

# Risk-Adjusted Performance Measurement – State of the Art

**Bachelor Thesis**  
of the University of St. Gallen  
School of Business Administration,  
Economics, Law and Social Sciences (HSG)  
to obtain the title of  
Bachelor of Arts (B.A.) in Business Administration

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St. Gallen, May 2010

## **Abstract**

More than 100 alternative risk-adjusted performance measures can be identified in literature, most of them attempting to remedy against the shortcoming of the Sharpe Ratio which relies on normally distributed returns. However, there is a fervent discussion in literature whether the choice of risk-adjusted performance measure actually matters or not.

This work firstly provides an overview of the current state of the art of risk-adjusted performance measurement by discussing the most frequently used risk-adjusted performance measures in scientific literature and secondly investigates the question whether different alternative performance measures lead to different investment decisions compared to each other and to the Sharpe Ratio.

The discussed risk-adjusted performance measures are applied to the return distribution of a portfolio of bank products. The analysis shows that the majority of the examined performance measures produce rankings of investment alternatives which are highly correlated to each other and to the Sharpe Ratio. Yet, contradicting the findings of some authors, it is shown that the choice of performance measure and of underlying parameters does matter. There are some measures which produce significantly different results, which for some of them is caused by the choice of parameters.

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## 1. Introduction

“Does the choice of risk-adjusted performance measure matter?” This is the question the current discussion in academic literature revolves around. Risk-adjusted performance measures are an important tool for investment decisions. Whenever an investor evaluates the performance of an investment he will not only be interested in the achieved absolute return but also in the risk-adjusted return – i. e. in the risk which had to be taken to realize the profit.

The first ratio to measure risk-adjusted return was the Sharpe Ratio introduced by William F. Sharpe in 1966. Although being frequently used in theory and practice the Sharpe Ratio has a major drawback as it is designed for the use in a  $\mu$ - $\sigma$ - framework and thus requires returns to be normally distributed. The events of the current financial crisis have shown clearly that this assumption does not hold true and that especially events at the tails of the distribution – most importantly high losses – are more likely than assumed by the normal distribution. As a result, the Sharpe Ratio might lead to inaccurate investment decisions.

In the past years a flood<sup>1</sup> of alternative risk-adjusted performance measures which mostly do not rely on normally distributed returns have been developed in literature in order to remedy against this shortcoming of the Sharpe Ratio. The creators of the new performance measures<sup>2</sup> try to prove that their respective ratios are able to produce more accurate results than the Sharpe Ratio. To verify this claim, some authors have attempted – mostly using hedge fund returns – to explore the differences in performance results between the Sharpe Ratio and the alternative performance measures in order to determine the usefulness of the newer performance measures. But so far the results have been ambiguous.

This work aims at adding on to the ongoing discussion about risk-adjusted performance measures. The first objective is to provide an overview of the current state of the art of risk-adjusted performance measurement by describing and discussing the most frequently used risk-adjusted performance measures in scientific literature. Secondly,

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<sup>1</sup> In a comprehensive study Cogneau and Hübner (2009) identified more than 101 ways to measure performance.

<sup>2</sup> For reasons of better legibility risk-adjusted performance measure in this work simply shall be referred to as performance measures.

following other authors, it shall be determined whether the described alternative performance measures indeed lead to other investment decisions than the Sharpe Ratio respectively if they yield different results in comparison to each other. This is done by applying the performance measures to a portfolio of bank products, an asset class which has not been used for this kind of analysis before.

This work is divided into four parts. The first part describes the fields of application of risk-adjusted performance measurement and the theoretical need for alternative performance measures is highlighted. Furthermore a literature review of studies which compare the results of traditional performance measures to those of alternative measures shall be provided. The second part is devoted to the description and critical discussion of risk-adjusted performance measures. In the third part the performance measures described in part two are applied to measure the performance of a portfolio of bank products and variations in the assumed parameters are conducted to test the resistance of the performance results. Finally, the findings of this work shall be summarized in the fourth part.

## 2. Risk-Adjusted Performance Measurement

### 2.1. The need for Risk-Adjusted Performance Measurement

Risk-adjusted performance Measurement encompasses a set of concepts. Those concepts may vary in detail depending on the context they are used in. However, all risk-adjusted performance measures have one thing in common: they compare the return on capital to the risk taken to earn this return – i.e. some kind of risk-adjustment is adopted. Generally speaking, return in risk-adjusted performance measures is measured either by absolute returns or by relative returns (i.e. excess returns), whereas disagreement prevails in literature on how exactly risk should be taken into account. This has given rise to the development of a considerable number of alternative risk-adjusted performance measures. Thus, risk-adjusted performance measures can take many forms as shall be shown in the following chapters.

In the past years risk-adjusted performance measures have gained great importance. The first reason for this development is the emergence of investment funds as an important investment category. Investors needed an effective tool to evaluate the respective performance of the various funds compared to the risk taken by the fund managers to choose the right option for capital allocation (Weisman, 2002, p. 80). The second reason is the introduction of the Basel II regulatory framework, which requires financial institutions to hold a certain amount of equity as a cushion against unexpected losses for each risky position taken. As a result financial institutions have a great interest in efficiently allocating capital not only according to the resulting return but also to the risk shouldered and

Therefore, banks more and more turn to concepts based on risk-adjusted performance measures like Risk-adjusted Return on Capital (RAROC) when evaluating their business activities (Smithson, Brannan, Mengle & Zmiewski, 2003, p. 5). Due to these developments, risk-adjusted performance measures gradually replace traditional performance measures like Return on Equity (ROE) or Return on Investment (ROI) when it comes to analyzing performance in financial contexts as these traditional measures do not take into account risk (Zimmermann & Wegmann, 2003, p. 33).

As indicated above, risk-adjusted performance measurement has two major fields of application- performance evaluation and capital allocation. In the field of performance

evaluation, risk-adjusted performance measures are used to rank competing investment strategies ex-ante and ex-post according to their respective risk-adjusted returns. The investment with the highest return and the lowest risk ranks first (Jorion, 2006, p. 291). A further application in this context is the design of compensation plans for traders or asset managers. If their performance is assessed according to the raw profits they make, they will seek to take risk to maximize their bonuses. In contrast, in a risk-adjusted compensation framework, excessive risk-taking can be avoided as employees are compensated according to their risk-adjusted profits. (Dowd, 2002, p. 210)

Risk-adjusted performance measurement is also used to guide management in efficient internal capital allocation. It helps the management of financial institutions to evaluate the risk-adjusted performance of their business units, traders or investment portfolios. Like this capital is only allocated to deals which are profitable from a risk-adjusted performance point of view as equity is rare and expensive due to the minimum capital requirements stated in the Basel II framework (McNeil, Embrechts & Frey, 2005, p. 44).

The focus of this composition is on the use of risk-adjusted performance measurement in the field of performance evaluation of investment opportunities. For an overview of the application of risk-adjusted performance measurement for risk capital allocation in banks and risk-adjusted management of entire financial institutions see for example Lister (1997).

## **2.2. Evolution of Risk-Adjusted Performance Measures**

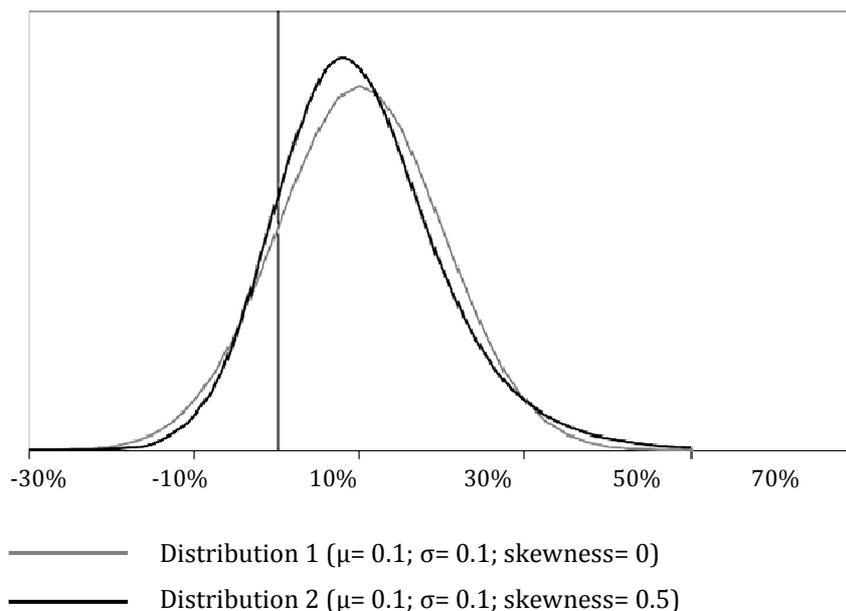
The Sharpe Ratio, introduced by William F. Sharpe in his seminal Journal of Business article in 1966, can be regarded as the first risk-adjusted performance measure. Due to its simplicity and thus easy application it has found widespread acceptance in literature and in practice. The Sharpe Ratio measures risk by volatility, which reflects the paradigm of the Modern Portfolio Theory prevalent during the genesis of the Sharpe Ratio.

The concept of volatility represents one of the major shortcomings of the Sharpe Ratio. Firstly, volatility does not treat variability in gains and variability in losses separately—i.e. the Sharpe Ratio penalizes for both downside and upside variability in returns. A rational investor, however, distinguishes between gains and losses and would rather consider high variability in gains to be an attractive reward potential (Zakamouline,

2010, p. 14). Yet, measuring risk-adjusted performance with the Sharpe Ratio might lead to the fact that an asset with a high upside volatility is ranked lower than an asset with a low downside volatility.

Secondly, if volatility is used to measure risk, normally distributed returns are assumed. However, starting with the research of Mandelbrot (1963), many empirical studies have proven that the theoretical assumption of normally distributed returns does not hold true for many asset classes. There is a considerable body of literature (e.g. Ekholm & Pasternack, 2005; Leland, 2002; Aparicio & Estrada, 2001) which demonstrates that returns are not normally distributed but often show 'fat tails' – i.e. that extreme events are more likely than predicted by the normal distribution. This has practically been proven by several financial crises in the past. For alternative investment forms like hedge funds researchers (e.g. Brooks & Kat, 2002; Malkiel & Saha, 2005; Kosowski, Naik & Teo, 2007) have shown that this asset class especially exhibits significant amounts of skewness with rare but extreme gains or losses due to their dynamic trading strategies and their holding of derivatives. That is why some authors (e.g. Hodges, 1998; Bernardo & Ledoit, 2000) conclude that performance evaluation with the Sharpe Ratio seems dubious if returns are non-normally distributed, which shall be illustrated by the following example from Guse and Rudolf (2008, p. 198).

In Figure 1 there are two return distributions depicted by a grey (Distribution 1) and a black line (Distribution 2). Both distributions have the parameters  $\mu=0.1$  and  $\sigma=0.1$ . The only difference is that Distribution 1 has zero skewness whereas Distribution 2 has a skewness of 0.5.



*Figure 1: Comparison of return distributions with different values of skewness  
(Source: Guse & Rudolf, 2008, p. 3)*

As both distributions have the same values for  $\mu$  and  $\sigma$ , they both produce the same Sharpe Ratio. An investor who makes his decision based on this performance ratio is only indifferent between both distributions. Yet, comparing distribution 1 and 2 graphically one can notice that Distribution 2 has more upward potential – i.e. that high gains are more probable than in distribution 1 which has no skewness. Similarly, Distribution 2 also has a lower downward potential as very low gains are less probable. Thus, Distribution 2 is more attractive for an investor - a fact which could not be determined based on the Sharpe Ratio. With the help of alternative performance measures which account for skewness like the Adjusted Sharpe Ratio (ASR), however, an investor is able to choose the best investment opportunity as the ASR<sup>3</sup> equals 1 for Distribution 1 and 1.83 for Distribution 2. This example shows that performance analysis based solely on the Sharpe Ratio – in some cases – can lead to incorrect investment decisions.

As a consequence of the drawbacks of the Sharpe Ratio, a considerable number of alternative performance measures have been developed. Researchers have tried to replace volatility by other measures of risk to overcome the described shortcomings of the underlying normal distribution assumption and to provide a more accurate picture of risk-adjusted performance. This has been done by encompassing information of

<sup>3</sup> The exact calculation of the Sharpe Ratio and the Adjusted Sharpe Ratio shall be described in Chapter 3.

higher moments of distributions like skewness and kurtosis as well as by developing measures which do not assume any distribution at all and therefore are generally applicable – regardless of the return distribution.

### **2.3.Literature Review**

As indicated in the introduction, there is a growing body of literature which tries to determine whether the information contained in higher moments of distributions is significant for performance results and thus, whether alternative performance measures indeed lead to different rankings compared to the Sharpe Ratio and compared to each other. In this section an overview of the most important contributions to this topic shall be given.

Pedersen and Rudholm-Alfvén (2003) examine the performance of financial institution stocks using a choice of traditional and alternative performance measures partly identical to the selection used in this work. They find that the rankings of the alternative performance measures are extremely positively correlated among each other and to the Sharpe Ratio. As the alternative performance measures do not lead to significantly different results compared to the Sharpe Ratio in their analysis, the authors recommend staying with this traditional measure as it is analytically convenient and traditionally supported by researchers and practitioners (Pedersen & Rudholm-Alfvén, p. 166).

Motivated by these findings, Eling and Schumacher (2006) analyze the performance of different categories of hedge funds using the Sharpe Ratio and a selection of the most documented alternative performance measures similar to those described in this work. Their results show high correlations in the rankings across all performance measures as well. They further prove that the rankings are very robust to changes in underlying parameters. Thus, they conclude that the choice of the performance measure does not matter and that the Sharpe Ratio is sufficient for appraising risk-adjusted performance.

Using another sample of hedge fund returns, Glawischnig (2007) attempts to refute Eling and Schumacher (2006) by showing that the choice of performance measure has a considerable influence on the ranking. His analysis, however, also yields highly correlated rankings for all performance measures. Nevertheless, this author warns against dismissing the alternative performance measures. He points out that it is necessary to include the information contained in the higher moments of distributions

even if they lead to the same result for the majority of observations. Yet, for some investment alternatives the additional information might lead to alterations in the ranking, which, even if small, might be significant for the decision of a particular investor (Glawischnig, 2007, p. 27).

Heidorn (2009) also intends to disprove Eling and Schumacher (2006) based on hedge fund returns. He partly succeeds as he is able to demonstrate that the rank order is significantly heterogeneous with some of the middle-ranking indices whereas the indices ranking best respectively worst are mostly the same. Furthermore he shows that some alternative performance measures, mainly those based on drawdown, lead to different rankings when a different method for evaluating ranking correlations is used. From an overall point of view, however, his findings are the same as those from Eling and Schumacher (2006), with highly correlated rankings for the alternative performance measures.

Zakamouline (2010) criticizes the database used by Eling and Schumacher (2006). He argues that their conclusion is based on a short sampling period and a small subset of performance measures and thus, is only of limited validity. Furthermore he critically notes that the majority of hedge fund returns examined by Eling and Schumacher (2006) were close to being normally distributed which might be an explanation for the high correlation in ranks. Zakamouline (2010) shows in his own analysis that, despite high rank correlations, the rankings produced by alternative performance measures are far from being identical to the ranking produced by the Sharpe Ratio. He points out that there are some performance measures – notably the Farinelli-Tibiletti Ratio – which yield rather low rank correlations to the Sharpe Ratio.

Summarizing the studies cited above, the findings show that alternative performance measures in general do not yield significantly different ranking results than the Sharpe Ratio. However, there seem to be some performance measures which – under certain circumstances – lead to significant differences in rankings and therefore, legitimate the existence of this group of performance measures.

Except for the studies of Pedersen and Rudholm-Alfvin (2003) the discussed studies were based on hedge fund returns. This asset class is often criticized for suffering from severe selection biases (for a detailed discussion see for example Kaiser, Heidorn &

Roder, 2009, p. 9) which might put the hitherto obtained statements about the usefulness of alternative performance measures into question. Hence, returns of the asset class of bank products shall be used in this work to determine whether alternative performance measures yield different results compared to the Sharpe Ratio.

### **3. State of the Art of Risk-Adjusted Performance Measures**

#### **3.1. Categorization of Performance Measures**

The performance measures discussed in this composition can be categorized according to the risk-measure applied in the respective measures. The particular groups are performance measures based on either (simple) volatility, Value-at-Risk, Lower Partial Moments or drawdown. Technically the performance measures based on Value-at-Risk also use volatility to measure risk but they additionally consider other factors, like the mean. Thus, for the performance measures using volatility two separate groups were formed, a common way of categorization in scientific literature.

Furthermore, one can distinguish between performance measures which assume normally distributed returns, measures which explicitly account for higher moments of distribution and measures which implicitly account for higher moments of distribution by not assuming any distribution at all. The first group consists of the Sharpe Ratio, Excess Return on Value-at-Risk and the Conditional Sharpe Ratio if the underlying Value-at-Risk and Conditional Value-at-Risk are computed with the parametric method. The second group is formed by the Adjusted Sharpe Ratio and the Modified Sharpe Ratio. The third group includes all performance measures based on Lower Partial Moments or drawdown.

As mentioned in Chapter 1, the first aim of this composition is to give an overview of the most frequently discussed and applied alternative performance measures in current literature of risk-adjusted performance measurement. Thus, the discussion of the performance measures in this piece of work does not claim to be a complete enumeration of the entire body of measures in literature. Instead, the performance measures have been selected by their prevalence in current literature.

## 3.2. Performance Measures based on Volatility

### 3.2.1. Sharpe Ratio

The first and most frequently used performance measure, which uses volatility as a risk measure is the Sharpe Ratio (Sharpe, 1966) The Sharpe Ratio (SR), also often referred to as “Reward to Variability”, divides the excess return of an asset  $i$  over a risk-free interest rate by the asset’s volatility.

$$(1) \quad SR_i = \frac{r_i^d - r_f}{\sigma_i}$$

$r_i^d$  ..... mean asset return<sup>4</sup>

$r_f$  ..... risk-free interest rate

$\sigma_i$  ..... standard deviation

As risk-averse investors prefer high returns and low volatility, the alternative with the highest Sharpe Ratio should be chosen when assessing investment possibilities (Scott & Horvath, 1980, p. 915).

Due to its simplicity and its easy interpretability the Sharpe Ratio has become one of the most widely used risk-adjusted performance measures (Weisman, 2002, p. 81). Yet, as indicated in the previous Chapter, there are some shortcomings of the Sharpe Ratio that need to be considered when employing it. Firstly, the Sharpe Ratio assumes normally distributed returns as it measures risk by volatility. Consequently, as shown in the example, the Sharpe Ratio might lead to wrong investment decisions when returns deviate from the normal distribution and in this case is not the right tool to measure risk-adjusted performance (Ingersoll, Spiegel & Goetzmann, 2007). Secondly, studies (Ingersoll et al., 2007, Leland, 1999) have shown that the Sharpe Ratio is prone to be manipulated through so-called information-free trading strategies. With these strategies a fund manager increases the fund’s performance by manipulative actions without actually adding value (Ingersoll et al., 2007, p. 1540). This is especially appealing for managers whose bonuses are correlated to the Sharpe Ratio of the assets they manage. In order to increase the Sharpe Ratio, they realize a gain in an early stage of the evaluation period and then invest the entire funds in a risk-free asset for the rest of the

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<sup>4</sup> This notation for the mean asset return and the risk-free interest rate -  $r_i^d$  respectively  $r_f$  shall henceforth be used in this work for all measures which require these parameters.

period. As the risk-free asset has a volatility of zero, the Sharpe Ratio converges towards infinity and so does the bonus of the manager (Ingersoll et al., 2007, p. 1541).

### 3.2.2. Adjusted Sharpe Ratio

As skewness and kurtosis of a return distribution, which both might influence an investor's decision, are the essential part of the Adjusted Sharpe Ratio and of other performance measures discussed in this composition, a brief description of these moments of distributions shall be given before the Adjusted Sharpe Ratio is introduced:

Skewness (S), the third standardized moment of a random variable, measures the asymmetry of the probability distribution of a random variable (r) around the mean ( $\mu$ ). It is defined by the formula below (Poddig, Dichtl & Petersmeier, 2003, p. 141).

$$(2) \quad S = \frac{\frac{1}{T} \sum_{t=1}^T (r_t - \mu)^3}{\sigma^3}$$

Skewness yields positive values for so-called right-skewed distributions in which the major part of values is concentrated on the left side of the distribution. Negative values for skewness indicate a left-skewed distribution with more values on the right side. Applied to an investment context, skewness can be interpreted as a risk parameter. Returns with a positively skewed distribution density exhibit a lower probability of losses (with the same expected return) and a higher probability for positive extreme values. Conversely, with negatively skewed distribution densities negative extreme values are more likely. Thus, a risk-averse investor prefers returns with positively skewed distributions (Poddig, Dichtl & Petersmeier, 2003, p. 142).

Kurtosis is the fourth standardized moment of a random variable and describes the amount of "peakedness" of a distribution compared to the normal distribution. A distribution with a high amount of "peakedness" shows a concentration of values around the mean and at the tails of a distribution. Thus, the probability of extreme values is higher than in the normal distribution. Consequently, risk-averse investors prefer return distributions with low kurtosis. Kurtosis (E) is calculated by the formula below (Poddig, Dichtl & Petersmeier, 2003, p. 143).

$$(3) \quad E = \frac{\frac{1}{T} \sum_{t=1}^T (r_t - \mu)^4}{\sigma^4} - 3$$

The expression more precisely describes the amount of excess kurtosis over the normal distribution as it already takes into account that the normal distribution has a kurtosis of 3. Summing up the findings above, risk-averse investors prefer positively skewed return distributions with low kurtosis.<sup>5</sup> This fact is either explicitly or implicitly accounted for in the risk-adjusted performance measures presented in the following chapters.

The Adjusted Sharpe Ratio (ASR) belongs to the group of measures in which skewness and kurtosis are explicitly included. Its creators Pezier and White (2006) were motivated by the drawbacks of the Sharpe Ratio, especially those caused by the assumption of normally distributed returns, and therefore suggested an Adjusted Sharpe Ratio to overcome this deficiency.

The measure is derived from a Taylor series expansion of expected utility with an exponential utility function. Keeping the first four terms of the expansion leads to the formula of the ASR stated below, where SR stands for the original Sharpe Ratio, S for skewness and E for excess kurtosis (Pezier & White, 2006, p. 15).

$$(4) \quad ASR_i = SR_i \left[ 1 + \left( \frac{S}{6} \right) SR_i - \left( \frac{E}{24} \right) SR_i^2 \right]$$

The ASR accounts for the fact that investors prefer positive skewness and negative excess kurtosis, as it contains a penalty factor for negative skewness and positive excess kurtosis. If S is negative and E is positive the ASR gets smaller compared to the traditional Sharpe Ratio. If the returns are normally distributed S and E are equal to zero and the formula for the Adjusted Sharpe Ratio yields the same values as the traditional Sharpe Ratio.

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<sup>5</sup> For a detailed proof of an investor's preference towards the moments of a distribution see Scott & Horvath (1980).

### 3.3. Performance Measures based on Value-at-Risk

#### 3.3.1. Excess Return on Value-at-Risk

The following group of risk-adjusted performance measures uses Value-at-Risk (VaR) and its modification the Conditional Value-at-Risk (CVaR) to account for risk.

The popular concept of VaR describes the expected maximum loss over a target horizon within a given confidence level  $\alpha$ .<sup>6</sup> For example, a 10-day VaR of 10 % at a  $\alpha=0.05$  confidence level means that the maximum loss in the next 10 days will not exceed 10% of an asset's value in 95% (=100(1- $\alpha$ )%) of all cases. Using these numbers for a total asset value of 10 million, the VaR is 1 Mio. So in 95% of all cases the loss will not exceed 1 million in the next 10 days.

VaR has become an essential tool for communicating risk to managers, directors and shareholders as it captures downside risk in a single figure which is easy to interpret. It is derived from probability distributions and it can, for instance, be modeled by considering the empirical distribution – i.e. by taking the  $[(1-\alpha) \cdot N]^{\text{th}}$  realization of a sample of  $n$  realizations as a measure for VaR. For a sample of 100 return observations sorted in descending order, for example, the empirical VaR is the 95<sup>th</sup> return observation. Other methods are simulation and parametric approximation - e.g. through the parameters of the normal distribution (Eisele, 2004, p. 113). In this work the parametric approximation shall be used first and then be compared to the results of the empirical VaR.

Assuming normally distributed returns, the VaR of a long-position is calculated as a quantile of the standard normal distribution at a certain confidence level  $\alpha$ , using the expected value – i.e. the mean - and the standard deviation (Jorion, 2006, p. 110).

$$(5) \quad VaR_i = -(r_f^d + z_\alpha \cdot \sigma_i)$$

$\alpha$  ..... confidence level

$z_\alpha$  ..... quantile of the standard normal distribution

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<sup>6</sup> For a comprehensive discussion of the concept of Value-at-Risk and its applications see for example Jorion (2006).

For the case of normally-distributed returns VaR can be graphically illustrated by the following figure:

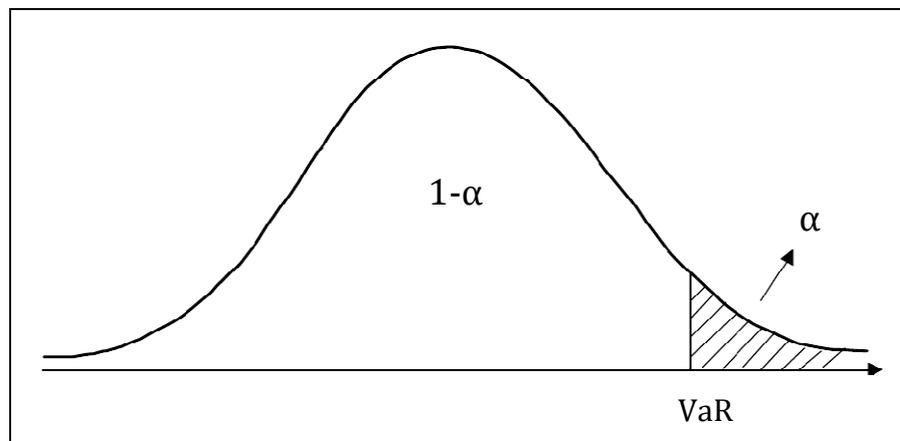


Figure 2: Density function of loss and Value-at-Risk (Source modified from Poddig, Dichtl & Petersmeier, 2003, p. 138)

However, there are some problems in the application of VaR. Firstly, VaR is heavily criticized (e.g. Embrechts, 2000, p. 453) for not being a coherent risk measure according to the requirements proposed by Artzner, Delbaen, Eber and Heath (1997). It is not, as it lacks the property of subadditivity. Subadditivity refers to the fact that the total risk of a portfolio should be smaller than or equal to the sum of the stand-alone risks of its components as it is supposed to reflect diversification effects (Kremer, 2008, p. 24). As the VaR lacks subadditivity it does not encourage diversification (Zakamouline, 2010, p. 8). This is however not true for elliptical distributions (like the normal distribution) as VaR does fulfill the subadditivity criterion for these distributions (McNeil et al., 2005, p. 114).

Secondly, the results of VaR are sensitive to the underlying parameters (for example  $\alpha$ ) and to the methods of calculation employed and the results therefore can vary substantially (Beder, 1995, p.12). This shall also be shown in Chapter 4.2. where different computation methods of VaR are compared. Thirdly, VaR does not make a statement about the loss if VaR is exceeded. To remedy against this shortcoming, Artzner et al. (1997) have introduced the Conditional Value-at-Risk which shall be discussed in the following subchapter.

When VaR is used to assess risk-adjusted performance, the measure Excess Return on VaR (EVAR) emerges. It was developed by Dowd (2002) and compares the excess return of an asset to the VaR of the asset. EVAR can be calculated by the following formula.

$$(6) \quad EVAR_i = \frac{r_i^d - r_f}{VaR_i}$$

### 3.3.2. Conditional Sharpe Ratio

As mentioned above, in order to overcome the shortcoming of VaR, which does not consider losses outside of the  $(1-\alpha)$ -confidence interval, the Conditional Value-at-Risk (CVaR) has been developed. CVaR describes the expected loss under the condition that VaR is exceeded. Therefore only the values of the distribution that exceed the VaR are considered when calculating CVaR. Following the interpretation of VaR as maximum expected loss in  $100 \cdot (1-\alpha)\%$  of cases, CVaR can be interpreted as average loss in  $100 \cdot \alpha\%$  of cases (Albrecht & Koryciorz, 2003, p. 2). Similar to the VaR, CVaR can be calculated either empirically or parametrically, using the parameters of the normal distribution. In the case of normally distributed returns CVaR is described by the following expression (Albrecht & Koryciorz, 2003, S. 6).

$$(7) \quad CVaR_i = r_i^d + \frac{\varphi(N_{1-\alpha})}{\alpha} \sigma_i$$

$N_{1-\alpha}$  ...  $(1-\alpha)$ - quantile of the standard normal distribution

$\varphi$  ..... density function of the standard normal distribution

It is shown by Acerbi & Tasche (2002) that CVaR is a coherent measure of risk as it fulfills all axioms proposed by Artzner et al. (1997). Therefore, CVaR is an effective response to the deficiency in coherence of VaR.

Using CVaR for assessing risk-adjusted performance yields the Conditional Sharpe Ratio (CSR). The CSR contrasts the excess return over a risk-free interest rate to the CVaR of an asset and has first been used for performance measurement by Argawal and Naik (2004). The CSR is described by the following expression.

$$(8) \quad CSR_i = \frac{r_i^d - r_f}{CVaR_i}$$

### 3.3.3. Modified Sharpe Ratio

If EVaR and CVaR are computed with the parametrical method normally distributed returns are assumed even if returns deviate from the normal distribution in reality. Only when these measures are calculated empirically deviations from the normal distribution are taken into account. This can only be done if empirical information on the return distribution is available. The performance measure presented in this section, however, accounts for the higher moments of distributions, by directly modifying a parameter of the normal distribution. Therefore no empirical data is necessary.

The performance measure is based on Modified Value-at-Risk (MVaR) which adjusts VaR for skewness and kurtosis. In particular, this is done by modifying the quantile of the standard normal distribution using a Cornish-Fisher-Expansion (Gregoriou & Gueyie 2002; Favre & Galeano, 2003). The modified quantile  $z_{CF}$  is calculated by the following expression with skewness (S) and excess kurtosis (E).

$$(9a) \quad z_{CF} = z_{\alpha} + \frac{1}{6}(z_{\alpha}^2 - 1)S + \frac{1}{24}(z_{\alpha}^3 - 3z_{\alpha})E - \frac{1}{36}(2z_{\alpha}^3 - 5z_{\alpha})S^2$$

The modified quantile is then used to calculate the Modified Value-at Risk.

$$(9b) \quad MVaR_i = -(r_i^d + z_{CF} * \sigma)$$

If MVaR is used to measure risk-adjusted performance the Modified Sharpe Ratio (MSR), described by the following equation, emerges.

$$(10) \quad MSR_i = \frac{r_i^d - r_f}{MVaR_i}$$

The MSR has first been employed by Gregoriou and Gueyie (2002) and yields the same results as the Excess Return on VaR if returns are normally distributed as in this case, S and E are equal to zero. It is also thinkable to modify the Conditional Value-at-Risk with the Cornish-Fisher-Expansion to create a measure which compares excess return to the “Modified Conditional Value-at-Risk”. This modification has not been used in literature yet but it would be a logical consequence to adjust for skewness and kurtosis of returns in order to create a counterpart to the Conditional Sharpe Ratio. Due to the lack of theoretical foundation, this measure shall not be considered in the calculations later in this work.

The common ground of EVaR, CVar and MSR is the fact that, similar to the Sharpe Ratio, a risk-averse investor prefers the alternative with the highest value as he or she prefers high returns and low risk.

### 3.4. Performance Measures based on Lower Partial Moments

#### 3.4.1. Omega

Another approach to overcome the deficiencies of traditional performance measures if returns deviate from the normal distribution, are performance measures based on Lower Partial Moments (LPM). Lower Partial Moments measure risk by considering only those deviations that fall below an ex-ante defined threshold. This can either be the mean of the distribution or another kind of minimum return. An LPM of order  $m$  can be empirically estimated from a sample of  $n$  returns using the formula for discrete observations presented below, where  $r_i$  is a single return realization and  $\tau$  is the minimum return threshold (Kaplan & Knowles, 2004, p. 3).

$$(11) \quad LPM_m = \frac{1}{n} \sum_{i=1}^n \max[\tau - r_i; 0]^m$$

The major advantage of measuring risk with LPM is that no parametric assumptions (e.g. on mean and standard deviation) are needed and that there are no constraints on the form of the underlying distribution (Shadwick & Keating, 2002, p. 8). At the same time, measuring risk by LPM is subject to criticism as the set of sample returns deviating negatively from the return threshold may be small or even empty. For these cases parametric methods such as the fitting of a three-parameter lognormal distribution can be deployed (Sortino, 2001; Forsey, 2001). Shadwick and Keating (2002, p. 9), however, point out possible shortcomings of this method and recommend cautiousness in assessing performance in such cases.

The choice of the order of the LPM determines how strongly negative deviations from the target return are weighted. The LPM of order 0 counts the relative amount of realizations below the return threshold and can be interpreted as default probability (Poddig, Dichtl & Petersmeier, 2003, p. 136). The LPM of order 1 is sometimes referred to as downside potential and can be seen as the average or expected loss (e.g. Bacon, 2008, p. 94). For the case where the target return is equal to the mean of the distribution, the LPM of order 2 corresponds to the semi-variance (Bürkler & Hunziker,

2008, p. 16). In all other cases it is referred to as downside variance (Bacon, 2008, p. 94). The order of the LPM shall be chosen according to an investor's preferences, whereby high orders stand for a high degree of risk-aversion. Integrity in the choice of numbers is not necessary (Poddig, Dichtl & Petersmeier, 2003, p. 135).

Applying the Lower Partial Moments of order 1, 2 and 3 to performance measurement yields the performance measures Omega, Sortino Ratio and Kappa 3. Originally Omega and the Sortino Ratio did not explicitly include LPM as a risk-measure. The categorization of Omega, Sortino Ratio and Kappa 3 according to the number of order of LPM traces back to the work of Kaplan and Knowles (2004), who tried to find a more general description of performance measures. In the following sections both, the general and the original versions of Omega and Sortino Ratio shall be discussed.

The first performance measure based on LPM is Omega. The original version was developed by Shadwick and Keating (2002) and is defined by the following expression, where  $x$  is the random one-period rate of return,  $F(x)$  is the cumulative distribution of the one-period return,  $L$  is a minimum return threshold and with  $a$  and  $b$  representing the upper and lower bound of the distribution (Kazemi, Gupta & Schneeweis, 2003, p. 1).

$$(12a) \quad \Omega(L) = \frac{\int_L^b [1-F(x)] dx}{\int_a^L F(x) dx}$$

Kazemi, Gupta and Schneeweis (2004, p. 2) argue that Omega is not an entirely new concept in finance as Omega can be described by the ratio between the price of a European call option  $C$  in the numerator and the price of a European put option  $P$  in the denominator. This yields expression (12b) with  $\tau$  being the strike price of the put option.

$$(12b) \quad \Omega(\tau) = \frac{C(\tau)}{P(\tau)} = \frac{E[\max(x-\tau;0)]}{E[\max(\tau-x;0)]}$$

The price of the put option can be interpreted as the LPM of order 1. Further transformations lead to expression (12c) – the version of Omega which shall be used henceforth in this composition<sup>7</sup>. In this version, which clearly resembles the original

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<sup>7</sup> For a detailed proof of the transformation see the Appendix of Kazemi, Gupta and Schneeweis (2004).

Sharpe Ratio, Omega is the ratio of the excess return over a threshold  $\tau$  and the LPM of order 1.

$$(12c) \quad \Omega_i(\tau) = \frac{r_i^d - \tau}{LPM_1(\tau)} + 1$$

Expression (12c) contains exactly the same information about the performance of an asset as the original expression (12a) and thus also leads to the same rankings. The main advantage of the transformed version is that it is more intuitive as it resembles the Sharpe Ratio in its structure. That is why Kazemi et al. (2004, p. 1) refer to the first part of expression (12c) as the Omega-Sharpe Ratio.

### 3.4.2. Sortino Ratio

The second LPM-based performance measure is the Sortino Ratio, which was first introduced by Sortino and van der Meer (1991). It is defined as the ratio of the excess return over a minimum threshold  $\tau$  and the downside deviation  $\delta^2$ . Originally, the Sortino Ratio (SOR) and  $\delta^2$  were calculated by the following expressions.

$$(13a) \quad SOR_i(\tau) = \frac{r_i^d - \tau}{\delta^2} \quad \text{with}$$

$$(13b) \quad \delta_i^2 = \int_{-\infty}^{\tau} (\tau - r)^2 f(r) dr$$

The Sortino Ratio can be regarded as a modification of the Sharpe Ratio as it replaces the standard deviation by downside deviation which only considers the negative deviations from the mean or a minimum return threshold. Similar to Omega, downside deviation can be interpreted as the square root of the LPM of order 2 which finally leads to the version of the Sortino Ratio below in which an LPM is used as a risk measure (Kaplan & Knowles, 2004, p. 3).

$$(13c) \quad SOR_i(\tau) = \frac{r_i^d - \tau}{\sqrt{LPM_2(\tau)}}$$

Compared to the Omega Ratio, negative deviations from the return threshold are more strongly weighted due to the LPM of order 2 and thus, express a higher risk-aversion of the investor (Poddig, Dichtl & Petersmeier, 2003, p. 135).

### 3.4.3. Kappa 3

Motivated to find a more generalized risk-adjusted performance measure, Kaplan and Knowles (2004) developed the Kappa-measure. They showed that Omega and the Sortino Ratio are only special cases of Kappa, whereby the parameter  $n$  of Kappa determines whether the Sortino Ratio, Omega, or another risk-adjusted return measure is generated. The general form of Kappa is described by the expression below.

$$(14) \quad K_n(\tau) = \frac{r_i^d - \tau}{\sqrt[n]{LPM_n(\tau)}}$$

Choosing the parameter so that  $n=1$  respectively  $n=2$  yields Omega ( $=K_1$ ) respectively the Sortino Ratio ( $=K_2$ ). In general, any number is possible for the parameter  $n$ . Kappa 3 ( $=K_3$ ) however, seems to be the most frequently used version of the Kappa measure in literature (e.g. Kaplan & Knowles, 2004; Eling & Schumacher, 2006; Glawischnig, 2007). Thus,  $K_3$  shall also be applied in the empirical examination of performance measures in this composition.

### 3.4.4. Gain-Loss Ratio and Upside-Potential Ratio

Two performance measures which both combine Lower Partial Moments (LPM) and Higher Partial Moments (HPM) to measure risk-adjusted performance are the Gain-Loss-Ratio by Bernardo-Ledoit (2000) and the Upside-Potential-Ratio by Sortino, van der Meer and Plantinga (1999).

The Gain-Loss Ratio (Bernardo-Ledoit, 2000) compares the expected value of positive returns  $E(R^+)$  to the expected value of negative returns  $E(R^-)$ . Positive returns are the returns in a distribution which surpass a certain return threshold. Consequently, negative returns are the ones that do not exceed the threshold. Similar to Omega, the Sortino Ratio and Kappa 3, negative expected returns are measured by LPMs. The difference is that the excess return in the numerator is replaced by the expected value of positive returns which can be regarded as the Higher Partial Moment of order 1 of the return distribution (Eling & Schumacher, 2006, p. 431). The Gain-Loss Ratio is described by the following expression.

$$(15) \quad GLR_i(\tau) = \frac{\text{Expected Value } (R^+)}{\text{Expected Value } (R^-)} = \frac{HPM_1(\tau)}{LPM_1(\tau)} = \frac{\frac{1}{n} \sum_{i=1}^n \max[r_i - \tau; 0]^m}{\frac{1}{n} \sum_{i=1}^n \max[\tau - r_i; 0]^m}$$

It can be shown that the GLR in the form presented above is equal to Omega (Kazemi, Gupta & Schneeweis, 2004, p. 6). This is only the case if a minimum return threshold  $\tau$  is assumed. To avoid confusion it should be pointed out that in the original version of the GLR, which Bernardo-Ledoit introduced at the beginning of his paper (2000, p. 10), no return threshold is explicitly defined and thus  $\tau=0$ . This version of the GLR can also be found in literature (e.g. Bacon, 2008, p. 95). For this case, Omega does not yield the same values as the GLR. For the application of this performance measure in Chapter 4 of this composition, the version with a minimum return threshold shall be used.

The creation of the Upside-Potential-Ratio (UPR) by Sortino et al. (1999) was motivated by the findings of Shefrin (1998) who argues that investors seek upside potential with downside protection – i.e. that investors are risk-seeking above a certain return threshold risk-averse below this threshold. As a result, Sortino et al. (1999) suggested the Upside-Potential Ratio (UPR), which reflects the investor’s wish for downside protection by more strongly weighting negative deviations from the minimum return threshold. More specifically, the UPR measures the upside potential of an asset in form of the expected value of the positive returns over a minimum threshold in the numerator relative to the downside deviation in the denominator. Analogously to the GLR, the average positive returns can be calculated by the HPM of order 1 and downside deviation can be described by the LPM of order 2 which leads to the formula below.

$$(16) \quad UPR_i(\tau) = \frac{HPM_1(\tau)}{\sqrt[2]{LPM_2(\tau)}}$$

Due to the use of the LPM of order 2, negative deviations from the mean are weighted more strongly and thus, compared to the GLR, a stronger risk-aversion of the investor is assumed (Bacon, 2008, p. 97).

### 3.4.5. Farinelli-Tibiletti Ratio

The Farinelli-Tibiletti Ratio named after their developers Farinelli and Tibiletti (2008) represents a more generalized measure of the Gain-Loss-Ratio and the Upside-Potential-Ratio presented above. In the original version of the Farinelli-Tibiletti Ratio (FTR), expected values of returns over and below a return threshold, raised to the power of  $p$  respectively  $q$ , are compared (Farinelli & Tibiletti, 2008, p. 4). Similar to the other LPM-based measures presented above, the FTR can also be described as the ratio of a Higher

Partial Moment (HPM) of order  $p$  and a Lower Partial Moment (LPM) of order  $q$ . Both versions are depicted in the expression below.

$$(17) \quad FTR_i(p, q, \tau) = \frac{E^{1/p}[\{(r_i - \tau)^+\}^p]}{E^{1/q}[\{(r_i - \tau)^-\}^q]} = \frac{\sqrt[p]{HPM_p(\tau)}}{\sqrt[q]{LPM_q(\tau)}}$$

The parameters  $p$  and  $q$  are some real numbers and are chosen according to the investor's preferences<sup>8</sup>. They determine whether an investor is risk-seeking, risk-neutral or risk-averse above (for  $p$ ) or below (for  $q$ ) a reference point or return threshold  $\tau$ . If  $p = 1$  and  $q = 1$  the investor is risk-neutral above and below  $\tau$ . If  $0 < p < 1$  the investor is risk-averse above  $\tau$ . Contrarily, if  $p > 1$  the investor is risk-seeking above  $\tau$ . Similarly, if  $0 < q < 1$  the investor is risk-seeking below  $\tau$  and risk-averse below  $\tau$  for  $q > 1$  (Zakamouline, 2010, p. 10).

The FTR can be flexibly tailored to the individual preferences of an investor. If an investor who is acting according to the Expected Utility Theory shall be modeled,  $q > 1$  and  $0 < p < 1$  are assumed as the investor is risk-averse below and above the reference point. The prospect theory of Kahneman and Tversky (1979) states that an investor is risk-seeking below and risk-averse above the reference point. In this case  $0 < q < 1$  and  $0 < p < 1$  are used. An investor acting according to Markovitz (1952) has a concave utility function (i.e. risk-averse) below and a convex (i.e. risk-seeking) utility function above the reference point, which would explain why people buy insurance and lottery tickets at the same time. This preference is modeled by  $q > 1$  and  $p > 1$  (Zakamouline, 2010, p. 10).

The Gain-Loss-Ratio (GLR) and the Upside-Potential-Ratio (UPR) are special cases of the FTR (Farinelli-Tibiletti, 2008, p. 1546). The GLR uses  $p = 1 / q = 1$  and thus assumes a risk-neutral investor. With  $p = 1 / q = 2$  the UPR models an investor who is risk-averse below the reference point – i.e. has a negative preference for losses – and risk-neutral above the reference point.

<sup>8</sup> The underlying utility function  $U(r)$  takes the form  $(r - \tau)^p$  when  $r \geq \tau$  and  $\gamma(\tau - r)^q$  when  $r < \tau$ .

### 3.5. Performance Measures based on Drawdown

#### 3.5.1. Calmar Ratio

The fourth category of risk-adjusted performance measures consists of performance measures that use drawdown in the denominator to measure risk. The drawdown of an asset describes the loss incurred over a certain period of time. With reference to Eling and Schumacher (2006, p. 432) the following notation shall be introduced:  $r_{it-\tau}$  describes the realized return of an asset within the period  $t - \tau$  ( $t < \tau < T$ ). Among the single return realizations in this period,  $MD_1$  is the smallest return (i.e. largest drawdown),  $MD_2$  the second smallest and so on. Using either the maximum drawdown ( $MD_1$ ), an average of the  $N$  largest drawdowns, or some kind of variance of the  $N$  largest drawdowns, the performance measures Calmar-Ratio, Sterling-Ratio and Burke-Ratio emerge.

The first performance measure based on drawdown is the Calmar-Ratio (CR). Introduced by Young (1991), it is defined by the excess return over the risk-free interest rate divided by the maximum loss – i.e. the maximum drawdown - incurred in the discussed period.

$$(18) \quad CR_i = \frac{r_i^d - r_f}{-MD_1}$$

The negative sign of the drawdown is a convention such that the denominator becomes positive and as a result high values of the denominator stand for a high amount of risk<sup>9</sup>. The risk-free interest rate was originally not part of the Calmar-Ratio. However, the notation above is the version generally applied in literature. According to Bacon (2008, p. 89) this alteration reflects the move from commodities and futures to traditional portfolio management.

#### 3.5.2. Sterling Ratio

The second performance measure based on drawdown discussed in this composition is the Sterling-Ratio. Instead of the maximum drawdown, the Sterling-Ratio uses the average of a certain number  $N$  of smallest drawdowns of an asset within a certain period of time to measure risk. As a result, the Sterling-Ratio is less sensitive to outliers than the Calmar-Ratio. The origins of this ratio are attributed to Deane Sterling Jones although there is no scientific paper or article written by this author that describes this

<sup>9</sup> The same convention applies to the Sterling Ratio and VaR.

ratio. This fact, however, has not hampered the widespread acceptance and use of this ratio in literature. The Original Sterling-Ratio (OSTR) takes the following form (Glawischnig, 2007, p.11).

$$(19a) \quad OSTR_i = \frac{r_i^d}{\left(\frac{1}{N} \sum_{j=1}^N -MD_j\right) + 10\%}$$

In the original version the denominator is the average of the N smallest drawdowns plus 10%. The choice of 10% is arbitrary and is intended to compensate for the fact that the average of the maximum drawdowns is always smaller than the maximum drawdown (Bacon, 2009, p. 11). Again, modifications to this measure have been made. The return has been replaced by the excess return over the risk-free interest rate and the addition of 10% has been omitted. The version of the Sterling-Ratio which is currently prevalent across literature – and thus shall be used for the calculations in Chapter 4 – is described by the following expression (Lhabitant, 2004, p.84).

$$(19b) \quad STR_i = \frac{r_i^d - r_f}{\frac{1}{N} \sum_{j=1}^N -MD_j}$$

### 3.5.3. Burke Ratio

The third ratio based on drawdown which has found widespread use in literature is the Burke Ratio. In this measure proposed by Burke (1994) risk is described by the square root of the sum of the squares of the N smallest drawdowns of an asset within a certain period of time.

$$(20) \quad BR_i = \frac{r_i^d - r_f}{\sqrt{\sum_{j=1}^N MD_j^2}}$$

Similar to the Sterling-Ratio, the Burke-Ratio is less sensitive to outliers than the Calmar-Ratio. Furthermore, as the square of the largest drawdowns is used, the major drawdowns among the N largest drawdowns are weighted more strongly compared to the smaller ones. This is done in order to account for the fact that few very large losses represent a bigger risk than several smaller ones (Bacon, 2008, p. 91).

## 4. Application of Risk- Adjusted Performance Measures

### 4.1. Performance Analysis

In this chapter the performance measures presented in the previous section of this work shall be applied to a portfolio of bank products. Following the research of other authors, which has been presented in section 2.3., this empirical analysis it shall answer two questions: First, it shall determine whether alternative performance measures yield results that differ significantly from those obtained when performance is measured with the Sharpe Ratio. And second, it is supposed to show if the computed results differ among the alternative performance measures.

The examined data consists of 210 continuous, annualized monthly returns of a portfolio of Swiss bank products in two scenarios E and F with three management strategies each (E0, E1, E2 respectively F0, F1, F2). First of all, the distribution of returns will be analyzed. Secondly, the performance measures shall be applied to the data. Based on these results, a ranking of the different strategies will be established for each performance measure. In the next step, the correlation between the rankings of the different performance measures shall be determined based on Spearman's rank correlation coefficient which is the most frequently used method for this purpose (e.g. Heidorn, 2009; Glawischnig, 2007; Eling & Schumacher; 2006). Finally, following the analysis methods of Eling and Schumacher (2006), the parameters, which needed to be assumed arbitrarily will be varied to determine their impact on the ranking results.

Table 1 summarizes the general analysis of the distribution of returns. The examined parameters are the mean, the standard deviation, skewness and excess kurtosis.

	Scenario E			Scenario F		
	E0	E1	E2	F0	F1	F2
Mean	0.47%	0.60%	0.69%	0.46%	0.64%	0.70%
Standard Deviation	0.51%	0.76%	0.77%	0.39%	0.61%	0.62%
Skewness	-0.39	0.03	-0.16	1.04	0.91	0.84
Excess Kurtosis	-0.31	-0.72	-0.67	0.16	-0.30	-0.48

Table 1: General analysis of the return distributions

Strategy 0 exhibits the lowest mean and the lowest standard deviation in both scenarios whereas strategy 2 shows the highest mean with the highest volatility in both scenarios. This implies that strategy 2 promises more reward in comparison to the other strategies but at the same time is riskier. As mentioned in Chapter 3, investors prefer positive skewness and negative excess kurtosis. Strategies E1, F1 and F2 possess the combination of these two positive features. This should positively influence their performance when measures accounting for higher moments are applied. No strategy exhibits the negative combination of negative skewness and positive excess kurtosis but strategy E0 and E2 show some amount of negative skewness whereas strategy F0 is the only strategy with positive excess kurtosis.

In Table 2 to 7 the returns of each strategy are analyzed for being normally distributed. For this purpose Q-Q-Plots depicting the expected quantile of the standard normal distribution on the x-axis and the observed quantile on the y-axis are used.

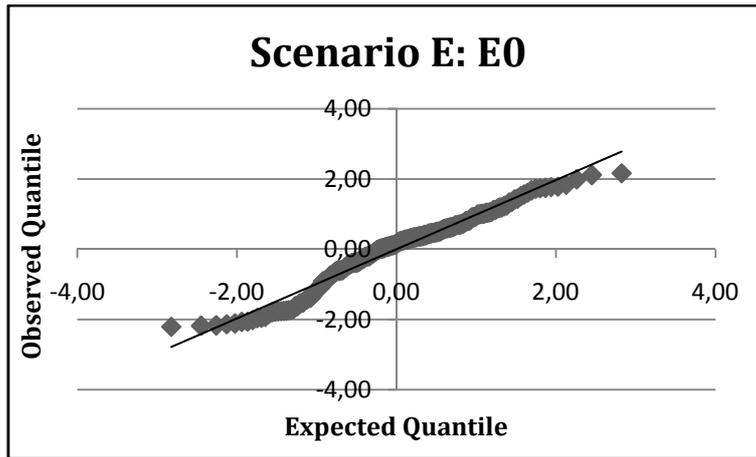


Table 2: Q-Q-Plot Scenario E: E0

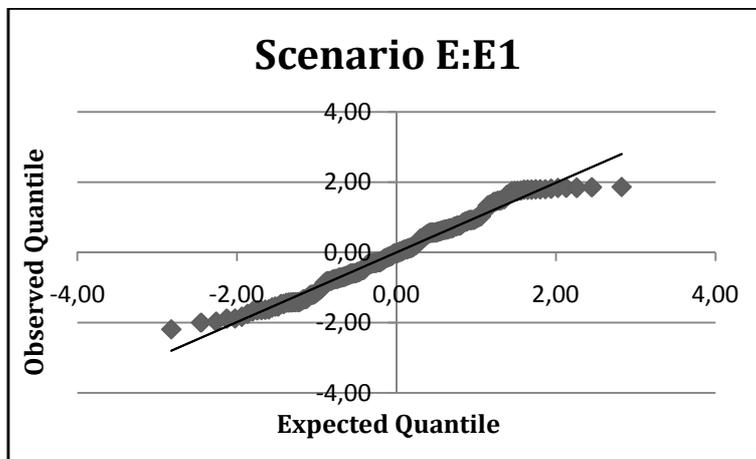


Table 3: Q-Q-Plot Scenario E: E1

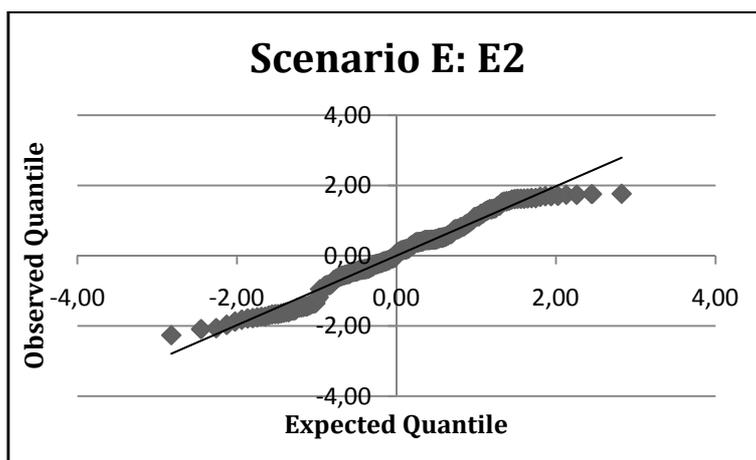


Table 4: Q-Q-Plot Scenario E: E2

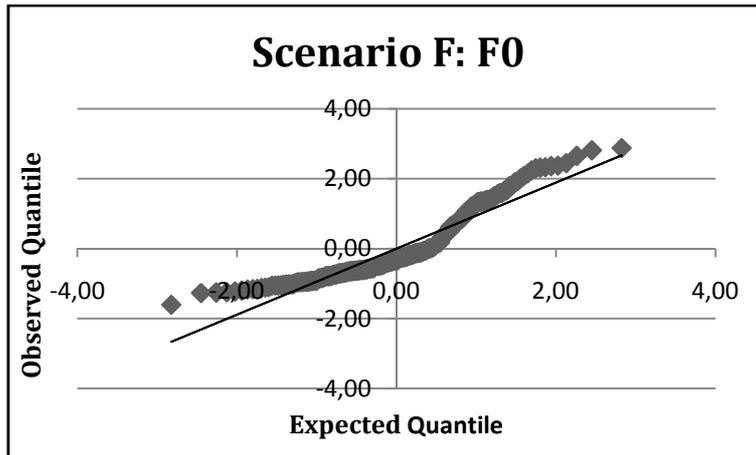


Table 5: Q-Q-Plot Scenario F: F0

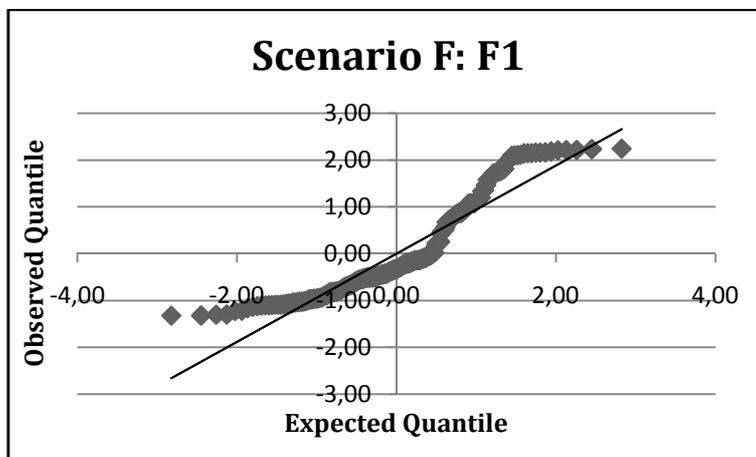


Table 6: Q-Q-Plot Scenario F: F1

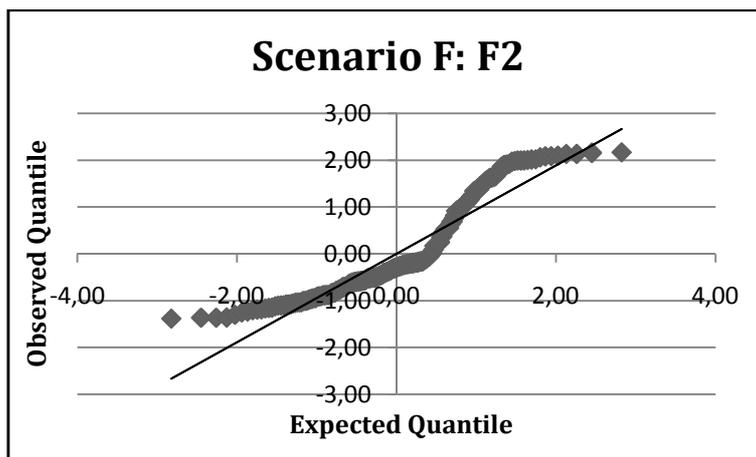


Table 7: Q-Q-Plot Scenario F: F2

For scenario E the Q-Q-Plots show that the observations only deviate little from the trend line. Therefore it can be assumed that the returns of all three strategies are normally distributed. This observation is confirmed analytically by the quality criterion  $\eta^2$  which takes values between 97 % and 99% in scenario E <sup>10</sup>. This range of  $\eta^2$  indicates a very good fit (Persike, 2008, p. 5). In scenario F,  $\eta^2$  is around 88% which is still a good fit. However, looking at the Q-Q-Plots the situation is less clear with significantly more observations deviating from the trend line, especially at the tails of the distribution. Hence, it shall be interesting to observe the impact of these outliers in scenario F by comparing the results of the alternative performance measures which account for divergences from the normal distribution to those obtained by the Sharpe Ratio which assumes a normal distribution of returns.

Table 8 shows the results of the performance analysis. The examined sample mainly contains positive returns. Yet, the measures based on VaR and on drawdown require negative returns. Thus, in order to obtain reasonable results, all observations have been corrected for a minimum return threshold of  $\tau = 0.34$ <sup>11</sup>. This is also the risk-free interest rate and the minimum return threshold that has been used in all the other ratios which require these parameters. Following Zakamouline (2010), the Farinelli-Tibiletti Ratio has been calculated for three different pairs of the parameters p and q. With  $p=0.5/q=2$  an investor acting according to the Expected Utility Theory has been modeled. The choice of  $p=1.5/q=2$  reflects Markovitz' theory on utility. Finally, the pair of parameters  $p=0.8/q=0.85$  represent the assumptions of the Prospect Theory. The ratios based on Value-at-Risk have been calculated for a confidence level of  $\alpha=0.01$  and the parameter-based method has been applied. For the ratios based on drawdown, i.e. the Sterling Ratio and the Burke Ratio, the N= 5 largest drawdowns have been considered.

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<sup>10</sup>The equation for calculating the criterion is  $\eta^2 = (1 - \frac{s_e^2}{s^2}) * 100$ ; where  $s^2$  is the total variance and  $s_e^2$  is the unexplained variance of the observations.

<sup>11</sup> LIBOR in CHF for 6-months on 12. 4. 2010

	Scenario E			Scenario F		
	E0	E1	E2	F0	F1	F2
SR	0.24	0.33	0.45	0.30	0.49	0.57
Adjusted SR	0.24	0.33	0.45	0.31	0.53	0.62
Excess Return on VaR	0.17	0.21	0.32	0.26	0.39	0.47
Conditional SR	0.07	0.10	0.13	0.31	0.28	0.30
Modified SR	0.15	0.25	0.33	16.23	2.27	2.82
Omega	7.75	7.24	8.46	150.54	117.41	139.66
Sortino Ratio	0.69	1.05	1.46	9.56	12.26	14.69
Kappa 3	0.49	0.74	1.02	4.12	6.95	8.29
Gain-Loss-Ratio	7.75	7.24	8.46	150.54	117.41	139.66
Upside-Potential Ratio	2.98	2.88	3.30	16.91	26.46	29.11
FTR <sub>0.5/2</sub>	2.24	1.96	2.43	6.57	19.76	22.70
FTR <sub>1.5/2</sub>	3.40	3.45	3.83	25.04	31.63	34.24
FTR <sub>0.8/0.85</sub>	9.88	8.81	10.67	233.31	172.37	217.22
Calmar Ratio	0.19	0.24	0.33	0.70	1.88	2.20
Sterling Ratio	0.19	0.28	0.39	2.09	2.10	2.48
Burke Ratio	0.09	0.12	0.18	0.66	0.93	1.10

*Table 8: Results of the performance analysis*

Based on the Sharpe Ratio, investment strategies E2 and F2 yield the best performance results whereas strategies E0 and F0 show the lowest performance in their scenarios. At first glance most of the other performance measures approximately lead to the same conclusion. A ranking of the strategies according to their performance in the two scenarios helps to clarify this impression. This ranking is depicted in Table 9.

	Scenario E			Scenario F		
	E0	E1	E2	F0	F1	F2
SR	3	2	1	3	2	1
Adjusted SR	3	2	1	3	2	1
Excess Return on VaR	3	2	1	3	2	1
Conditional SR	3	2	1	1	3	2
Modified SR	2	3	1	1	3	2
Omega	2	3	1	1	3	2
Sortino Ratio	3	2	1	3	2	1
Kappa 3	3	2	1	3	2	1
Gain-Loss-Ratio	2	3	1	1	3	2
Upside-Potential Ratio	2	3	1	3	2	1
FTR <sub>0.5/2</sub>	2	3	1	3	2	1
FTR <sub>1.5/2</sub>	3	2	1	3	2	1
FTR <sub>0.8/0.85</sub>	2	3	1	1	2	3
Calmar Ratio	3	2	1	3	2	1
Sterling Ratio	3	2	1	3	2	1
Burke Ratio	3	2	1	3	2	1

Table 9: Ranking of the investment strategies

The ranking of scenario E draws a rather clear picture. Strategy E2 ranks first across all performance measures. However, for Omega respectively the Gain-Loss Ratio<sup>12</sup>, the Upside-Potential Ratio, the FTR<sub>0.5/2</sub> and the FTR<sub>0.8/0.85</sub> ranks two and three are interchanged. These results are not astonishing as the returns of scenario E are largely normally distributed. As it was shown by Zakamouline (2010, p. 4), if returns are normally distributed all alternative performance measures show high, nearly perfect rank correlations to the Sharpe Ratio.

In scenario F the rankings vary slightly but strategy F2 still ranks first in 11 out of 16 ratios and strategy F0 predominantly ranks last. Remarkably, the Conditional Sharpe Ratio, the Modified Sharpe Ratio, Omega respectively the Gain-Loss Ratio and the FTR<sub>0.8/0.85</sub> yield rankings which are the complete opposite of the Sharpe Ratio with strategy F0 in the first rank and strategy F2 in the last rank.

For the Conditional Sharpe Ratio, the deviation of the ranking might depend on the fact that the CSR has been calculated by the parametric method. Thus, in Chapter 4.2., when examining the influence of variations in parameters, the rankings obtained by the

<sup>12</sup> As mentioned in chapter 3.4.4. Omega and the Gain-Loss Ratio are calculated by a different but equivalent formula so that the results are identical when a minimum return threshold is assumed.

parametric method should be compared to the rankings which result when the Conditional Value-at-Risk is calculated with the empirical method in order to verify if the difference to the Sharpe Ratio depends on the applied method.

For the Modified Sharpe Ratio the driver of the different ranking results compared to the Sharpe Ratio is mainly skewness. Strategy F0 has the highest positive skewness in scenario F, which leads to a lower Modified Value-at-Risk as in this ratio positive skewness is rewarded. As a result strategy F0 possesses the highest value for the Modified Sharpe Ratio. As strategy F2 has the lowest positive skewness of all strategies in scenario F, it also yields the lowest Modified Sharpe Ratio and therefore ranks last.

The explanation for the inverted ranking of Omega respectively the Gain-Loss Ratio and the  $FTR_{0.8/0.85}$  is not that clear at first sight. As both ratios largely depend on the assumed return threshold the ranking may change if the return threshold is changed. This hypothesis shall be investigated further in Chapter 4.2. when the underlying parameters are varied.

The overall results of the rankings for scenario E in Table 9 suggest that the rankings obtained by the respective performance measures are to a large extent the same which is explained by the largely normally distributed returns of scenario E. In scenario F the rankings are the same for the majority of performance measures but with significant exceptions. To support these observations analytically, the correlation of the ranks was calculated using Spearman's rank correlation coefficient  $\rho$ . It is calculated by the following formula (Zöfel, 2000, p. 127).

$$(21) \quad \rho = 1 - \frac{6 \cdot \sum_{i=1}^n d_i^2}{n^3 - n}$$

d ..... difference between the ranks of each observation

n ..... number of observations

The results of this analysis are shown in Table 10 for scenario E and in Table 11 for scenario F.

	Sharpe Ratio	Adjusted SR	EVaR	Conditional SR	Modified SR	Omega	Sortino Ratio	Kappa 3	GLR	UPR	FTR <sub>0.5/2</sub>	FTR <sub>1.5/2</sub>	FTR <sub>0.8/0.85</sub>	Calmar Ratio	Sterling Ratio	Burke Ratio	
Sharpe Ratio																	
Adjusted SR	1.00																
EVaR	1.00	1.00															
CSR	1.00	1.00	1.00														
Modified SR	1.00	1.00	1.00	1.00													
Omega	0.92	0.92	0.92	0.92	0.92												
Sortino Ratio	1.00	1.00	1.00	1.00	1.00	0.92											
Kappa 3	1.00	1.00	1.00	1.00	1.00	0.92	1.00										
GLR	0.92	0.92	0.92	0.92	0.92	1.00	0.92	0.92									
UPR	0.92	0.92	0.92	0.92	0.92	1.00	0.92	0.92	1.00								
FTR <sub>0.5/2</sub>	0.92	0.92	0.92	0.92	0.92	1.00	0.92	0.92	1.00	1.00							
FTR <sub>1.5/2</sub>	1.00	1.00	1.00	1.00	1.00	0.92	1.00	1.00	0.92	0.92	0.92						
FTR <sub>0.8/0.85</sub>	0.92	0.92	0.92	0.92	0.92	1.00	0.92	0.92	1.00	1.00	1.00	0.92					
Calmar Ratio	1.00	1.00	1.00	1.00	1.00	0.92	1.00	1.00	0.92	0.92	0.92	1.00	0.92				
Sterling Ratio	1.00	1.00	1.00	1.00	1.00	0.92	1.00	1.00	0.92	0.92	0.92	1.00	0.92	1.00			
Burke Ratio	1.00	1.00	1.00	1.00	1.00	0.92	1.00	1.00	0.92	0.92	0.92	1.00	0.92	1.00	1.00		
Average	0.97	0.97	0.97	0.97	0.95	0.97	0.97	0.94	0.95	0.94	0.93	0.94	0.96	0.97	0.97	0.97	0.97

Table 10: Rank correlations for scenario E

In general, all performance measures show high rank correlations to the Sharpe Ratio and to each other. As indicated above, the picture in scenario E is very clear with all ratios being perfectly correlated to the Sharpe Ratio except for Omega resp. the Gain-Loss Ratio and the Upside-Potential Ratio. The average rank correlation to the Sharpe Ratio in scenario E is 0.97. Comparing the alternative performance measures to each other, the lowest ranking coefficient is 0.92. This results if ranks 2 and 3 are interchanged in some measures. The total average of all ranking coefficients is 0.96.

	SR	Adjusted SR	EVaR	Conditional SR	Modified VaR	Omega	Sortino Ratio	Kappa 3	UPR	GLR	FTR <sub>0.5/2</sub>	FTR <sub>1.5/2</sub>	FTR <sub>0.8/0.85</sub>	Calmar Ratio	Sterling Ratio	Burke Ratio
SR																
Adjusted SR	1.00															
EVaR	1.00	1.00														
CSR	0.75	0.75	0.75													
Modified VaR	0.75	0.75	0.75	1.00												
Omega	0.75	0.75	0.75	1.00	1.00											
Sortino Ratio	1.00	1.00	1.00	0.75	0.75	0.75										
Kappa 3	1.00	1.00	1.00	0.75	0.75	0.75	1.00									
GLR	0.75	0.75	0.75	1.00	1.00	1.00	0.75	0.75								
UPR	1.00	1.00	1.00	0.75	0.75	0.75	1.00	1.00	0.75							
FTR <sub>0.5/2</sub>	1.00	1.00	1.00	0.75	0.75	0.75	1.00	1.00	0.75	1.00						
FTR <sub>1.5/2</sub>	1.00	1.00	1.00	0.75	0.75	0.75	1.00	1.00	0.75	1.00	1.00					
FTR <sub>0.8/0.85</sub>	0.67	0.67	0.67	0.92	0.92	0.92	0.67	0.67	0.92	0.67	0.67	0.67				
Calmar Ratio	1.00	1.00	1.00	0.75	0.75	0.75	1.00	1.00	0.75	1.00	1.00	1.00	0.67			
Sterling Ratio	1.00	1.00	1.00	0.75	0.75	0.75	1.00	1.00	0.75	1.00	1.00	1.00	0.67	1.00		
Burke Ratio	1.00	1.00	1.00	0.75	0.75	0.75	1.00	1.00	0.75	1.00	1.00	1.00	0.67	1.00	1.00	
Average	0.91	0.91	0.91	0.81	0.81	0.91	0.91	0.91	0.81	0.91	0.91	0.91	0.73	0.91	0.91	0.91

Table 11: Rank correlations for scenario F

For scenario F, where returns are less normally distributed, the results are less conclusive as only 11 out of 16 performance measures show a perfect correlation to the traditional Sharpe Ratio. The other five performance measures possess, as a result of the different ranking partly explained above, a correlation coefficient to the Sharpe Ratio of 0.67 resp. 0.75 which is significantly less than the lowest coefficient in scenario E. The average rank correlation coefficient to the Sharpe Ratio is 0.91, which is slightly lower than in scenario E. This indicates that alternative performance measures do not lead to rankings which are significantly different from the Sharpe Ratio. However, the lowest correlation among the alternative performance measures is 0.67, which means that the results among alternative performance measures vary significantly. Especially the FTR<sub>0.8/0.85</sub> shows comparatively low correlations to all other alternative performance measures and with 0.73, actually the lowest average correlation to all other measures. As a result, the FTR<sub>0.8/0.85</sub> would lead to significantly different performance decisions.

The overall average of all ranking coefficients in scenario F is 0.88 which is lower than in scenario E.

All in all, the analysis above allows for the preliminary conclusion that in general all performance measures lead to highly correlated rankings but that there are some alternative performance measures which yield significantly different rankings compared to the Sharpe Ratio but also compared to the other alternative performance measures. In order to be able to make a more sound conclusion about the relationship between the different performance measures the underlying parameters assumed in the computation above shall be varied in the following chapter in order to determine their impact on the performance results.

#### **4.2.Variation of Methods and Parameters**

The results of the performance analysis in the previous chapter are all based on assumptions concerning the underlying parameters like the confidence level, the minimum return threshold respectively the risk-free interest rate and the number of drawdowns considered. Thus, it is necessary to test the resistance of the results by varying the assumed parameters to see if the preliminary hypothesis stated above holds true in general.

First of all the results for the Excess Return on VaR and the Conditional VaR shall be compared when either the parametric method or the empirical method is employed. Secondly, the confidence level  $\alpha$  in the VaR-based measures will be varied. Thirdly, different minimum return thresholds shall be assumed for the LPM-based measures and lastly, the number of drawdowns considered for calculating the Sterling Ratio and the Burke Ratio will be modified.

For calculating the Excess Return on Value-at-Risk (EVaR) and the Conditional Sharpe Ratio (CSR) in section 4.1., Value-at-Risk and the Conditional Value-at-Risk were calculated by the parametric method which assumes a normal distribution of returns. When returns deviate substantially from the normal distribution, the parametric VaR and CVaR yield biased results. In order to verify if this is the case for the considered portfolio of bank products, VaR and CVaR were also calculated by the empirical method. Table 12 presents the results for a confidence level  $\alpha=0.01$  in comparison to the ranking obtained by the parametric method.

	Scenario E			Scenario F		
	E0	E1	E2	F0	F1	F2
SR	3	2	1	3	2	1
Parametric EVaR	3	2	1	3	2	1
Empirical EVaR	3	2	1	1	3	2
Parametric CSR	3	2	1	1	3	2
Empirical CSR	3	2	1	3	2	1

Table 12: Parametric vs. Empirical EVaR and CSR

In scenario E it does not make a difference whether the Parametric or the Empirical VaR and CVaR are employed. Both methods and both performance measures lead to the same rankings. And those rankings coincide with the rankings obtained when using the traditional Sharpe Ratio (SR).

In scenario F, however, the empirical and the parametric method yield the reversed ranking for both performance measures. For the Empirical EVaR the resulting ranking is completely contrary to the ranking obtained by using the Sharpe Ratio, whereas the Empirical CSR leads to exactly the same ranking as the Sharpe Ratio. This supports the assumption made in section 4.1., that the opposing ranking of the CSR is a result of the applied calculation method. As the empirical and parametric method yield significantly different results for scenario F, the empirical method should be used to calculate VaR and CVaR whenever there is enough empirical data available in order to obtain exact results.

So far, a confidence level of  $\alpha=0.01$  was assumed for the VaR- based performance measures Excess Return on VaR, Conditional Sharpe Ratio and Modified Sharpe Ratio. In order to test the resistance of the results, the confidence levels were varied from  $\alpha=0.01$  to  $\alpha=0.1$  in order to examine their impact on the performance results. The rankings for different confidence levels are summarized in Table 13.

	$\alpha$	Scenario E			Scenario F		
		E0	E1	E2	F0	F1	F2
SR		3	2	1	3	2	1
Excess Return on VaR	0.01	3	2	1	3	2	1
	0.03	3	2	1	3	2	1
	0.05	3	2	1	3	2	1
	0.07	3	2	1	3	2	1
	0.10	3	2	1	2	3	1
Conditional SR	0.01	3	2	1	1	3	2
	0.03	3	2	1	1	3	2
	0.05	3	2	1	1	3	2
	0.07	3	2	1	1	3	2
	0.10	3	2	1	1	3	2
Modified SR	0.01	3	2	1	1	3	2
	0.03	3	2	1	1	3	2
	0.05	3	2	1	2	3	1
	0.07	3	2	1	1	3	2
	0.10	3	2	1	1	3	2

Table 13: Rankings of EVar, Conditional SR and Modified SR for different confidence levels

According to Table 13, the ranking in scenario E is not affected by the changes in confidence level with the ranking being equal to the ranking obtained by the traditional Sharpe Ratio.

The majority of the rankings in scenario F stay the same compared to the ranking obtained at a confidence level of  $\alpha=0.01$ , which had been assumed in the original analysis. There are only changes in rank for Excess Return on VaR and the Modified Sharpe Ratio at a confidence level of 0.1 respectively of 0.05. Hence, it can be assumed that the confidence level does not have a major impact on the ranking for this data set and that the rankings for a confidence level of 0.01 obtained in Chapter 4.1. are representative. However, for the Conditional Sharpe Ratio it shall be noted that the use of the empirical CVaR leads to a significantly different ranking as observed in the analysis above.

For the performance measures based on Lower Partial Moments, i.e. Omega resp. the Gain-Loss Ratio, the Sortino Ratio, Kappa 3, the Upside-Potential Ratio and the versions of the Farinelli-Tibiletti Ratio, a minimum return threshold  $\tau$  needed to be assumed. To observe the influence of this parameter on the ranking results the assumed interest rate as well was varied from 0.2% to 1% whereby the original return threshold used in Chapter 4.1. was 0.34%. The results of this analysis are entirely clear for the Sortino

Ratio and Kappa 3 and the Upside-Potential Ratio with no changes in ranking for the whole range of assumed interest rates.<sup>13</sup> So the rankings of the Sortino Ratio, Kappa 3 and the Upside-Potential Ratio coincide with the ranking of the traditional Sharpe Ratio for all assumed interest rates except for a minor deviation of rank two and three in the Upside-Potential Ratio in scenario E.

The ranking for different interest rates for Omega respectively the Gain-Loss Ratio and the versions of the Farinelli-Tibiletti Ratio, however, give interesting insights and are depicted in Table 14.

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<sup>13</sup> As there are no changes in ranking, the results for Omega, the Sortino Ratio and Kappa 3 shall not be presented separately at this point of the work. For the detailed results of the variation of interest rates and number of drawdowns please see the Appendix.

	$\tau$	Scenario E			Scenario F		
		E0	E1	E2	F0	F1	F2
SR		3	2	1	3	2	1
Omega /Gain-Loss Ratio	0.20%	1	3	2	3	2	1
	0.34%	2	3	1	1	3	2
	0.40%	2	3	1	3	2	1
	0.50%	3	2	1	3	2	1
	0.60%	3	2	1	3	2	1
	0.70%	3	2	1	3	2	1
	0.80%	3	2	1	3	2	1
	0.90%	3	2	1	3	2	1
	1.00%	3	2	1	3	2	1
FTR <sub>0.5/2</sub>	0.20%	1	2	3	3	2	1
	0.34%	2	3	1	3	2	1
	0.40%	2	3	1	3	2	1
	0.50%	2	3	1	3	2	1
	0.60%	3	2	1	3	2	1
	0.70%	3	2	1	3	2	1
	0.80%	3	2	1	3	2	1
	0.90%	3	2	1	3	2	1
	1.00%	3	2	1	3	2	1
FTR <sub>1.5/2</sub>	0.20%	1	3	2	3	2	1
	0.34%	3	2	1	3	2	1
	0.40%	3	2	1	3	2	1
	0.50%	3	2	1	3	2	1
	0.60%	3	2	1	3	2	1
	0.70%	3	2	1	3	2	1
	0.80%	3	2	1	3	2	1
	0.90%	3	2	1	3	2	1
	1.00%	3	2	1	3	2	1
FTR <sub>0.8/0.85</sub>	0.20%	1	3	2	3	2	1
	0.34%	2	3	1	1	3	2
	0.40%	2	3	1	3	2	1
	0.50%	2	3	1	3	2	1
	0.60%	3	2	1	3	2	1
	0.70%	3	2	1	3	2	1
	0.80%	3	2	1	3	2	1
	0.90%	3	2	1	3	2	1
	1.00%	3	2	1	3	2	1

Table 14: Rankings of Omega/Gain-Loss Ratio and the Farinelli-Tibiletti Ratio for different interest rates

As indicated in Chapter 4.1., Omega resp. the Gain-Loss Ratio, the  $FTR_{0.5/2}$  and the  $FTR_{0.8/0.85}$  were among the few ratios in scenario E which did not comply with the ranking of the traditional Sharpe Ratio. In scenario F some of these ratios even led to an inverted ranking compared to the Sharpe Ratio. Table 14 shows that the rankings produced by these four performance measures vary significantly for small interest rates (up to 0.5%) For higher interest rates the rankings are stable and coincide with the ranking produced by the Sharpe Ratio. Thus, the influence of the interest rate may not be neglected as it might lead to significantly different investment decisions when small values of  $\tau$  are chosen.

The last parameter which needs to be varied and examined is the number of drawdowns used in the Sterling Ratio and in the Burke Ratio. In the original analysis in Chapter 4.1. the number of drawdowns considered was  $N=5$ . To test the resistance of the results the number of drawdowns was varied from  $N=2$  to  $N=10$ . The results show a very clear picture as the ranking in both ratios does not change at all for both scenarios when the number of drawdowns is altered so that the ranking of the Sterling and the Burke Ratio corresponds to the ranking of the traditional Sharpe Ratio for each number of drawdowns observed<sup>14</sup>.

Summarizing the variations in methods and parameters, it can be stated that the results have proven to be relatively resistant against changes of the underlying parameters, especially considering the confidence level and the number of drawdowns. The change in the computation method of VaR and CVaR, however, led to significant differences in ranks. The same applies to the variation of the interest rate for Omega resp. the Gain-Loss Ratio and the versions of the Farinelli-Tibiletti Ratio when small values for the interest rate were chosen.

Both the original performance analysis and the variation of parameters and methods confirm the preliminary conclusion stated above that – in general – risk-adjusted performance measures produce highly correlated rankings. But some performance measures produce significantly different rankings compared to the Sharpe Ratio and to other alternative performance measures. Furthermore, the computation method and the

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<sup>14</sup> For the exact results please see the Appendix.

choice of interest rate under certain circumstances have a significant influence on the ranking results for some performance measures. Thus, based on the analysis of bank products in this work, it seems hasty to dismiss alternative performance measures which account for higher moments of distributions and to declare performance measurement with the Sharpe Ratio to be sufficient as it was done by Eling and Schumacher (2006).

The author of this work agrees with the argumentation of Glawischnig (2007, p. 27) which states that the information contained in the higher moments of distributions needs to be considered even if it does not lead to significantly different rankings for most performance measures. The reason for this is that, as shown in the empirical analysis above, there are some performance measures and some calibrations of parameters that lead to differences in rankings which might be important to a particular investor. Moreover, the choice between different performance measures, despite producing similar results in many cases, helps the investor to find a performance measure which exactly reflects his preferences and thus leads to an optimal decision from his point of view (Bacon, 2009, p. 12).

## 5. Conclusion

Risk-adjusted performance measures can take many forms and their main fields of application lie within performance evaluation of investment strategies and efficient internal capital allocation especially in financial institutions. The selection of the in the scientific literature most frequently applied and discussed risk-adjusted performance measures presented in Chapter 3 was categorized according to the risk-measure used in the respective ratios. The categories are performance measures based on volatility, Value-at Risk, Lower Partial Moments and drawdown. The Sharpe Ratio, Excess Return on Value-at Risk and the Conditional Sharpe Ratio in its parametric forms assume normally distributed returns. All other measures try to overcome this strong assumption by explicitly or implicitly considering the information contained in the higher moments of distributions.

To verify whether alternative performance measures lead to significantly different investment decisions than the Sharpe Ratio and other performance measures which assume normally distributed returns, the presented ratios have been applied to a portfolio of bank products. The analysis of the results has shown that both traditional and alternative performance ratios produce highly correlated rankings. As the results strongly depend on the underlying parameters, like the minimum return threshold, the resistance of the results was tested. This was done by altering the calculation method of Value-at-Risk and Conditional Value-at-Risk, the confidence level, the minimum return threshold and the number of drawdowns. The analysis shows that the performance results in general are not very sensitive to changes especially in confidence level and in the number of drawdowns considered and thus, in general lead to the same rankings. Different computation methods for VaR and CVaR and low values for the interest rate, however, lead to significant changes in rankings for some performance measures.

These findings support the conclusion of other authors that risk-adjusted performance measurement based on the Sharpe Ratio only is not sufficient and that alternative performance measures deliver additional, important information for investment decisions. Still, as it was the case in studies conducted in the current literature of performance analysis, the analysis conducted in this work cannot tell exactly which performance measures under which circumstances and for what reasons lead to different performance results compared to the Sharpe Ratio and to each other. So far all studies, including this work, were based on a rather small selection of performance measures and asset classes. This is why the conclusions derived so far might only be of

limited general validity. A study which covers all alternatives to the Sharpe Ratio based on a broad range of asset classes seems an enormous task as there are more than 100 performance measures identified in literature (Cogneau & Hübner, 2010). Yet, it seems to be the only way to bring more transparency into the existing discussion concerning traditional and alternative risk-adjusted performance measures.

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## Appendix

	$\tau$	Scenario E			Scenario F		
		E0	E1	E2	F0	F1	F2
Omega/Gain-Loss Ratio	0.20%	14.55	11.79	13.22	2597.56	4062.82	5415.95
	0.34%	7.75	7.24	8.46	150.54	117.41	139.66
	0.40%	6.12	5.97	7.15	41.49	48.16	65.70
	0.50%	4.06	4.23	5.30	7.64	16.53	22.81
	0.60%	2.65	3.00	3.95	2.38	7.74	10.30
	0.70%	1.69	2.17	2.91	0.95	4.16	5.45
	0.80%	1.06	1.56	2.12	0.43	2.37	3.06
	0.90%	0.64	1.13	1.54	0.21	1.46	1.85
	1.00%	0.37	0.82	1.12	0.11	0.95	1.18
Sortino	0.20%	0.98	1.36	1.90	71.85	183.35	258.50
	0.34%	0.69	1.05	1.46	9.56	12.26	14.69
	0.40%	0.62	0.96	1.34	5.50	7.54	9.47
	0.50%	0.51	0.82	1.16	2.29	3.93	5.15
	0.60%	0.44	0.72	1.02	1.23	2.49	3.25
	0.70%	0.37	0.63	0.90	0.77	1.76	2.28
	0.80%	0.33	0.55	0.80	0.53	1.32	1.70
	0.90%	0.29	0.49	0.72	0.40	1.03	1.33
	1.00%	0.25	0.44	0.65	0.31	0.84	1.07
Kappa 3	0.20%	0.67	0.93	1.28	29.47	86.62	115.27
	0.34%	0.49	0.74	1.02	4.12	6.95	8.29
	0.40%	0.44	0.68	0.94	2.85	4.74	5.72
	0.50%	0.37	0.60	0.83	1.52	2.77	3.45
	0.60%	0.32	0.53	0.73	0.92	1.86	2.34
	0.70%	0.28	0.47	0.66	0.62	1.37	1.72
	0.80%	0.25	0.43	0.59	0.45	1.06	1.34
	0.90%	0.22	0.39	0.54	0.35	0.86	1.08
	1.00%	0.20	0.35	0.49	0.28	0.71	0.89
Upside-Potential Ratio	0.20%	5.14	4.36	4.93	179.25	479.80	612.80
	0.34%	2.98	2.88	3.30	16.91	26.46	29.11
	0.40%	2.43	2.46	2.84	8.51	15.03	17.38
	0.50%	1.70	1.88	2.19	2.85	6.77	8.22
	0.60%	1.18	1.44	1.70	1.22	3.68	4.48
	0.70%	0.82	1.10	1.32	0.59	2.21	2.69
	0.80%	0.55	0.85	1.03	0.31	1.43	1.72
	0.90%	0.36	0.65	0.80	0.16	0.96	1.15
	1.00%	0.22	0.50	0.62	0.08	0.68	0.81

Table 15: Performance Results for Omega/Gain-Loss Ratio, the Sortino Ratio, Kappa 3 and the Upside-Potential Ratio for different interest rates

	$\tau$	Scenario E			Scenario F		
		E0	E1	E2	F0	F1	F2
FTR <sub>0.5/2</sub>	0.20%	4.06	3.20	2.79	100.52	406.43	525.60
	0.34%	2.24	1.96	2.43	6.57	19.76	22.70
	0.40%	1.77	1.61	2.04	2.84	10.54	12.80
	0.50%	1.16	1.14	1.51	0.78	4.26	5.44
	0.60%	0.75	0.82	1.10	0.30	2.06	2.67
	0.70%	0.47	0.58	0.78	0.13	1.02	1.37
	0.80%	0.28	0.41	0.55	0.06	0.56	0.74
	0.90%	0.15	0.29	0.39	0.03	0.31	0.41
	1.00%	0.07	0.20	0.28	0.01	0.20	0.25
FTR <sub>1.5/2</sub>	0.20%	5.74	5.08	5.61	238.97	545.76	690.63
	0.34%	3.40	3.45	3.83	25.04	31.63	34.24
	0.40%	2.80	2.99	3.33	13.10	18.36	20.87
	0.50%	2.02	2.34	2.62	4.60	8.61	10.25
	0.60%	1.45	1.84	2.08	2.05	4.89	5.83
	0.70%	1.04	1.45	1.66	1.04	3.10	3.68
	0.80%	0.74	1.15	1.33	0.57	2.09	2.47
	0.90%	0.52	0.91	1.06	0.32	1.47	1.73
	1.00%	0.36	0.72	0.85	0.19	1.06	1.25
FTR <sub>0.8/0.85</sub>	0.20%	19.34	14.98	17.07	5618.28	7935.33	10638.22
	0.34%	9.88	8.81	10.67	233.48	172.48	217.30
	0.40%	7.65	7.08	8.89	51.22	62.82	92.06
	0.50%	4.88	4.77	6.40	7.50	19.27	28.01
	0.60%	3.03	3.23	4.61	2.03	8.31	11.56
	0.70%	1.83	2.24	3.24	0.73	4.09	5.61
	0.80%	1.07	1.54	2.24	0.31	2.14	2.89
	0.90%	0.59	1.07	1.55	0.14	1.23	1.62
	1.00%	0.31	0.74	1.08	0.06	0.76	0.97

Table 16 : Performance Results for the versions of the Farinelli-Tibiletti Ratio for different interest rates

	N	Scenario E			Scenario F	
		E0	E1	E2	F1	F2
Sterling Ratio	2	0.19	0.25	0.36	1.89	2.29
	3	0.19	0.26	0.37	1.93	2.32
	4	0.19	0.27	0.38	1.96	2.34
	5	0.19	0.28	0.39	2.10	2.48
	6	0.20	0.28	0.41	2.22	2.67
	7	0.20	0.29	0.42	2.40	2.86
	8	0.20	0.30	0.43	2.60	3.10
	9	0.20	0.31	0.44	2.81	3.34
	10	0.21	0.31	0.45	3.03	3.56
Burke Ratio	2	0.13	0.18	0.25	1.33	1.62
	3	0.11	0.15	0.21	1.11	1.34
	4	0.10	0.13	0.19	0.98	1.17
	5	0.09	0.12	0.18	0.93	1.10
	6	0.08	0.11	0.17	0.89	1.07
	7	0.07	0.11	0.16	0.87	1.04
	8	0.07	0.10	0.15	0.87	1.03
	9	0.07	0.10	0.14	0.86	1.03
	10	0.06	0.10	0.14	0.86	1.02

Table 17: The Sterling Ratio and the Burke Ratio for different numbers of drawdowns

Strategy F0 has not been included in the analysis as there were only five negative observations which would not lead to reasonable results for  $N > 5$ .

## **Eigenständigkeitserklärung**

Ich erkläre hiermit,

– dass ich die vorliegende Arbeit ohne fremde Hilfe und ohne Verwendung anderer als der angegebenen Hilfsmittel verfasst habe,

– dass ich sämtliche verwendeten Quellen erwähnt und gemäss gängigen wissenschaftlichen Zitierregeln nach bestem Wissen und Gewissen korrekt zitiert habe.

St. Gallen, 21.Mai 2010

Alexandra Wiesinger