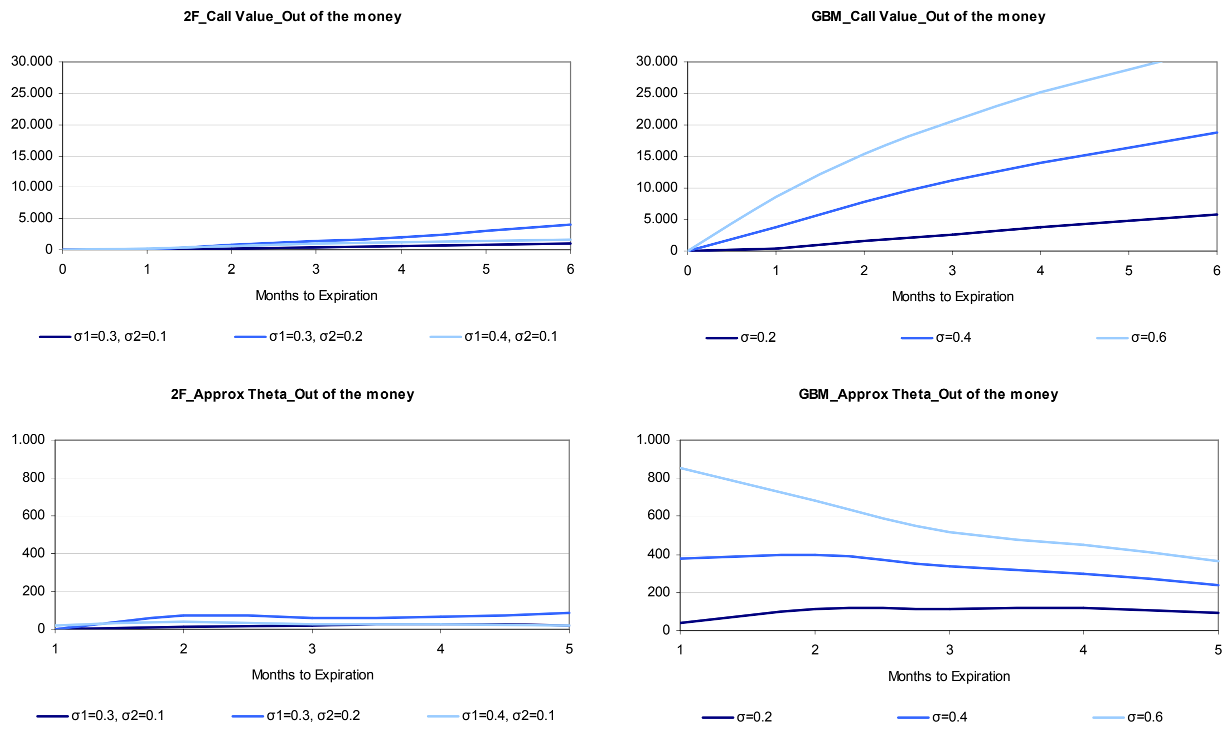
In the Money K=50



At the Money K=55



Out of the Money K=60



Starting from the intrinsic value, the option price rises as time to maturity. *Figure 7* shows that time value is maximum at the money. We can demonstrate this numerically, analysing the standard contract subject to a time decay of 6 months. The value of an in the money call on future at maturity equals the intrinsic value, namely 22.080 euro. The option value 6 months prior to expiration totals 26.930 euro (GBM) respectively 23.189 euro (2F). We observe a decrease in value equal to 4.850 euro (GBM) respectively 1.109 (2F). At the money, the call on future exhibits a time decay equal to 13.666 euro (GBM) respectively to 7.326 euro (2F). For an out of the money option the numbers are 5.785 euro (GBM) respectively 1.008 euro (2F). As expected, the option value based on GBM exhibits a much stronger decay along the passage of time than the option value based on 2F. The sensitivity of the option value with respect to time is stronger for the GBM price dynamics in general and for higher levels of volatility in particular. In contrast to this, the theta profile based on 2F does not exhibit strong deviations subject to volatility due to mean reversion. Moreover, we observe the parameters estimated by the market (α=5, σ_1=0.3 and σ_2=0.1) leading to the lowest theta and thus approximating the 'true' volatility best.

3.3. Sensitivity with Respect to Volatility: Vega

Vega denotes the sensitivity of an option with regard to change in volatility. The holder of a call option benefits from a volatility increase and at the same time has a limited downside risk due to the convex payoff structure. In the following, we define vega as the rate of change of the option's value with respect to a finitely small up and down change in volatility.

$$ApproxVega = \frac{1}{2}\left(\frac{\Delta V}{\Delta\sigma^{-}} + \frac{\Delta V}{\Delta\sigma^{+}}\right)$$

As the dynamics of 2F consists of a short and a long term volatility, we need to calculate a short term vega (vega 1) and a long term vega (vega 2). To simplify matters we will use a one sided change in volatility in order to calculate the vegas for the 2F model. We approximate vega 1 and vega 2 as follows:

$$Vega_1 = \frac{\Delta V}{\Delta\sigma_1}$$

$$Vega_2 = \frac{\Delta V}{\Delta\sigma_2}$$

The price dynamics based on GBM assumes a constant volatility level along the passage of time. The volatility presented in the 2F model is determined through the interaction of the long and the short term volatility. Unlike GBM, 2F does not assume a constant but a mean reverting volatility with the short term volatility fluctuating around a long term level[44]. To state it in other words, every forward price exhibits an 'individual' volatility[45]. Both price dynamics, however, have the deterministic specification of the volatility in common.

A deterministic specification of volatility withdraws the uncertainty and eliminates the vega risk[46]. Thus, calculating vega from models with deterministic volatility seems inappropriate. However, one can simulate changes in volatility through parallel shifts of the volatility level. Referring to this procedure, *Hull and White* have shown that the sensitivity results display no substantial difference between models with stochastic and deterministic volatility[47].

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[44] Pilipovic (1998), p. 114.

[45] Pilipovic (1998), p. 117.

[46] Hull (2006), p. 359, p. 361.

[47] Hull (2006), p. 361, Hull and White (1987), p. 146.

Taleb, however, warns that a parallel shift of the volatility level in order to simulate vega risk bears little relevant information due to the mean reverting nature of volatility[48]. To understand the implication, consider the electricity market being hit by an unpredicted shock followed by an excessive increase in prices and volatility[49]. Market participants assume the shock to be temporary und thus to affect prices of options close to expiration only. Therefore, it is rational to assume that options with shorter maturities are more sensitive to volatility shifts than options with longer maturities.

In order to evaluate the vega risk according to the conceptual considerations presented above, we simulate volatility risk through parallel shifts of the volatility level. Unfortunately, a direct comparison between the two price dynamic models is impossible, as the payoff with respect to volatility is charted in a two-dimensional space for GBM and in a three-dimensional space for 2F respectively. Therefore, the vega analysis is performed separately for each price dynamics.

3.3.1. GBM Model

Generally, vega is expressed with respect to the moneyness of an option. *Taleb* postulates a convex vega when the option is from the money and a linear vega for at the money options[50]. *Figure 8* shows the payoff structure and the associated vega profile for the standard contract based on GBM for different maturities (3 months, 6 months and 1 year). The underlying price forward curve is flat at 55 euro and options are referred to as in the money (K=45), at the money (K=55) and out of the money (K=65). The vega risk is simulated through parallel shifts in sigma. The numerical results are presented in the annex.

Figure 8 shows a price increase subject to a rise in volatility, independent of the moneyness of the option. The GBM dynamics implies that the variance is growing proportional to the passage of time. Therefore, we observe the strongest price increase for the option with the longest holding period. The associated vega is higher and thus more sensitive to volatility changes for options with long maturities.

When we analyse *Figure 8*, we find the relationship between the moneyness of an option and its vega as stated by *Taleb* reconfirmed. At the money options show indeed a linear payoff structure indicating an approximately constant volatility sensitivity. As the forward price moves away from the strike price, we obtain a convex relationship between option price and

[48] Taleb (1997), p. 150, Malz (2001), p. 7.

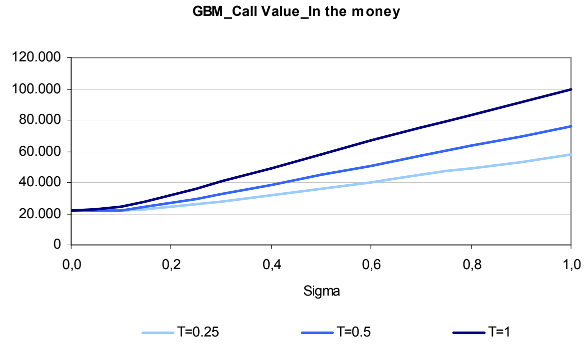
[49] This situation is only valid under the assumption of a positive correlation between price and volatility. Hull and White (1987).

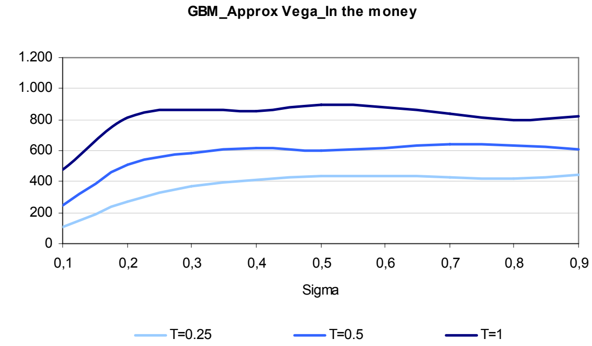
[50] Taleb (1997), p. 148.

volatility. *Figure 8* also shows, that vega is maximum at the money as the option becomes more sensitive when the forward price approaches the strike price.

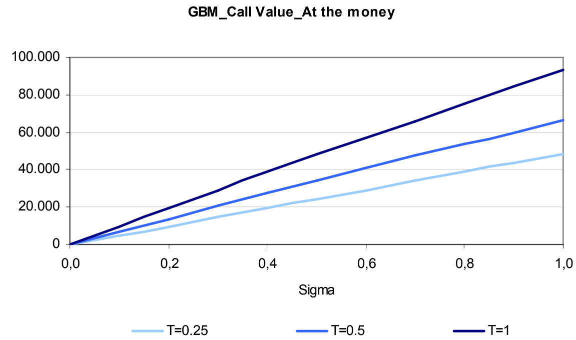
Figure 8: Value and Vega Profile for a Call on Future based on GBM for Different Maturities

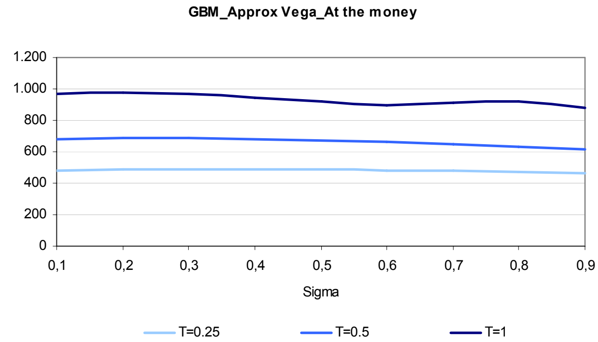
a) In the Money K=45



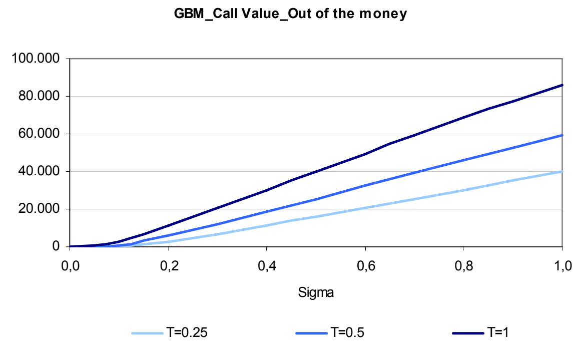


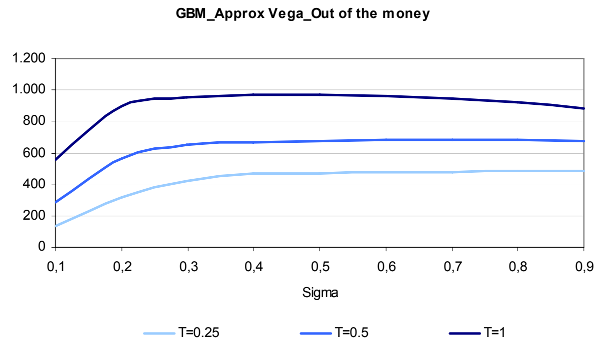
b) At the Money K=55





c) Out of the Money K=65





In the beginning of this chapter, we explained that it is rational to assume options with shorter maturities to be more sensitive to volatility shifts than options with longer maturities. However, *Figure 8* shows that it is not possible to capture the mean reversion feature of volatility with parallel shift of the volatility level. As a matter of fact, our numerical results prove the effect to be converse as they show that the sensitivity with respect to volatility is lower for options that expires in the near future than for options with longer maturities. The price dynamics underlying GBM is not able to replicate more realistic volatility risk.

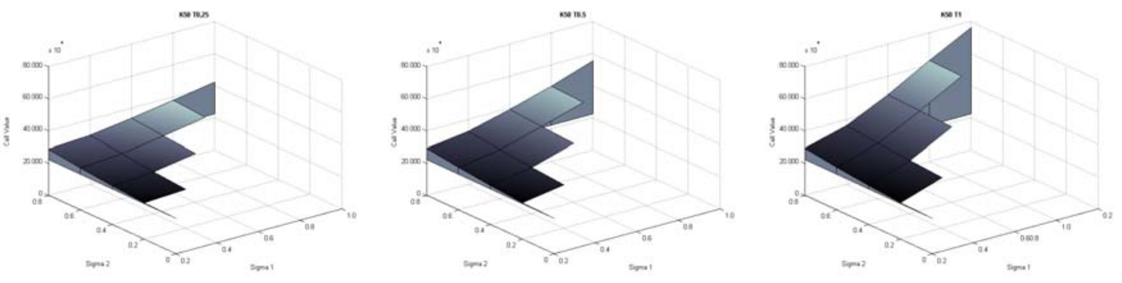
3.3.2. 2F Model

As mentioned before, the volatility of the underlying is composed of the interaction between the short term and the long term volatility linked by the mean reversion parameter α. *Pilipovic*, however, shows that not only the underlying but also the volatility itself follows a mean reversion path[51]. In order to analyze the sensitivity of the call on future with respect to the short term and the long term volatility we price the standard contract for the maturities of 3 months, 6 months and 12 months. The underlying price forward curve is flat at 55 euro and options are referred to as in the money (K=45), at the money (K=55) and out of the money (K=65). The vega risk is simulated through parallel shifts in sigma.

Further, we expect the short term volatility to be lager than the long term volatility due to the mean reverting character as volatility converges toward its mean level. We define the relationship between the short term volatility and the long term volatility strictly as $\sigma_1 > \sigma_2$. The numerical results for vega 1 and vega 2 are presented in the annex. Graphically, we limit our analysis to the demonstration of the option price evolution with respect to σ_1 and σ_2. *Figure 9* displays the obtained results.

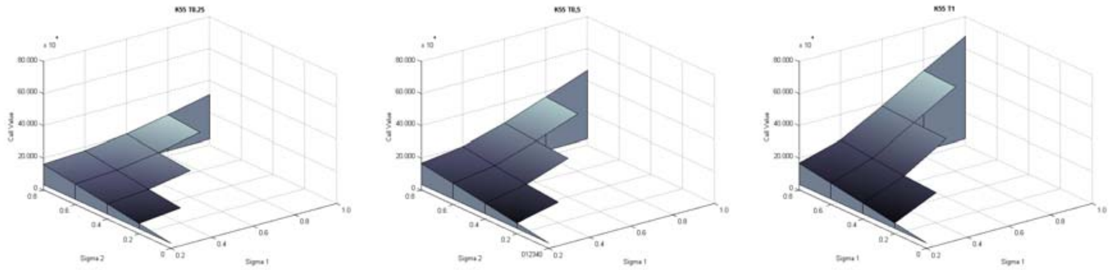
Figure 9: Payoff from a Call on Future based on 2F for Different Maturities

In the Money K=50



[51] Pilipovic (1998), p. 115.

At the Money K=55



Out of the Money K=60

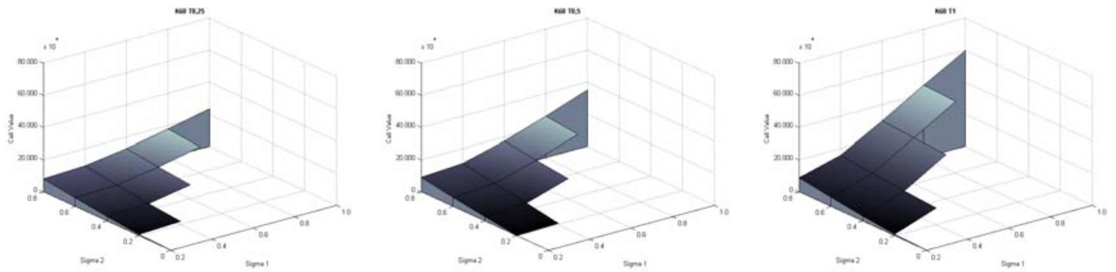


Figure 9 shows the development of the call option value along two axes, sigma 1 and sigma 2. We observe that an increase in short term volatility has a stronger effect on the value of the call on future than an increase in long term volatility. The effect is even more obvious when we look at the time to maturity of one year (left column). In general, the passage of time seems to affect the option value mainly through the short term volatility as we see the option value growing strong along the sigma 1 axis. Therefore, we conclude, that vega 1, the sensitivity with respect to the short term risk, is bigger than vega 2, the sensitivity with respect to long term risk. Moreover, only vega 1 seems to be affected by maturity while vega 2 remains approximately constant.

Furthermore, we observe in *Figure 9* that in the money and out of the money options are convex with respect to both, the short term and the long term volatility. However, when the option nears maturity we notice a decline in convexity along the sigma 1 axis. The vega of an at the money option is expected to be stable to volatility[52]. This statement is only approved with respect to sigma 2 as we observe a linear relationship between option price and long term volatility. The option value, however, is convex with respect to sigma 1. The convexity becomes even stronger when we consider options with longer maturities.

[52] Taleb (1997), p. 148.

4. Value-at-Risk (VaR)

In the previous chapter, we have discussed how electricity companies can track their exposure to individual market risk factors by applying the sensitivity analysis often referred to as ‘the Greeks’. However, the variety of risk factors boosts the complexity of risk management. Value at Risk (VaR) provides a methodology which expresses the total risk exposure in one single number. The concept of VaR has become widely successful among banks and corporations in the first place. Nowerdays, companies involved in electricity trading use the VaR concept as an essential risk management tool[53].

Steiner, Wenninger and Willinsky define VaR as the maximum loss for a portfolio at a confidence level over a specified time frame.[54]. However, the application of the VaR concept to electricity markets should be handled with caution. In the following, we first demonstrate the limits of the traditional VaR approach when applied to electricity markets. Then, we undertake a quasi-sensitivity analysis of the VaR with respect to the risk factors price and volatility.

4.1. Variance-Covariance Methodology

VaR has become widely used since the release of JP Morgan’s Risk Metrics in 1994 providing benchmark data for a series of VaR methodologies. The accuracy of the VaR estimation strongly depends on the chosen technique and the underlying assumptions. *Henney* and *Keers* come to the following conclusion: *"The closer the models match economic reality, the more accurate are the estimated VaR numbers, but they are never completely accurate."*[55]

There are three techniques for calculating VaR: variance-covariance, historical simulation and Monte Carlo simulation. Due to its simplicity, the most commonly used approach is the *variance-covariance* methodology also referred to as *Delta normal* or *analytical* method. The analysis is based on variance-covariance matrix deducted from the volatilities and correlations among the different risk factors of the portfolio. A portfolio’s VaR is given by:

$$VaR_P = z_{(1-\alpha)}\sqrt{\delta^T \sum \delta}$$

where $z_{(1-\alpha)}$ specifies the confidence factor. The variable δ represents the deltas and Σ the variance-covariance matrix of the market risk factors. As we assume the risk factors to be

[53] Weber und Schuler (2006), p. 59.

[54] Steiner, Wenninger und Willinsky (2002), p. 69.

[55] Henney und Keers (1998), p. 38.

normally distributed, the portfolio's P&L distribution is also normal with the following parameters[56]:

$$\Delta V : \aleph(0, \delta ' \sum \delta)$$

The parametric approach of the variance-covariance method bears an important implication: The VaR for any given confidence level α is given by the (1-α)-percentile of a portfolio's return distribution. The implementation of the method involves three major assumptions:[57]: (1) the portfolio is maintained neutral either through a linear delta hedge or a convex delta-gamma hedge, (2) market prices for every product are available and allow for a „mark to market" valuation and (3) the portfolio's P&L is normally distributed[58]. In order to measure the accuracy of the variance-covariance method, we examine the validity of the underlying assumptions when applied electricity markets.

(1) Standard products are insufficient in order to rebalance complex electricity contracts such as swing options. A full neutrality of a position with respect to the underlying price is difficult to obtain. Therefore, risk neutral valuation is not practicable making assumption 1 invalid in electricity markets.

(2) No electricity company marks to market its generation assets. Market prices are rather simulated through the calibration of the price forward curve as mentioned in chapter 2. Strictly speaking, electricity contracts are priced to model rather than to market restricting the validity of assumption 2.

(3) Most electricity derivatives will not map perfectly to a parametric model due to unique P&L distributions. The parametric model, where VaR is calculated under the normal assumption does not represent the true distribution, for example does not account for the probability of a call on future not being exercised. Therefore, assumption 3 is invalid in electricity markets.

The choice of the variance-covariance method to calculate VaR in electricity markets lacks accuracy as we find the three major assumptions violated. Indeed, *Barnwell* points out that if energy companies use VaR the way financial institutions use it, they could essentially underestimate their risk exposure[59].

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[56] Mina und Yi Xiao (2001), p. 24.

[57] Weber und Schuler (2006), p. 60.

[58] Steiner, Wenninger und Willinsky (2002), p. 70.

[59] Barnwell (2001), p. 15.

BIT@EPI.VPP provides an alternative approach for calculating VaR. The multistage stochastic optimization methodology maps the underlying P&L distribution for every electricity contract calculating the VaR as a percentile from the generated distribution.

In the following, we demonstrate the error resulting from the assumption of normal distribution. As an example, we consider two contract types, a call on future and a swing option. *Table 3* provides an summary of the contract details. The swing option is characterised by additional volumetric flexibility allowing the total amount to swing between 3.000 MWh and 3.264 MWh. The strike price is set at 82,55 Euro. The valuation date is October 1st 2006; the delivery period lasts from January 1st to December 31st 2007. The option can be exercised in peak hours only, ranging from 8.00 am to 8.00 pm. The underlying price dynamics is based on the 2F and the GBM model with the parameter values $\sigma1$=30%, $\sigma2$=10% and α=5, respectively σ=20%. We analyze the two contract types with respect to their VaR at 1%, 5% and 10% level of significance.

Table 3: Contract Details

Contract Details	
Contract Specification	Call on Future and Swing Option
Delivery Period	1. July 2007 - 31 December 2007
Valuation Date	1. Apr 07
Power	1 MW
Quality	Peak: Mo-Fr 8.00am - 8.00pm
Model Parameter GBM	σ=0.2
Model Parameter 2F	σ1=0.3, σ2=0.1, α=5

We now compare the P&L distribution calculated by BIT@EPI.VPP with a normal distribution generated with the same parameters. Subsequently, we calculate the VaR estimation difference between these two methodologies demonstrating the error extension under the assumption of normal distribution. *Figure 10* plots the different shapes of probability distributions for the call on future. The corresponding values for VaR at 1%, 5% and 10% confidence level are presented in *table 4.*

Figure 10: P&L Distribution of the Call on Future vs. Normal Distribution

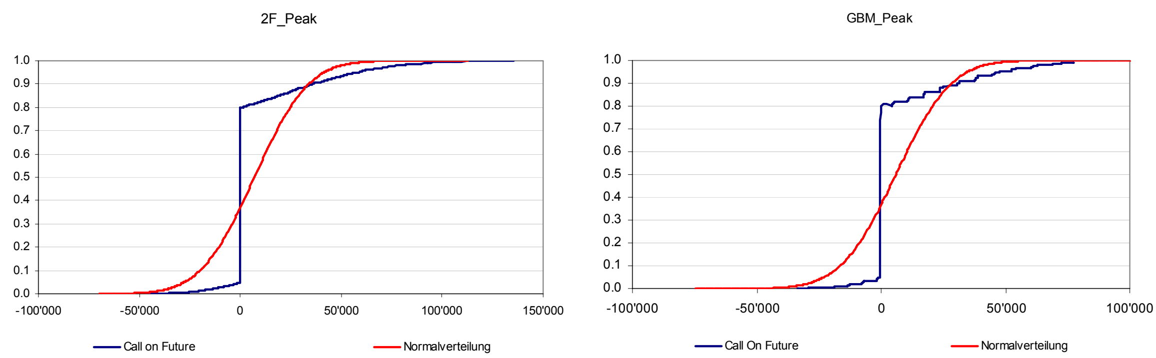


Table 4: Call on Future vs. Normal Distribution: Difference in VaR

	2F			GBM		
Mean	7'501			6'460		
STD	21'016			17'505		
VaR	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**
Call on Future	-22'515	2	2	-18'731	1	1
Normal Distribution	-41'500	-27'500	-19'500	-34'500	-22'500	-16'000
Difference	**18'985**	**27'502**	**19'502**	**15'769**	**22'501**	**16'001**

Figure 10 shows the probability distributions for the call on future with the corresponding values for VaR at 1%, 5% and 10% confidence level presented in *table 4.* We observe a big difference in the shape of the generated probability distributions. The normal distribution is incapable of capturing the step within the P&L distribution of an call on future, which measures the probability of an option not being exercised. The error is maximum at a 5% confidence level. Furthermore, we observe that the error is bigger for the valuation based on the 2F model than on GBM. The tails of the distribution generated by BIT@EPI.VPP are narrower than the tails of the normal distribution.

Figure 11**:** P&L Distribution of the Swing Option vs. Normal Distribution

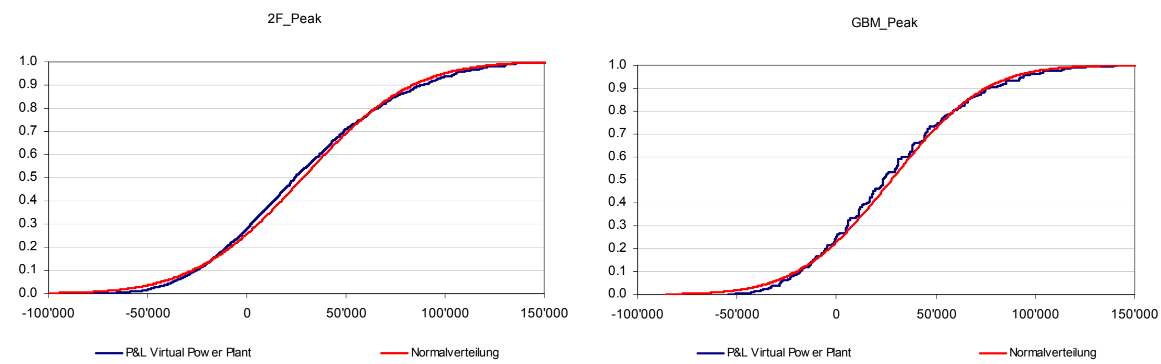


Table 5: Swing Option vs. Normal Distribution: Difference in VaR

	2F			GBM		
Mean	28'275			27'669		
STD	43'349			37'182		
VaR	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**
Call on Future	-55'355	-37'310	-25'490	-42'441	-28'401	-18'482
Normal Distribution	-71'500	-43'000	-27'000	-58'500	-33'500	-20'000
Difference	**17'145**	**-5'690**	**-1'510**	**-16'059**	**-5'099**	**-1'518**

In *figure 11*, we observe a convergence in the shape of the distributions generated with different methodologies. The error is maximum at 1% confidence level an shrinks when the confidence level increases. Similar to the call on future, we observe the tails being narrower for the distribution generated by BIT@EPI.VPP. Moreover, the difference in VaR obtained by the price dynamics 2F and GBM is only slightly distinctive which can be observed in *table 5*.

4.2. Sensitivity of VaR

The software package BIT@EPI.VPP generates not only the fair value of an electricity contract but also the associated P&L distribution including all relevant risk ratios. This feature enables us to carry out a sensitivity analysis with respect to the VaR. In the following, we examine the rate of change of the 10%-VaR with respect to the underlying price and volatility. Therefore, we use the standard contract from *chapter 3* as a basis for our valuation further on.

4.2.1. Sensitivity with Respect to the Price of the Underlying

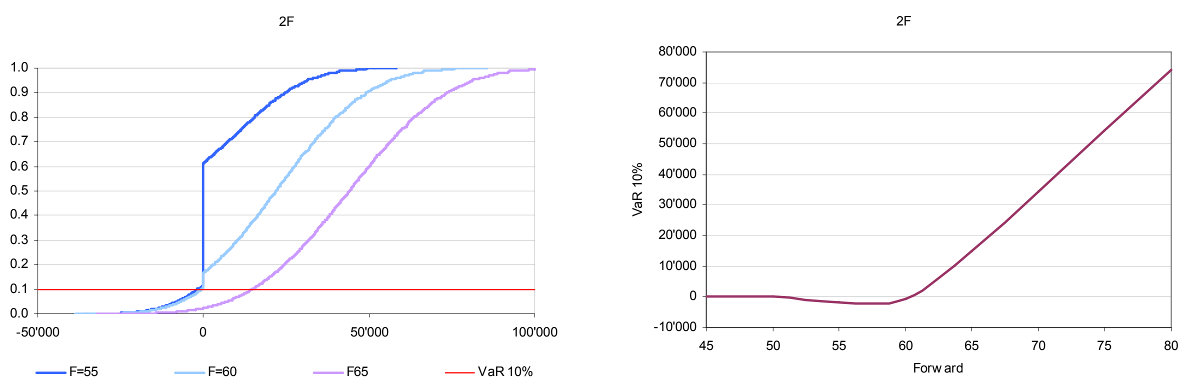
The fair value of an option equals the calculated mean from the underlying risk profile. The modeling of the P&L curve enables us to undertake a risk analysis based on the distribution information. In the following, we examine the rate of change of the VaR at 10% confidence level with respect to the price of the underlying. We set the strike price at 55 Euro and simulate the change in the underlying price through a parallel shift of the price forward curve. The numerical results are presented in the annex.

First, we consider the evolution of VaR under the assumption of 2F as represented in *figure 12 a)*. The relationship between the VaR and the forward price is not monotonic in contrast to the relationship between option price and the price of the underlying. With an increase in the underlying price chances are rising that the option will be exercised at maturity. Therefore, the VaR can become negative, which is the case for the forward price of 55 Euros.

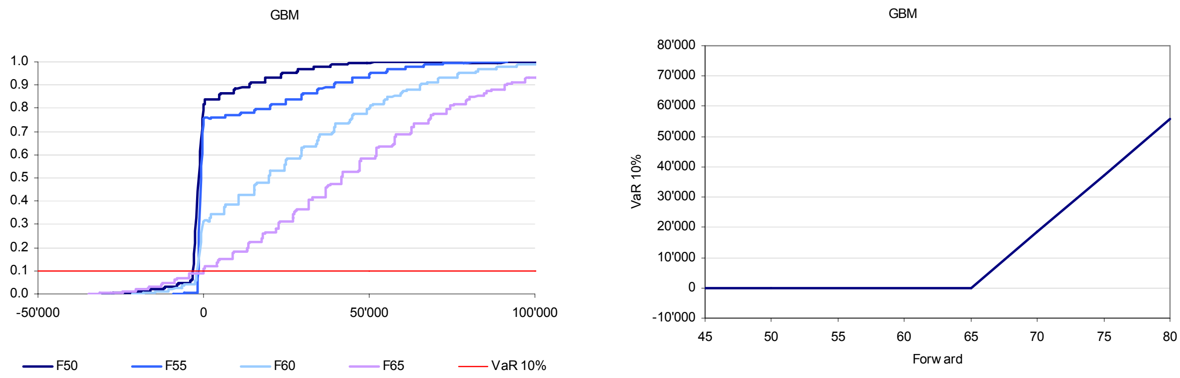
When we consider the P&L distribution of options priced under the assumption of GBM in *figure 12 b)*, the 10th percentile is located within the step of all functions in question. Therefore, we observe the VaR being monotonic at 10% confidence level. However, this result does not hold when it comes to VaR at 5% confidence level.

Figure 12: VaR 10% with Respect to the Forward Price

a) 2F



b) GBM



4.2.2. VaR Sensitivity with Respect to Volatility

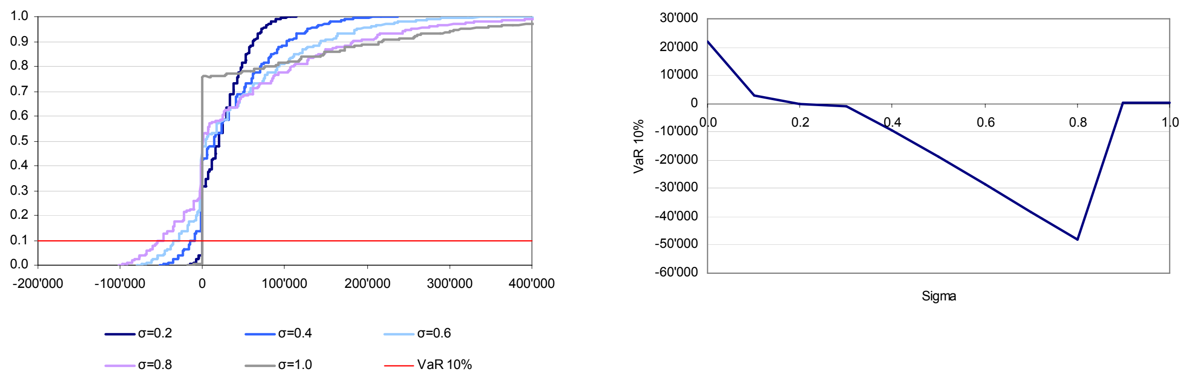
In order to evaluate the VaR sensitivity with respect to volatility, we simulate volatility risk through parallel shifts of the volatility level. Unfortunately, a direct comparison between the two price dynamic models is impossible, as the payoff with respect to volatility is charted in a two-dimensional space for GBM and in a three-dimensional space for 2F respectively. Therefore, we perform the analysis separately for each price dynamics.

4.2.2.1 GBM

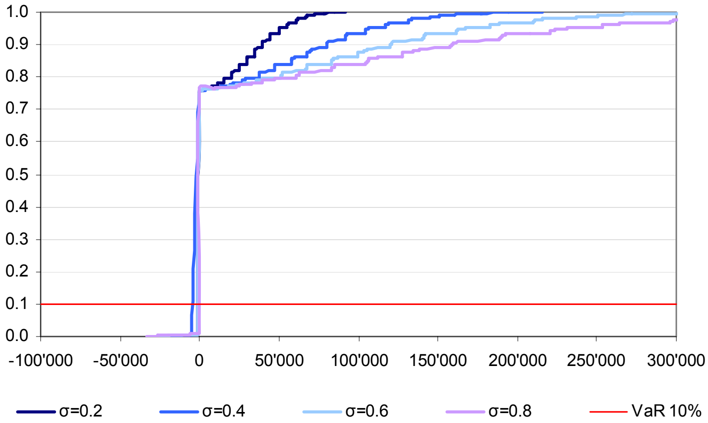
Generally, the sensitivity is expressed with respect to the moneyness of an option. Therefore, options are referred to as in the money (K=50), at the money (K=55) and out of the money (K=60). The underlying price forward curve is flat at 55 euro and the volatility risk is simulated through parallel shifts in sigma. The numerical results are presented in the annex.

Figure 13: GBM: VaR 10% with Respect to Volatility

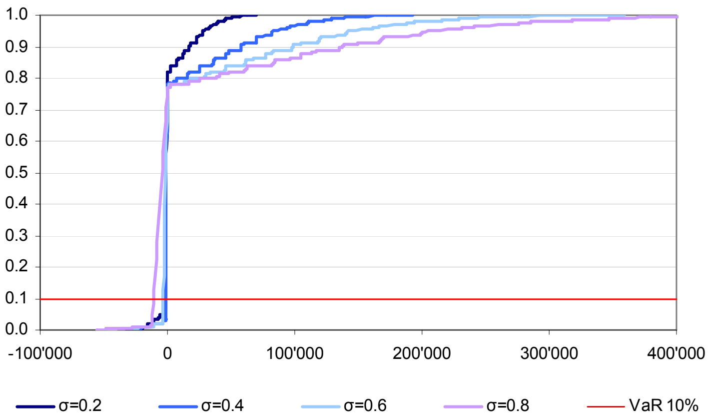
a) In-the-Money K=50



b) At-the-Money K=55



c) Out-of-the-Money K=60



The VaR at 10% confidence level is equal to the 10^{th} percentile of the distribution. The exact location is represented by the intersection of the distribution function with the horizontal red line at plotted in the figure. When it comes to at the money and out of the money options as represented in *figure 13b)* and *13c)* we see that the 10^{th} percentile is located in the step of the distribution, therefore is approximately equal to zero.

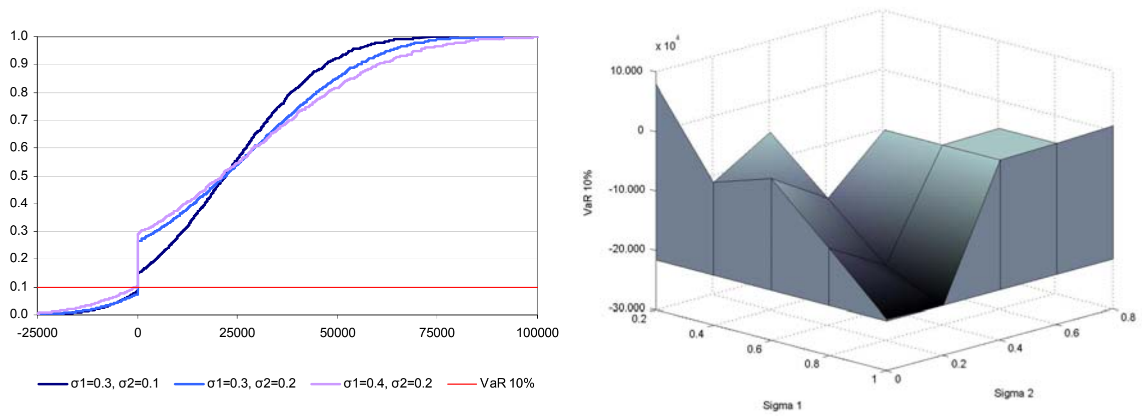
Figure 13a) shows the P&L distribution of an in the money option. A volatility increase results in a parallel upward shift of the distribution curve with the exercise probability remaining constant. As the 10^{th} percentile is located in the left part of the distribution, the VaR changes with respect to volatility in a non-monotonic way.

4.2.2.2 2F

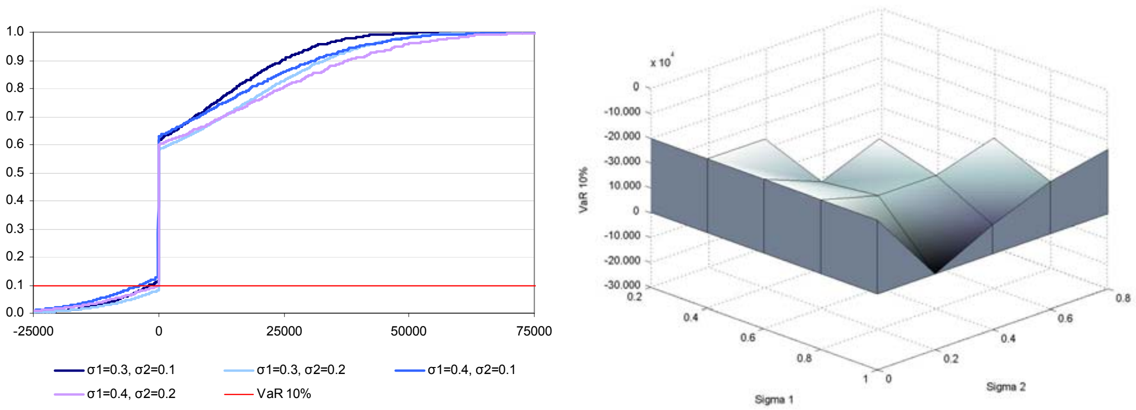
The P&L distribution for options in the money, at the money and out of the money as well as the associated VaR pattern are shown in *figure 14*. The options are split according to their moneyness. The numerical results are presented in the annex.

Figure 14: 2F: VaR 10% with Respect to Volatility

a) In-the-Money K=50



b) At-the-Money K=55



c) Out-of-the-Money K=60

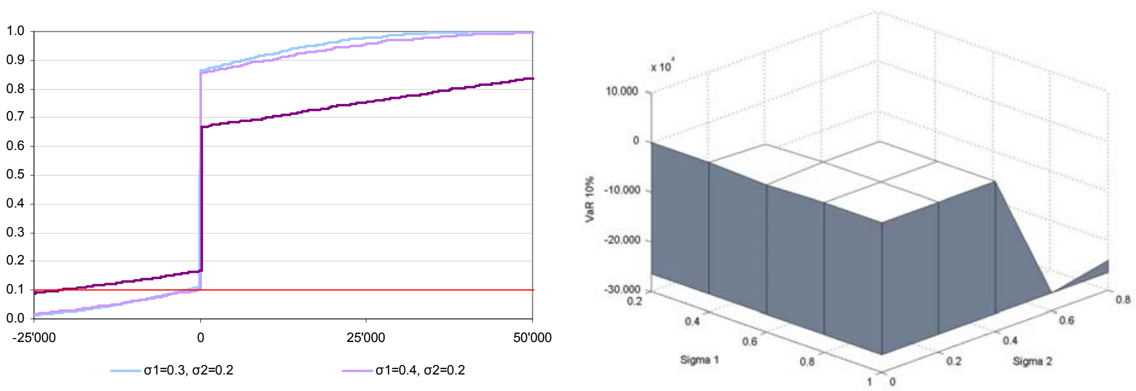


Figure 14 a) plots the P&L distribution of options in the money. An increase in short term or long term volatility raises the chance of an option to expire without being exercised. Therefore, the associated VaR pattern is not monotonic.

At the money, as shown in *figure 14 b)*, a change in volatility has approximately no effect on the exercise probability of a call on future. The VaR sensitivity is therefore lower than for options from the money.

F*igure 14 c)* shows out of the money options. Only a strong increase in volatility, namely to $\sigma 1=0.8$ and $\sigma 2=0.6$, has an impact on the exercise probability of the out of the money option. Therefore, the VaR profile is approximately at zero for lower levels of volatility and collapses when parameter values grow significantly.

5. Model Tests

5.1. Test with Respect to Additivity

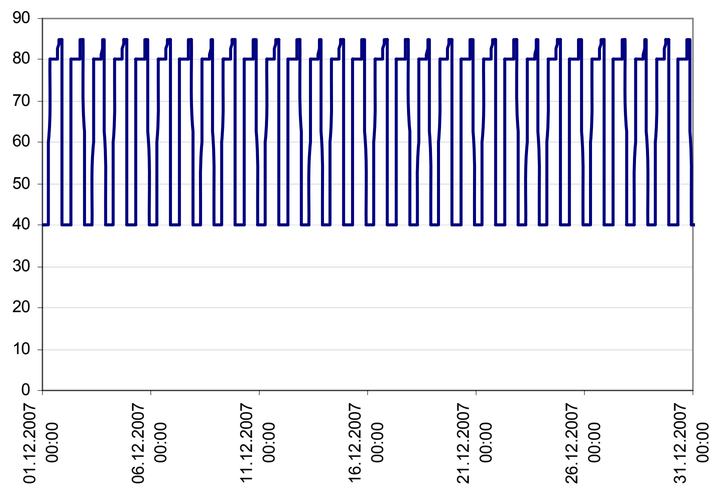
In the following, we examine the additivity of electricity contracts priced under different price models. The underlying price dynamics are based on the 2F model and the GBM model with the parameter values σ1=30%, σ2=10% and α=5, respectively σ=20%. We analyze the contracts 1-4 with the characteristics presented in *table 6*.

Table 6: Characteristic of Contracts 1-4 with Delivery in December 2007

	Valuation Date	Delivery Period	Quality	Power	Strike
Contract 1	1. Nov 07	Dec 07	Mo-Fr 08.00-17.79	1MW	80 Euro
Contract 2	1. Nov 07	Dec 07	Mo-Fr 18.00-21.59	1MW	85 Euro
Contract 3	1. Nov 07	Dec 07	Mo-Fr 22.00-07.59	1MW	40 Euro
Contract 4	1. Nov 07	Dec 07	Mo-Fr 00.00-23.59	1MW	variable

Contracts 1-3 are constructed in a way that no overlapping in quality time occurs whereas the sum of quality hours of contracts 1 to 3 equals the number of quality hours of contract 4. Moreover, the strike prices for contracts 1-3 differ according to the quality time they cover. When it comes to valuation 4, we build a variable strike price function which captures the strike price differences in the block hours. *Figure 15* illustrates the run of the variable strike price which is taken as a basis for valuation of contract 4.

Figure 15: Variable Strike Price



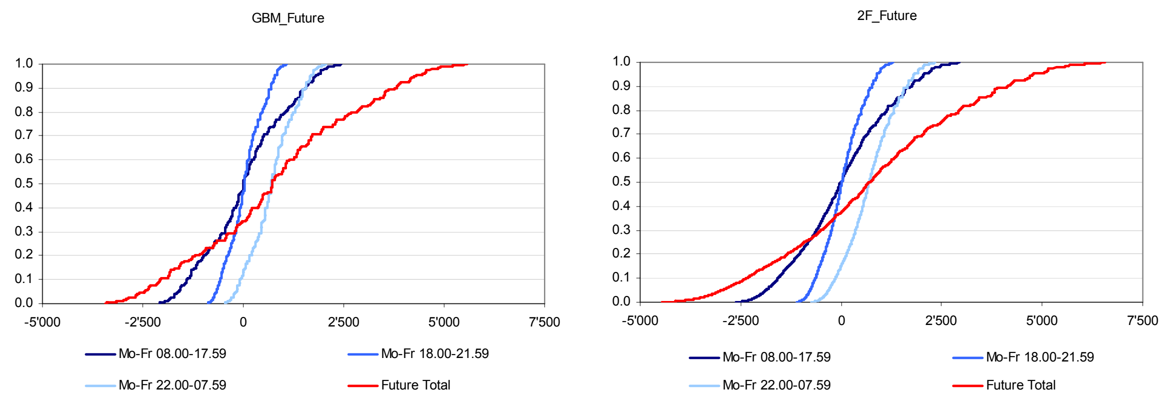
5.1.1. Future

In this chapter, we analyze future contracts with the characteristics described above. All results are presented in *table 7* and *figure 16*.

Table 7: Summary of the Future Contract Evaluation with Delivery in December 2007

Future December	Quality	Volume MWh	Price	VaR 10%	STD
GBM	Mo-Fr 08.00-17.79	210	43	-1'415	1'044
	Mo-Fr 18.00-21.59	84	39	-586	446
	Mo-Fr 22.00-07.59	210	724	-69	569
	Sum	504	806	-2'071	2'059
	Mo-Fr 00.00-23.59	504	806	-2'080	2'059
2F	Mo-Fr 08.00-17.79	210	43	0	1'190
	Mo-Fr 18.00-21.59	84	39	-644	508
	Mo-Fr 22.00-07.59	210	724	0	646
	Sum	504	806	-2'349	2'344
	Mo-Fr 00.00-23.59	504	806	-2'348	2'344

Figure 16: P&L- Distribution of Future Contracts 1-4 based on GBM and 2F



In a first step, we sum up volume, price, VaR at 10% confidence level and standard deviation of contracts 1-3. In a second step, we contrast the obtained sums with the results we calculated for contract 4. *Table 7* shows that the mean and the standard deviations are equal resulting from a perfect correlation among the contracts. The values for 10%-VaR slightly diverge from each other.

We assume that perfect correlation among future contracts does only appear within narrow time intervals. In order to prove the assumption, we construct four additional contracts with delivery time in January 2008. *Table 8* sums up the characteristics of the newly created contracts 5-8 and table 9 shows their evaluation. Again, we observe additivity in price, standard deviation and VaR for contracts with adjacent block hours.

Table 8: Characteristic of Contracts 5-8 with Delivery in January 2008

	Valuation Date	Delivery Period	Quality	Power	Strike
Contract 5	1. Nov 07	Jan 07	Mo-Fr 08.00-17.79	1MW	80 Euro
Contract 6	1. Nov 07	Jan 07	Mo-Fr 18.00-21.59	1MW	85 Euro
Contract 7	1. Nov 07	Jan 07	Mo-Fr 22.00-07.59	1MW	40 Euro
Contract 8	1. Nov 07	Jan 07	Mo-Fr 00.00-23.59	1MW	variable

Table 9: Summary of the Future Contract Evaluation with Delivery in January 2008

Future January	Quality	Volume MWh	Price	VaR 10%	STD
GBM	Mo-Fr 08.00-17.79	230	1'340	-1'028	1'728
	Mo-Fr 18.00-21.59	92	425	-564	721
	Mo-Fr 22.00-07.59	230	1'187	-57	908
	Sum	552	2'952	-1'649	3'357
	Mo-Fr 00.00-23.59	552	2'952	-1'649	3'357
2F	Mo-Fr 08.00-17.79	230	1'340	-865	1'657
	Mo-Fr 18.00-21.59	92	425	-496	691
	Mo-Fr 22.00-07.59	230	1'187	28	871
	Sum	552	2'952	-1'332	3'219
	Mo-Fr 00.00-23.59	552	2'952	-1'332	3'219

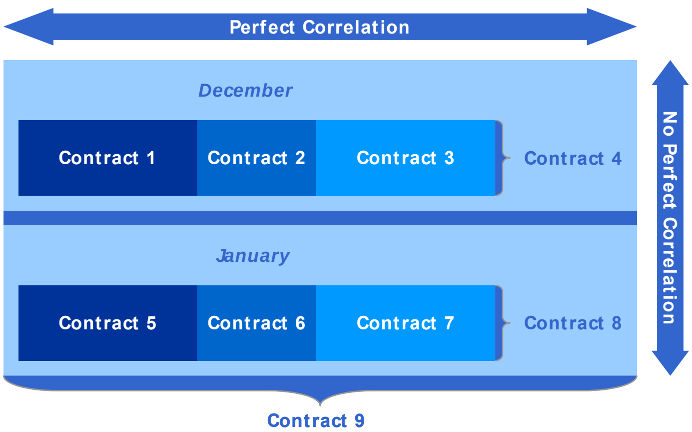
In a third step, we compare the sum of contracts 4 and 8 which deliver electricity in December and January, respectively with contract 9, which delivers electricity in both months, December and January. As valuation date, we chose 1st November 2007 and the strike price is variable allowing for different strike prices at different quality hours.

Table 10: Summary of the Future Contract Evaluation with Delivery in December 07 and January 08

Future Dec/Jan	Quality Mo-Fr 00.00-23.59	Volume MWh	Price	VaR 10%	STD
GBM	Dec	504	806	-2'028	2'059
	Jan	552	2'952	-1'649	3'357
	Sum	1056	3'758	-3'729	5'416
	Dec & Jan	1056	3'758	-2'799	5'052
2F	Dec	504	806	-2'348	2'344
	Jan	552	2'952	-1'332	3'319
	Sum	1056	3'758	-3'680	5'562
	Dec & Jan	1056	3'758	-2'752	5'063

Table 10 shows that the prices are still additive while the values for standard deviation and the VaR at 10% confidence level diverge due to the lack of perfect correlation. Perfect correlation only occurs when contracts' quality blocks are adjacent. If the delivery period lies apart, the criteria for perfect correlation are invalid. Then, we only observe additivity in price but not in standard deviation or VaR. *Figure 17* provides an overview over the obtained results.

Figure 17: Summary of the Results with Regard to Additivity



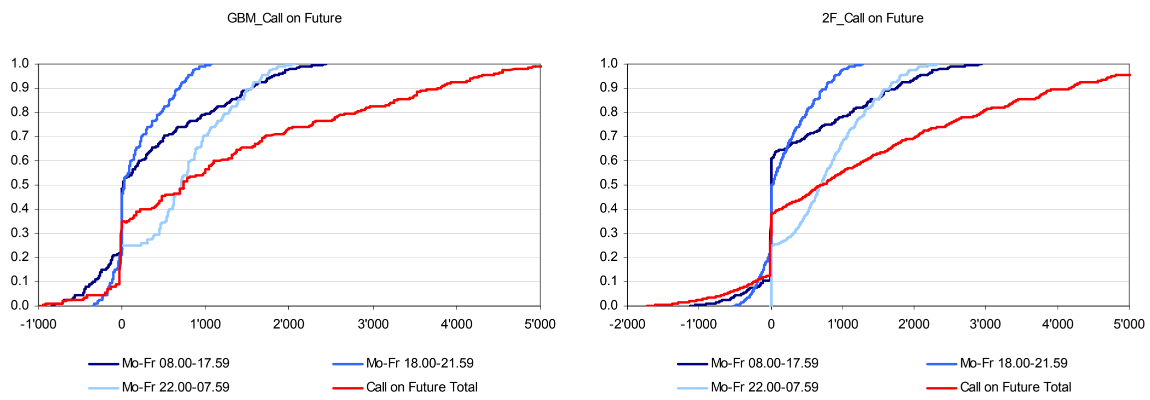
5.1.2. Call on Future

The evaluation of the call on future contracts 1-4 is summed up in *table 11* and *figure 18.*

Table 11: Evaluation of the Call on Future Contracts 1-4

Call on Future	Quality	Volume MWh	Price	VaR 10%	STD
GBM	Mo-Fr 08.00-17.79	210	367	-328	711
	Mo-Fr 18.00-21.59	84	172	-117	307
	Mo-Fr 22.00-07.59	210	724	0	560
	Sum	504	1'262	-445	1'577
	Mo-Fr 00.00-23.59	504	1'262	0	1'526
2F	Mo-Fr 08.00-17.79	210	412	-113	781
	Mo-Fr 18.00-21.59	84	186	-181	357
	Mo-Fr 22.00-07.59	210	740	0	610
	Sum	504	1'338	-295	1'748
	Mo-Fr 00.00-23.59	504	1'326	-211	1'750

Figure 18: P&L Distribution of the Call on Future Contracts 1-4 based on GBM and 2F



We first consider the valuation based on GBM as shown in *table 11.* The results suggest that call on future contracts are additive in price but not in risk. However, the result, which we obtain for the price, is specific to the contract and does not apply in general. Hence, *table 11* also shows, that call on future contracts based on 2F do not exhibit additivity in price.

When it comes to call on future, the additivity of contracts is not naturally given due to different exercise probabilities of the contracts. *Figure 18* shows that the exercise probability of contracts – as measured by the step in the function – differ substantially preventing the price from being additive. Moreover, the call on future contracts lack additivity in both, standard deviation and VaR.

5.1.3. Virtual Power Plant

When the contracts are priced as Virtual Power Plants or swing options additional volumetric flexibility is embedded in the contracts by allowing the amount to swing between ± 10 MWh around the deterministic volume of the future contract. When it comes to flexibility with respect to timing, the power constraints are allowed to swing between 0 and 2 MW. Table 12 presents the evaluation of the VPP contracts 1-4.

Table 12: Summary of the VPP Contracts 1-4

Virtual Power Plant	Quality	Power MW	Volume MWh	Price	VaR 10%	STD
GBM	Mo-Fr 08.00-17.79	0-2	200-220	3'472	1'880	1'199
	Mo-Fr 18.00-21.59	0-2	74-94	1'540	834	535
	Mo-Fr 22.00-07.59	0-2	200-220	2'545	1'665	658
	Sum	0-2	474-534	7'557	4'378	2'392
	Mo-Fr 00.00-23.59	0-2	474-534	7'008	3'739	2'518
2F	Mo-Fr 08.00-17.79	0-2	200-220	3'479	1'563	1'434
	Mo-Fr 18.00-21.59	0-2	74-94	1'545	690	630
	Mo-Fr 22.00-07.59	0-2	200-220	2'552	1'517	767
	Sum	0-2	474-534	7'576	3'770	2'830
	Mo-Fr 00.00-23.59	0-2	474-534	1'028	3'095	2'979

Due to the flexibility in volume and timing, it is difficult to derive a conclusion about additivity as we do not know the exact future exercise strategy of the contract. From the theoretical point of view, the arguments we derived for a call on future must also hold for a swing option.

5.2. Test with Respect to Put-Call Parity

The put-call parity describes the relationship between the price of a call option and the price of a put option when they have the same strike price and time to maturity. Hull defines the put-call parity with the following equation:[60]:

$$C - P = S_0 - Ke^{-rT}$$

This formula reflects the relationship between the call option C and the put option P from today's point of view. As we base our option valuation on the forward price rather than on the spot price, we must modify the equation. First, we assume that the risk-free interest rate is zero. Second, the relationship must hold irrespective of the delivery period and we must adjust the equation by the number of exercise hours *h.* In the following the put-call parity for electricity is given by the following equation:

$$\frac{C - P}{h} = F - K$$

Geman states that the key feature of the put-call parity is that it is based on the no-arbitrage assumption only, making statements about the future probability distribution and price dynamics redundant.[61]. In practice, however, filtration of the price process plays an important role within the stochastic optimization as it generates different price scenarios. Moreover, vectors of different dimensions represent the hourly exercise strategy for the contracts in

[60] Vgl. Hull (2006), S. 212.

[61] Vgl. Geman (2005), S. 82.

question[62]. A delivery period of 1 month is characterized by a 253-dimensional vector whereas a vector for a delivery period of 12 months (4th Quarter) has the dimension 792. By extending the length of the delivery period we simultaneously extend the dimension of the decision problem. Note that the multistage stochastic programming generates an individual non-trivial result for the call option and put option, respectively.

In the following, we examine the accuracy of put-call parity relationship when electricity contracts are priced with BIT@EPI.VPP. We consider the following contract: The delivery period is set to December 2007 (1 month) and 4th Quarter 2007 (3 months), respectively. The option can be exercised in peak hours from 8.00 am to 8.00 pm. Thus, the number of exercise hours totals 252 in December and 792 in the 4th Quarter. The power limit equals 1 MW and the forward price is held constant at 55 Euro. Moreover, we price the contracts at two different points in time so that the time to expiration is set to 1 month and to 12 months, respectively.

The valuation is based on the 2F model introduced in chapter 2.2. The volatility coefficients corresponding to the short and long term fluctuations are set to σ1=30% and σ2=10%, to σ_1=40% and σ_2=20% as well as to σ_1=60% and σ_2=40%, respectively. The mean-reversion parameter α is set to 5. In order to benchmark the obtained results, we undertake an additional valuation based on the GBM with constant volatilities of 10%, 20%, 30%, 40% and 50%.

5.2.1. GBM: Put-Call Parity for Delivery in December 2007

The results for delivery in December 2007 with the time to maturity of 1 month and 12 months are represented in *table 13* and *table 14*. The valuation is based on the GBM price dynamics. We observe that the put-call parity holds for all electricity contracts in question with 100% accuracy.

[62] Blöchlinger, Frauendorfer, Haarbrückner (2006), p. 10.

Table 13:Put-Call Parity based on GBM: Delivery Period December 2007/ Holding Period 1 Month

Call on Future		Volatility				
	Strike	10%	20%	30%	40%	50%
	40	3'780.01	3'780.01	3'780.01	3'780.01	3'780.01
	45	2'520.01	2'520.01	2'520.01	2'520.01	2'554.67
	50	1'260.00	1'260.01	1'353.16	1'483.03	1'609.67
	55	141.89	286.51	433.72	573.26	735.38
	60	0.00	0.00	118.72	268.39	420.38
	65	0.00	0.00	0.00	0.00	105.35
Put on Future		Volatility				
	Strike	10%	20%	30%	40%	50%
	40	0.00	0.00	0.00	0.00	0.00
	45	0.00	0.00	0.00	0.00	34.66
	50	0.00	0.00	93.15	223.03	349.66
	55	141.89	286.51	433.72	573.26	735.38
	60	1'260.00	1'260.00	1'378.72	1'528.40	1'680.38
	65	2'520.01	2'520.01	2'520.01	2'520.01	2'625.38

(C-P)/252		Volatility					
	Strike	10%	20%	30%	40%	50%	**F-K**
	40	15.00	15.00	15.00	15.00	15.00	15.00
	45	10.00	10.00	10.00	10.00	10.00	10.00
	50	5.00	5.00	5.00	5.00	5.00	5.00
	55	0.00	0.00	0.00	0.00	0.00	0.00
	60	-5.00	-5.00	-5.00	-5.00	-5.00	-5.00
	65	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00

Table 14: Put-Call Parity based on GBM: Delivery Period December 2007/ Holding Period 12 Months

Call on Future		Volatility				
	Strike	10%	20%	30%	40%	50%
	40	3'780.01	3'780.01	4'131.63	4'479.68	4'783.55
	45	2'520.01	2'795.13	3'186.62	3'534.68	3'838.55
	50	1'416.94	1'850.12	2'241.62	2'589.68	3'060.39
	55	506.47	1'041.88	1'599.24	2'170.24	2'745.39
	60	191.47	726.88	1'284.24	1'855.24	2'430.39
	65	0.00	411.88	969.24	1'540.24	2'115.39
Put on Future		Volatility				
	Strike	10%	20%	30%	40%	50%
	40	0.00	0.00	351.61	699.67	1'003.54
	45	0.00	275.12	666.61	1'014.67	1'318.54
	50	156.94	590.12	981.62	1'329.67	1'800.39
	55	506.47	1'041.88	1'599.24	2'170.24	2'745.39
	60	1'451.48	1'986.89	2'544.25	3'115.24	3'690.40
	65	2'520.01	2'931.89	3'489.25	4'060.25	4'635.40

(C-P)/252		Volatility					
	Strike	10%	20%	30%	40%	50%	**F-K**
	40	15.00	15.00	15.00	15.00	15.00	15.00
	45	10.00	10.00	10.00	10.00	10.00	10.00
	50	5.00	5.00	5.00	5.00	5.00	5.00
	55	0.00	0.00	0.00	0.00	0.00	0.00
	60	-5.00	-5.00	-5.00	-5.00	-5.00	-5.00
	65	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00

5.2.2. GBM: Put-Call Parity for Delivery in 4th Quarter 2007

In the following, we analyze the impact of a delivery period extension on the accuracy of the previously obtained results. Therefore, we extend the delivery period to 3 months (4th Quarter 2007) with time to maturities of 1 month and 12 months, respectively. The results are represented in *table 15* and *table 16*. The valuation is based on the GBM price dynamics. We observe that the put-call parity still holds for all electricity contracts in question with 100% accuracy.

Table 15:: Put-Call Parity based on GBM: Delivery Period 4th Quarter 2007/ Holding Period 1 Month

Call on Future		Volatility					
	Strike	10%	20%	30%	40%	50%	
	40	11'880.00	11'880.00	11'880.00	11'880.00	11'880.00	
	45	7'920.00	7'920.00	7'920.00	7'920.00	8'028.93	
	50	3'960.00	3'960.00	4'252.77	4'660.94	5'058.93	
	55	445.94	900.45	1'363.12	1'833.52	2'311.16	
	60	0.00	0.00	373.13	843.52	1'321.16	
	65	0.00	0.00	0.00	0.00	331.17	
Put on Future		Volatility					
	Strike	10%	20%	30%	40%	50%	
	40	0.00	0.00	0.00	0.00	0.00	
	45	0.00	0.00	0.00	0.00	108.93	
	50	0.00	0.00	292.77	700.94	1'098.93	
	55	445.94	900.45	1'363.12	1'833.52	2'311.16	
	60	3'960.00	3'960.00	4'333.13	4'803.52	5'281.16	
	65	7'920.00	7'920.00	7'920.00	7'920.00	8'251.17	
(C-P)/792		Volatility					
	Strike	10%	20%	30%	40%	50%	**F-K**
	40	15.00	15.00	15.00	15.00	15.00	15.00
	45	10.00	10.00	10.00	10.00	10.00	10.00
	50	5.00	5.00	5.00	5.00	5.00	5.00
	55	0.00	0.00	0.00	0.00	0.00	0.00
	60	-5.00	-5.00	-5.00	-5.00	-5.00	-5.00
	65	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00

Table 16: Put-Call Parity based on GBM: Delivery Period 4th Quarter 2007/ Holding Period 12 Months

Call on Future	Volatility					
Strike	10%	20%	30%	40%	50%	
40	11'880.00	11'880.00	12'985.09	14'078.96	15'033.94	
45	7'920.00	8'784.68	10'015.09	11'108.95	12'063.93	
50	4'453.25	5'814.67	7'045.08	8'138.95	9'618.27	
55	1'591.78	3'274.49	5'026.18	6'820.72	8'628.28	
60	601.78	2'284.50	4'036.19	5'830.73	7'638.28	
65	0.00	1'294.51	3'046.20	4'840.73	6'648.29	
Put on Future	Volatility					
Strike	10%	20%	30%	40%	50%	
40	0.00	0.00	1'105.09	2'198.96	3'153.94	
45	0.00	864.68	2'095.09	3'188.95	4'143.93	
50	493.25	1'854.67	3'085.08	4'178.95	5'658.27	
55	1'591.78	3'274.49	5'026.18	6'820.72	8'628.28	
60	4'561.78	6'244.50	7'996.19	9'790.73	11'598.28	
65	7'920.00	9'214.51	10'966.20	12'760.73	14'568.29	
(C-P)/792	Volatility					
Strike	10%	20%	30%	40%	50%	**F-K**
40	15.00	15.00	15.00	15.00	15.00	15.00
45	10.00	10.00	10.00	10.00	10.00	10.00
50	5.00	5.00	5.00	5.00	5.00	5.00
55	0.00	0.00	0.00	0.00	0.00	0.00
60	-5.00	-5.00	-5.00	-5.00	-5.00	-5.00
65	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00

5.2.3. 2F: Put-Call Parity for Delivery in December 2007

Analogue to the analysis based on the GBM price dynamics, we price the contract under the assumption of the 2F model. First, we consider the contracts with the delivery period in December 2007. The results are presented in *table 17* and *table 18*.

Table 17: Put-Call Parity based on 2F: Delivery Period December 2007/ Holding Period 1 Month

Call on Future	Volatility			
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%	
45	2'520.01	2'520.01	2'532.35	
50	1'261.41	1'339.52	1'534.13	
55	315.82	433.75	670.75	
60	0.00	103.64	329.59	
65	0.00	0.00	14.59	
Put on Future	Volatility			
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%	
45	0.00	0.00	12.34	
50	1.40	79.51	274.12	
55	315.82	433.75	670.75	
60	1'260.00	1'363.64	1'589.60	
65	2'520.01	2'520.01	2'534.60	
(C-P)/252	Volatility			
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%	**F-K**
45	10.00	10.00	10.00	10.00
50	5.00	5.00	5.00	5.00
55	0.00	0.00	0.00	0.00
60	-5.00	-5.00	-5.00	-5.00
65	-10.00	-10.00	-10.00	-10.00

Table 18: Put-Call Parity based on 2F: Delivery Period December 2007/ Holding Period 12 Months

Call on Future	Volatility		
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%
45	2'520.01	2'802.96	3'459.91
50	1'496.90	1'857.96	2'534.21
55	612.67	1'052.43	2'040.79
60	284.89	737.43	1'725.79
65	0.00	422.43	1'410.79

Put on Future	Volatility		
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%
45	0.00	282.95	939.00
50	236.89	597.95	1'274.21
55	612.67	1'052.43	2'040.79
60	1'544.89	1'997.44	2'985.79
65	2'520.01	2'942.44	3'930.80

(C-P)/252	Volatility			
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%	**F-K**
45	10.00	10.00	10.00	10.00
50	5.00	5.00	5.00	5.00
55	0.00	0.00	0.00	0.00
60	-5.00	-5.00	-5.00	-5.00
65	-10.00	-10.00	-10.00	-10.00

We observe that the put-call parity holds for all electricity contracts in question with 100% accuracy under the assumption of 2F price dynamics.

5.2.4. 2F: Put-Call Parity for Delivery in 4th Quarter 2007

In this paragraph, we extend the delivery period to 3 months in order to examine the impact of longer delivery periods on the accuracy of the put-call parity relationship, keeping the assumption of 2F price dynamics. *Table 19* and *table 20* show the numerical results.

Table 19: Put-Call Parity based on 2F: Delivery Period 4th Quarter 2007/ Holding Period 1 Month

Call on Future	Volatility		
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%
45	7'920.00	7'920.00	7'976.31
50	3'960.00	4'052.05	4'508.09
55	832.22	1'201.42	1'944.78
60	0.00	18.98	581.43
65	0.00	0.00	0.00

Put on Future	Volatility		
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%
45	0.00	0.00	56.31
50	0.00	92.05	548.09
55	832.22	1'201.42	1'944.78
60	3'960.00	3'978.98	4'541.43
65	7'920.00	7'920.00	7'920.00

(C-P)/792	Volatility			
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%	**F-K**
45	10.00	10.00	10.00	10.00
50	5.00	5.00	5.00	5.00
55	0.00	0.00	0.00	0.00
60	-5.00	-5.00	-5.00	-5.00
65	-10.00	-10.00	-10.00	-10.00

Table 20: Put-Call Parity based on 2F: Delivery Period 4th. Quarter 2007/ Holding Period 12 Months

Call on Future	Volatility			
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%	
45	7'935.92	8'652.26	10'723.00	
50	4'509.67	5'668.05	8'242.58	
55	1'893.23	3'374.98	6'262.58	
60	669.48	2'093.91	5'172.73	
65	0.00	1'103.92	4'182.74	
Put on Future	Volatility			
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%	
45	15.92	732.26	2'803.00	
50	549.67	1'708.05	4'282.58	
55	1'893.23	3'374.98	6'262.58	
60	4'629.48	6'053.91	9'132.73	
65	7'920.00	9'023.92	12'102.74	
(C-P)/792	Volatility			
Strike	σ_1=30%, σ_2=10%	σ_1=40%, σ_2=20%	σ_1=60%, σ_2=40%	**F-K**
45	10.00	10.00	10.00	10.00
50	5.00	5.00	5.00	5.00
55	0.00	0.00	0.00	0.00
60	-5.00	-5.00	-5.00	-5.00
65	-10.00	-10.00	-10.00	-10.00

We observe that the put-call parity holds for longer delivery periods and under the assumption of 2F price dynamics with 100% accuracy.

6. Conclusion

In this paper, we have introduced electricity derivatives by describing the modelling framework and outlined the difficulties when it comes to pricing power contracts. We have discussed GBM, the standard model from financial markets and have introduced a more realistic model for the approximation of electricity prices, the 2F model. The key extension of 2F was the inclusion of a stochastic long term level when modelling a mean reverting price path. Finally, we showed an alternative approach to measure risk, which is commonly referred to as the ‘Greeks’. We have undertaken a detailed sensitivity analysis of each relevant risk dimension. In the following we highlight the main results.

Delta: Sensitivity with respect to price

- At the money, the delta becomes flatter as the option maturity and the volatility increase. For deep from the money options the exact opposite is the case.
- At the money, the delta based on the 2F model reacts more sensitive to small changes in the underlying price than the delta based on GBM. For deep in the money and deep out of the money options the relationship is reversed.

Gamma: Sensitivity of second order with respect to price

- At the Money, gamma increases along the passage of time and a decline in volatility. The relationship is reversed when the option is deep in the money or deep out of the money.
- Gamma calculated under the assumption of 2F is bigger at the money and smaller from the money than the equivalent gamma based on GBM.

Theta: Sensitivity with respect to maturity

- The value of an option decreases with the passage of time. The time value of an option is maximum when the option nears expiration.
- The analysis based on GBM presents a higher theta than the analysis based on 2F. According to Taleb, this verifies the better fit of the 2F model.

Vega: Sensitivity with respect to volatility

- At the money, the option value is linear with respect to volatility, from the money, the relationship is convex.
- The numerical analysis demonstrate that the vega declines with the passage of time. However, this result bears only limited implication for the practice, as volatility shocks tend to be temporarily and volatility returns back to its mean level.

In the fourth part, we examined the suitability of traditional VaR methodologies in electricity markets. As the tree major assumptions of the commonly used variance-covariance method - risk free portfolio replication, normal distribution and mark-to-market asset pricing – were proven invalid in electricity markets, we introduced the methodology of stochastic optimization, which provided a more accurate approximation of the VaR through the generation of individual P&L distributions. The generation of all important risk ratios allowed us to undertake a quasi-sensitivity analysis of the VaR with respect to price and volatility. We found out, that in contrast to option prices, the VaR is related to the underlying risk factors in a non-monotonic way.

In the fifth part, we have undertaken model test in order to examine the accuracy of BIT@EPI.VPP. In a first step, we analyzed the contracts with respect to their additivity. We found out that future contracts are additive in price, standard deviation and VaR when the contracts' delivery blocks were adjacent. For future contracts with the delivery periods lying apart, we could only prove additivity with respect to price. Call on Futures and Virtual Power Plants, however, lacked additivity with respect to price and risk. In a second step, we provided evidence that the put-call parity holds with a 100% accuracy independent of the delivery period length or the underlying price dynamic model. With the model tests, we demonstrated the high computational accuracy of the tool BIT@EPI.VPP.

Annex

Value, Delta and Gamma of a Call on Future based on 2F for Different Maturities

		σ1=0.3, σ2=0.1					σ1=0.3, σ2=0.2					σ1=0.4, σ2=0.1				
	PFC	Call Value	Delta	Appr Delta	Gamma	Appr Gamma	Call Value	Delta	Appr Delta	Gamma	Appr Gamma	Call Value	Delta	Appr Delta	Gamma	Appr Gamma
T=0.25	30	0					0					0				
	35	0	0	0			0	0	0			0	0	0		
	40	0	0	0	0	5	0	0	0	0	27	0	0	0	0	16
	45	0	0	18	9	144	0	0	108	54	200	0	0	65	32	179
	50	183	37	577	279	559	1.078	216	799	346	544	645	129	717	326	551
	55	5.770	1.117	2.256	839	816	7.991	1.383	2.284	742	711	7.173	1.305	2.267	775	748
	60	22.738	3.394	3.840	792	524	23.918	3.185	3.644	680	488	23.313	3.228	3.710	722	507
	65	44.173	4.287	4.350	255	144	44.430	4.102	4.234	295	186	44.274	4.192	4.293	291	174
	70	66.240	4.413	4.415	32	16	66.259	4.366	4.389	77	45	66.243	4.394	4.405	56	31
	75	88.320	4.416	4.416	1		88.320	4.412	4.414	13		88.320	4.415	4.416	6	
	80	110.400	4.416				110.400	4.416				110.400	4.416			
T=0.5	30	0					0					0				
	35	0	0	0			0	0	0			0	0	0		
	40	0	0	0	0	18	0	0	25	13	79	0	0	0	0	30
	45	0	0	73	37	184	251	50	316	145	286	1	0	122	61	214
	50	733	147	735	331	556	3.159	582	1.170	427	519	1.215	243	855	367	542
	55	7.353	1.324	2.299	782	740	11.952	1.758	2.393	611	557	8.556	1.468	2.288	716	684
	60	23.724	3.274	3.697	699	488	27.089	3.027	3.399	503	391	24.092	3.107	3.590	651	482
	65	44.322	4.120	4.252	278	176	45.937	3.770	3.955	278	213	44.455	4.073	4.217	313	199
	70	66.248	4.385	4.400	74	41	66.644	4.141	4.251	148	106	66.261	4.361	4.387	85	49
	75	88.320	4.414	4.415	8		88.443	4.360	4.378	64		88.320	4.412	4.414	14	
	80	110.400	4.416				110.421	4.396				110.400	4.416			
T=1	30	0					0					0				
	35	0	0	0			0	0	0			0	0	0		
	40	0	0	3	1	48	0	0	243	122	205	0	0	11	5	62
	45	28	6	193	95	248	2.435	487	822	289	332	109	22	249	119	267
	50	1.932	381	996	401	533	8.217	1.156	1.572	375	400	2.494	477	1.078	414	525
	55	9.987	1.611	2.325	665	625	18.156	1.988	2.423	426	392	10.891	1.679	2.348	635	592
	60	25.184	3.039	3.494	585	449	32.449	2.858	3.139	358	308	25.979	3.018	3.446	549	426
	65	44.931	3.949	4.121	313	212	49.542	3.419	3.654	258	218	45.349	3.874	4.052	303	213
	70	66.395	4.293	4.340	110	70	68.984	3.889	4.009	178	135	66.499	4.230	4.300	124	85
	75	88.336	4.388	4.400	30		89.631	4.129	4.195	93		88.346	4.369	4.390	45	
	80	110.400	4.413				110.935	4.261				110.400	4.411			

Value, Delta and Gamma of a Call on Future based on GBM for Different Maturities

		σ1=0.2					σ1=0.4				
	PFC	Call Value	Delta	Appr Delta	Gamma	Appr Gamma	Call Value	Delta	Appr Delta	Gamma	Appr Gamma
T=0.25	30	0					9				
	35	0	0	0			136	25	91		
	40	3	1	17	8	51	920	157	349	129	192
	45	169	33	204	94	233	3.623	541	859	255	307
	50	2.046	375	950	373	518	9.505	1.176	1.578	360	373
	55	9.673	1.525	2.275	662	637	19.408	1.981	2.349	385	373
	60	24.796	3.025	3.499	612	469	32.995	2.717	3.069	360	320
	65	44.660	3.973	4.151	326	217	50.096	3.420	3.628	280	225
	70	66.301	4.328	4.367	108	65	69.277	3.836	3.969	171	138
	75	88.325	4.405	4.410	22		89.791	4.103	4.180	105	
	80	110.400	4.415				111.076	4.257			
T=0.5	30	0					254				
	35	3	1	10			1.208	191	346		
	40	101	20	107	49	116	3.711	501	749	201	234
	45	1.077	195	475	184	288	8.694	997	1.282	266	280
	50	4.853	755	1.259	392	460	16.526	1.566	1.868	293	291
	55	13.666	1.763	2.314	527	502	27.373	2.170	2.447	290	271
	60	27.991	2.865	3.266	476	394	40.999	2.725	2.953	253	227
	65	46.322	3.666	3.890	312	238	56.908	3.182	3.356	201	182
	70	66.894	4.114	4.219	164	116	74.555	3.529	3.681	163	146
	75	88.507	4.323	4.355	68		93.715	3.832	3.940	130	
	80	110.445	4.387				113.955	4.048			
T=1	30	10					1.914				
	35	138	26	92			4.750	567	776		
	40	925	158	350	129	192	9.676	985	1.228	226	236
	45	3.637	542	860	255	307	17.033	1.472	1.719	245	235
	50	9.529	1.178	1.580	360	372	26.863	1.966	2.168	225	200
	55	19.435	1.981	2.349	385	372	38.718	2.371	2.518	175	186
	60	33.017	2.717	3.068	360	320	52.047	2.666	2.912	197	182
	65	50.115	3.419	3.627	280	225	67.841	3.159	3.245	166	139
	70	69.290	3.835	3.968	171	138	84.495	3.331	3.468	112	109
	75	89.799	4.102	4.179	105		102.520	3.605	3.679	105	
	80	111.081	4.256				121.282	3.752			

Value, Delta and Gamma of a Call on Future based on 2F for Different Volatilities

		σ1=0.3, σ2=0.1					σ1=0.3, σ2=0.2					σ1=0.4, σ2=0.1				
	PFC	Call Value	Delta	Appr Delta	Gamma	Appr Gamma	Call Value	Delta	Appr Delta	Gamma	Appr Gamma	Call Value	Delta	Appr Delta	Gamma	Appr Gamma
T=0.25	30	0					0					0				
	35	0	0	0			0	0	0			0	0	0		
	40	0	0	0	0	5	0	0	0	0	27	0	0	0	0	16
	45	0	0	18	9	144	0	0	108	54	200	0	0	65	32	179
	50	183	37	577	279	559	1.078	216	799	346	544	645	129	717	326	551
	55	5.770	1.117	2.256	839	816	7.991	1.383	2.284	742	711	7.173	1.305	2.267	775	748
	60	22.738	3.394	3.840	792	524	23.918	3.185	3.644	680	488	23.313	3.228	3.710	722	507
	65	44.173	4.287	4.350	255	144	44.430	4.102	4.234	295	186	44.274	4.192	4.293	291	174
	70	66.240	4.413	4.415	32	16	66.259	4.366	4.389	77	45	66.243	4.394	4.405	56	31
	75	88.320	4.416	4.416	1		88.320	4.412	4.414	13		88.320	4.415	4.416	6	
	80	110.400	4.416				110.400	4.416				110.400	4.416			
T=0.5	30	0					0					0				
	35	0	0	0			0	0	0			0	0	0		
	40	0	0	0	0	18	0	0	25	13	79	0	0	0	0	30
	45	0	0	73	37	184	251	50	316	145	286	1	0	122	61	214
	50	733	147	735	331	556	3.159	582	1.170	427	519	1.215	243	855	367	542
	55	7.353	1.324	2.299	782	740	11.952	1.758	2.393	611	557	8.556	1.468	2.288	716	684
	60	23.724	3.274	3.697	699	488	27.089	3.027	3.399	503	391	24.092	3.107	3.590	651	482
	65	44.322	4.120	4.252	278	176	45.937	3.770	3.955	278	213	44.455	4.073	4.217	313	199
	70	66.248	4.385	4.400	74	41	66.644	4.141	4.251	148	106	66.261	4.361	4.387	85	49
	75	88.320	4.414	4.415	8		88.443	4.360	4.378	64		88.320	4.412	4.414	14	
	80	110.400	4.416				110.421	4.396				110.400	4.416			
T=1	30	0					0					0				
	35	0	0	0			0	0	0			0	0	0		
	40	0	0	3	1	48	0	0	243	122	205	0	0	11	5	62
	45	28	6	193	95	248	2.435	487	822	289	332	109	22	249	119	267
	50	1.932	381	996	401	533	8.217	1.156	1.572	375	400	2.494	477	1.078	414	525
	55	9.987	1.611	2.325	665	625	18.156	1.988	2.423	426	392	10.891	1.679	2.348	635	592
	60	25.184	3.039	3.494	585	449	32.449	2.858	3.139	358	308	25.979	3.018	3.446	549	426
	65	44.931	3.949	4.121	313	212	49.542	3.419	3.654	258	218	45.349	3.874	4.052	303	213
	70	66.395	4.293	4.340	110	70	68.984	3.889	4.009	178	135	66.499	4.230	4.300	124	85
	75	88.336	4.388	4.400	30		89.631	4.129	4.195	93		88.346	4.369	4.390	45	
	80	110.400	4.413				110.935	4.261				110.400	4.411			

Value, Delta and Gamma of a Call on Future based on GBM for Different Volatilities

		σ1=0.2					σ1=0.4				
	PFC	Call Value	Delta	Appr Delta	Gamma	Appr Gamma	Call Value	Delta	Appr Delta	Gamma	Appr Gamma
T=0.25	30	0					9,38				
	35	0	0	0			136,44	25	91		
	40	3	1	17	8	51	920	157	349	129	192
	45	169	33	204	94	233	3.623	541	859	255	307
	50	2.046	375	950	373	518	9.505	1.176	1.578	360	373
	55	9.673	1.525	2.275	662	637	19.408	1.981	2.349	385	373
	60	24.796	3.025	3.499	612	469	32.995	2.717	3.069	360	320
	65	44.660	3.973	4.151	326	217	50.096	3.420	3.628	280	225
	70	66.301	4.328	4.367	108	65	69.277	3.836	3.969	171	138
	75	88.325	4.405	4.410	22		89.791	4.103	4.180	105	
	80	110.400	4.415				111.076	4.257			
T=0.5	30	0					254				
	35	3	1	10			1.208	191	346		
	40	101	20	107	49	116	3.711	501	749	201	234
	45	1.077	195	475	184	288	8.694	997	1.282	266	280
	50	4.853	755	1.259	392	460	16.526	1.566	1.868	293	291
	55	13.666	1.763	2.314	527	502	27.373	2.170	2.447	290	271
	60	27.991	2.865	3.266	476	394	40.999	2.725	2.953	253	227
	65	46.322	3.666	3.890	312	238	56.908	3.182	3.356	201	182
	70	66.894	4.114	4.219	164	116	74.555	3.529	3.681	163	146
	75	88.507	4.323	4.355	68		93.715	3.832	3.940	130	
	80	110.445	4.387				113.955	4.048			
T=1	30	10					1.914				
	35	138	26	92			4.750	567	776		
	40	925	158	350	129	192	9.676	985	1.228	226	236
	45	3.637	542	860	255	307	17.033	1.472	1.719	245	235
	50	9.529	1.178	1.580	360	372	26.863	1.966	2.168	225	200
	55	19.435	1.981	2.349	385	372	38.718	2.371	2.518	175	186
	60	33.017	2.717	3.068	360	320	52.047	2.666	2.912	197	182
	65	50.115	3.419	3.627	280	225	67.841	3.159	3.245	166	139
	70	69.290	3.835	3.968	171	138	84.495	3.331	3.468	112	109
	75	89.799	4.102	4.179	105		102.520	3.605	3.679	105	
	80	111.081	4.256				121.282	3.752			

Value and Theta of a Call on Future based on 2F for Different Volatilities

		σ1=0.3, σ2=0.1			σ1=0.3, σ2=0.2			σ1=0.4, σ2=0.1		
	MtE	Call Value	Theta	Appr Theta	Call Value	Theta	Appr Theta	Call Value	Theta	Appr Theta
In-the-money	0	22.080			22.080			22.080		
K=50	1	22.132	10	5	22.231	30	15	22.278	40	20
	2	22.271	28	14	22.935	141	70	22.711	87	43
	3	22.499	46	23	23.580	129	65	23.026	63	32
	4	22.760	52	26	24.184	121	60	23.270	49	24
	5	22.971	42	21	25.077	179	89	23.462	38	19
	6	23.189	44		26.109	206		23.683	44	
At-the-money	0	0			0			0		
K=55	1	4.011	802	401	4.670	934	467	5.201	1.040	520
	2	5.099	217	109	6.526	371	186	6.481	256	128
	3	5.770	134	67	7.991	293	147	7.173	138	69
	4	6.342	114	57	9.403	283	141	7.702	106	53
	5	7.064	145	72	10.653	250	125	8.120	84	42
	6	7.326	52		11.952	260		8.556	87	
Out-of-the-money	0	0			0			0		
K=60	1	6	1	1	5	1	0	189	38	19
	2	140	27	13	749	149	74	619	86	43
	3	357	43	22	1.379	126	63	909	58	29
	4	591	47	23	2.033	131	65	1.171	52	26
	5	791	40	20	2.931	180	90	1.400	46	23
	6	1.008	43		4.062	226		1.640	48	

Value and Theta of a Call on Future based on GBM for Different Volatilities

		σ=0.2			σ=0.4			σ=0.6		
	MtE	Call Value	Theta	Appr Theta	Call Value	Theta	Appr Theta	Call Value	Theta	Appr Theta
In-the-money	0	22.080			22.080			22.080		
K=50	1	22.336	51	26	25.125	609	304	29.381	1.460	730
	2	23.230	179	89	28.718	719	359	35.390	1.202	601
	3	24.197	193	97	31.517	560	280	40.196	961	481
	4	25.184	197	99	34.131	523	261	44.282	817	409
	5	25.961	155	78	36.386	451	226	47.472	638	319
	6	26.930	194		38.624	448		50.606	627	
At-the-money	0	0			0			0		
K=55	1	5.541	1.108	554	11.116	2.223	1.112	16.706	3.341	1.671
	2	7.912	474	237	15.878	952	476	23.843	1.427	714
	3	9.673	352	176	19.408	706	353	29.105	1.053	526
	4	11.208	307	154	22.479	614	307	33.656	910	455
	5	12.435	245	123	24.925	489	245	37.260	721	360
	6	13.666	246		27.373	490		40.844	717	
Out-of-the-money	0	0			0			0		
K=60	1	410	82	41	3.798	760	380	8.566	1.713	857
	2	1.533	225	112	7.782	797	398	15.411	1.369	685
	3	2.688	231	116	11.138	671	336	20.567	1.031	516
	4	3.869	236	118	14.090	590	295	25.102	907	454
	5	4.821	190	95	16.464	475	237	28.745	729	364
	6	5.785	193		18.860	479		32.414	734	

Value and Vega of a Call on Future based on GBM for Different Maturities

	σ	T=0.25 Call Value	T=0.25 Vega	T=0.25 Appr Vega	T=0.5 Call Value	T=0.5 Vega	T=0.5 Appr Vega	T=1 Call Value	T=1 Vega	T=1 Appr Vega
In-the-money	0,0	22.080			22.080			22.080		
K=50	0,1	22.204	12	106	22.204	12	242	24.206	213	473
	0,2	24.197	199	272	26.930	473	509	31.538	733	817
	0,3	27.639	344	366	32.393	546	585	40.538	900	861
	0,4	31.517	388	407	38.624	623	617	48.752	821	855
	0,5	35.782	426	434	44.727	610	599	57.639	889	895
	0,6	40.196	441	436	50.606	588	613	66.655	902	881
	0,7	44.507	431	425	56.989	638	639	75.267	861	837
	0,8	48.705	420	422	63.393	640	631	83.403	814	798
	0,9	52.945	424	443	69.611	622	611	91.225	782	823
	1,0	57.573	463		75.615	600		99.856	863	
At-the-money	0,0	0			0			0		
K=55	0,1	4.823	482	484	6.810	681	683	9.686	969	972
	0,2	9.673	485	486	13.666	686	686	19.435	975	973
	0,3	14.539	487	487	20.532	687	685	29.145	971	964
	0,4	19.408	487	486	27.373	684	681	38.718	957	946
	0,5	24.268	486	485	34.155	678	674	48.057	934	918
	0,6	29.105	484	482	40.844	669	663	57.073	902	898
	0,7	33.908	480	478	47.407	656	648	66.021	895	915
	0,8	38.665	476	473	53.811	640	631	75.371	935	919
	0,9	43.363	470	466	60.029	622	617	84.405	903	883
	1,0	47.991	463		66.142	611		93.037	863	
Out-of-the-money	0,0	0			0			0		
K=60	0,1	205	20	134	925	93	289	2.699	270	558
	0,2	2.688	248	316	5.785	486	564	11.163	846	895
	0,3	6.520	383	422	12.215	643	654	20.607	944	953
	0,4	11.138	462	465	18.860	665	669	30.232	962	967
	0,5	15.824	469	471	25.605	674	678	39.940	971	970
	0,6	20.567	474	477	32.414	681	682	49.631	969	963
	0,7	25.356	479	481	39.253	684	684	59.201	957	946
	0,8	30.178	482	483	46.087	683	681	68.551	935	919
	0,9	35.020	484	485	52.880	679	675	77.586	903	883
	1,0	39.871	485		59.595	672		86.217	863	

Value, Vega 1 and Vega 2 for a Call on Future based on 2F for Different Maturities

			Call Value					Vega 1				Vega 2			
			σ_2					σ_2				σ_2			
		σ_1	0,0	0,2	0,4	0,6	0,8	0,0	0,2	0,4	0,6	0,0	0,2	0,4	0,6
T=0.25	**K=50**	0,2	22.082												
		0,4	22.525	23.989				22				73			
		0,6	23.960	25.692	30.019			72	85			87	216		
		0,8	25.933	27.798	31.684	36.162		99	105	83		93	194	224	
		1,0	28.408	29.958	33.494	37.612	41.936	124	108	90	73	78	177	206	216
	K=55	0,2	3.150												
		0,4	6.316	9.008				158				135			
		0,6	9.491	11.548	16.145			159	127			103	230		
		0,8	12.665	14.340	18.181	23.513		159	140	102		84	192	267	
		1,0	15.832	17.230	20.498	25.341	30.925	158	145	116	91	70	163	242	279
	K=60	0,2	9												
		0,4	724	1.827				36				55			
		0,6	2.644	3.738	8.391			96	96			55	233		
		0,8	5.175	6.249	10.367	15.759		127	126	99		54	206	270	
		1,0	7.901	8.925	12.586	17.587	23.172	136	134	111	91	51	183	250	279
T=0.5	**K=50**	0,2	22.084												
		0,4	22.596	26.772				26				209			
		0,6	24.200	28.428	36.221			80	83			211	390		
		0,8	26.339	30.319	37.481	45.426		107	95	63		199	358	397	
		1,0	28.898	32.316	38.893	46.389	55.254	128	100	71	48	171	329	375	443
	K=55	0,2	3.278												
		0,4	6.573	12.666				165				305			
		0,6	9.876	14.651	23.793			165	99			239	457		
		0,8	13.178	17.092	25.342	35.075		165	122	77		196	413	487	
		1,0	16.470	19.772	27.191	36.466	46.096	165	134	92	70	165	371	464	482
	K=60	0,2	12												
		0,4	798	2.998				39				110			
		0,6	2.570	4.085	12.262			89	54			76	409		
		0,8	5.505	7.153	14.521	23.514		147	153	113		82	368	450	
		1,0	8.804	10.476	17.154	25.553	34.845	165	166	132	102	84	334	420	465
T=1	**K=50**	0,2	22.084												
		0,4	22.603	31.261				26				433			
		0,6	24.221	32.745	46.461			81	74			426	686		
		0,8	26.375	34.221	47.177	61.817		108	74	36		392	648	732	
		1,0	28.942	35.891	48.102	62.480	75.938	128	84	46	33	347	611	719	673
	K=55	0,2	3.289												
		0,4	6.596	18.613				165				601			
		0,6	9.911	19.925	35.777			166	66			501	793		
		0,8	13.224	21.711	36.646	51.913		166	89	43		424	747	763	
		1,0	16.527	23.840	37.877	52.924	67.264	165	106	62	51	366	702	752	717
	K=60	0,2	16												
		0,4	857	10.165				42				465			
		0,6	2.974	11.625	27.215			106	73			433	779		
		0,8	5.628	13.417	28.308	43.887		133	90	55		389	745	779	
		1,0	8.574	15.478	29.670	44.816	59.363	147	103	68	46	345	710	757	727

VaR Sensitivity with respect to the Underlying Price

	GBM		2F	
PFC	VaR 5%	Var 10%	VaR 5%	Var 10%
45	0	0	0	0
50	-3'855	0	0	0
55	0	0	8'711	-487
60	-2'429	1	-6'731	14'769
65	8'808	-116	6'995	34'588
70	9'413	18'558	2'626	54'407
75	27'434	37'232	45'437	74'226
80	45'454	55'906	64'659	94'045

10%-VaR Sensitivity with respect to Volatility based on GBM

σ	In the Money K=50	At the Money K=55	Out of the Money K=60
0.0	22'080	0	0
0.1	3'029	0	0
0.2	1	0	0
0.3	-784	2	2
0.4	-9'602	5	5
0.5	-18'854	13	13
0.6	-28'436	27	27
0.7	-38'241	49	49
0.8	-48'164	84	84
0.9	134	134	134
1.0	203	203	203

10%-VaR Sensitivity with respect to Volatility based on 2F

		10%-VaR σ2				
	σ1	0.0	0.2	0.4	0.6	0.8
It the Money	0.2	7'872				
K=50	0.4	-6'088	-221			
	0.6	-2'886	-8'801	96		
	0.8	-11'999	-17'536	117	315	
	1.0	-21'379	-21'680	148	359	735
At the Money	0.2	0				
K=55	0.4	0	-59			
	0.6	2	-4'553	86		
	0.8	6	-3'208	-3'208	280	
	1.0	14	-14'862	-8'978	-4'379	-1'908
Out of the Money	0.2	0				
K=60	0.4	0	-440			
	0.6	-512	9	43		
	0.8	6	15	52	-20'770	
	1.0	14	25	66	-26'459	-23'988

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