THE PRICING OF SPARK SPREAD CONTINGENT CLAIMS

MASTER'S THESIS

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SEPTEMBER 13TH 2010

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"THE SPREAD, BOTH AS A PRODUCT AND AS A CONCEPT,

IS PROBABLY THE MOST USEFUL, PREVALENT, AND IMPORTANT

STRUCTURE IN THE WORLD OF ENERGY."

(Eydeland & Wolyniec, 2003, p. 48)

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ABBREVIATIONS

EEX	European Energy Exchange
GPL	Gaspool
IPE	International Petroleum Exchange
NCG	NetConnect Germany
OTC	over-the-counter
PCA	Principal Component Analysis

PARAMETERS

C	call option value
$C(u)$	copula function
d	dimension
dz	Wiener process
e	index for electricity
F_t^T	futures/forward contract at time point t with maturity T
$F^{\leftarrow}$	generalized inverse of distribution function F
g	index for natural gas
H	heat rate / inverse of power plant efficiency
K	strike price of an option
$N(.)$	cumulative standard normal distribution function
P	put option value
$\boldsymbol{P}$	correlation matrix
r	riskless interest rate
r^*	continuous return (log-return)
Ran	range
S	spot price of an asset
S^*	spot spread between two assets
t	time of valuation
T	maturity of a contract, greater than t
T^*	maturity of a contract, greater than T
u	uniformly distributed random variable
V	delivery volume of the commodity under a given contract
x	multiple of the at-the-money strike price
X	swap price / fixed leg of a swap
θ	parameter of the Gumbel copula
μ	mean / drift
ρ	correlation coefficient
σ	volatility
Σ	covariance matrix
Φ	cumulative standard normal distribution function

ABSTRACT

With the energy markets liberalization and increased energy trading on organized exchanges new price risk management needs as well as opportunities have arisen for energy market players. The subject of this thesis is the pricing of spark spread futures options and swaptions. These instruments be used to hedge the margin a power plant operator makes from burning natural gas and selling the so produced electricity. The evolution of the commodity forward curves is modeled through a multifactor forward curve model based on Principal Component Analysis. EEX natural gas and power prices are used as input data. Various closed form spread option formulas are presented and their performance compared to Monte Carlo simulation results. It is found that some of the existing closed form formulas represent good alternatives to the time consuming simulation methods. For the pricing of spark spread swaptions the use of Monte Carlo methods is shown. Approaches to incorporate seasonality as well as inter-commodity dependence structures into the model are presented and discussed.

1 INTRODUCTION

Over the past decade the German energy market has undergone substantial changes triggered by the market liberalization initiative of the European Union. A considerable portion of power and natural gas trading has come to take place on organized exchanges with transparent pricing mechanisms. This shift to exchange based power and natural gas trading has entailed on the one hand the necessity for market participants to hedge their exposure to the exchange energy prices, and on the other hand the emergence of a host of new hedging and risk management opportunities in the form of derivative instruments. Although the liquidity of the latter is still somewhat limited, it can be expected that their importance and trading volume will continue to increase in the future.

A key concept in the energy markets for which hedging and risk management considerations are essential is the spark spread. It describes the margin that can be earned by buying fuels, using them to produce power, and then selling the power (Carmona & Durrleman, 2003). A power plant operator could use various spark spread derivatives to hedge this margin, such as spark spread options or swaptions. As can be inferred from the introductory quote of this thesis, these derivatives play a major role in the energy business. Since the energy trading market in Germany is still very young it can be expected that their use and importance will increase with the expansion of energy trading. Given this outlook, the pricing of spark spread derivatives represents an interesting field of research. The main goal of this thesis is to demonstrate how futures options and swaptions on the spark spread can be priced. Various pricing approaches are introduced and their performance is compared. Since an important input to these valuation models are the dynamics of the forward curve, a multifactor model based on Principal Component Analysis (PCA) is used to capture and model these dynamics. Its use provides the necessary volatility input parameters for the option and swaption pricing models. This analysis and the model performance comparison is carried out using natural gas and power price data from the European Energy Exchange (EEX), the main energy exchange in Germany. In this framework, some important issues such as the incorporation of seasonality and the modeling of inter-temporal and inter-commodity correlations are illustrated and discussed. This facilitates an outlook on a potential starting point for further research: through the use of copulas more complex dependence structures between the commodities can be modeled and incorporated into option pricing schemes. The overall goal of this thesis is thus to present and illustrate the importance of spark spread derivatives, as well as the various approaches and issues with respect to their pricing.

The thesis is structured in seven chapters. In this first chapter the overall topic and goals of the thesis are presented. Chapter 2 gives an overview of the development and status quo of natural gas and power trading in Germany. Furthermore the use of spark spread options and swaptions as hedging instruments in the energy markets is explained. In chapter 3 the various valuation techniques for the instruments are introduced. The multifactor forward curve model is presented in chapter 4, and the incorporation of seasonality is explained. Chapter 5 summarizes some characteristics of natural gas and power prices, and describes the data used in the analysis. In chapter 6 the results of the analysis are shown, the performance of the models is compared, and issues with respect to correlation are discussed. Chapter 7 gives an outlook on the incorporation of copulas in the framework, chapter 8 summarizes and concludes.

2 ENERGY MARKETS AND HEDGING

To lay the foundation for the analysis of pricing methods for spark spread options and swaptions, in the first section of this chapter some facts on power and natural gas trading in Germany are presented. The subject of the second section is the use and application of hedging instruments written on the spread between two energy commodities.

2.1 POWER AND NATURAL GAS TRADING IN GERMANY

Empirical examples for spread option and swaption pricing in the German natural gas and power markets represent the core of this thesis. The data which is used consists of market prices observed at the European Energy Exchange in Leipzig. This section serves to give an overview of the German natural gas and power markets, as well as the development of the trading infrastructure and its current state. To lay an argumentative foundation for this thesis, the relevance of spreads in the German energy markets as well as hedging opportunities are illustrated.

With respect to liberalization and development the European energy markets have come a long way. The USA and Canada already liberalized their natural gas sectors in the late 1970s, entailing the emergence of energy risk management products (OECD/IEA, 1998). In Europe this development took place much later. Natural gas markets in Europe, which have always been characterized by high import rates, were traditionally based on long term supply contracts. This was mainly due to the high investments which natural gas exporters had to make into transmission systems and infrastructure. The contracts often had a duration of 20 to 30 years and were indexed to an alternative fuel (IEA, 2008). The first European country to completely liberalize its natural gas market was the United Kingdom. In 1997 the International Petroleum Exchange (IPE) put a trading system for natural gas futures into place (OECD/IEA, 1998). On the European continent, however, oil-indexed natural gas supply contracts still dominate. The number of hub-priced contracts has increased though, and this is largely due to the developments triggered by the European energy directives. These were a consequence of the objective established in the EC Treaty to build a common market for natural gas and electricity. The targeted effect was to increase competition and efficiency to support European industries in their global competitiveness (IEA, 2008). It was to be realized through "common rules for transmission, distribution, supply and storage" (OECD/IEA, 1998, p. 44). As a consequence, the first Electricity Directive was passed in 1996, and the first Gas Directive in 1998. The latter was specifically designed to further third party network access by unbundling the historically vertically integrated operators (IEA, 2008). For electricity, successes in Germany came quite fast: demarcations were prohibited effectively and the foundations to power trading were laid (Lokau & Ritzau, 2009). In 2000 two power exchanges were founded in Leipzig and Frankfurt (Maibaum, 2009). The Gas Directive, however, did at first not have much impact. One reason was the insufficient monitoring by the regulating Bundeskartellamt. Another was the fact that the German natural gas market was characterized by a multitude of networks, and that regional players made the propagation of third party access considerably more difficult. To improve the situation the second Gas and Electricity Directives (passed in 2003 and 2005, respectively) amongst others reinforced the regulation activities (IEA, 2008). Although in Germany the Bundeskartellamt made substantial progress in curbing long-term supply contracts in 2003 (Däuper & Lokau, 2009) the 2004 EU reports were not satisfactory. In fact, even by 2006 there was no liquid market in which natural gas could be traded (Lohmann, 2006). It was only in 2007 when natural gas contracts started to be traded on the European Energy Exchange in Leipzig. The EEX had resulted from the merger of the two German power exchanges mentioned above, and is today the main exchange and OTC marketplace for spot and derivatives contracts on power and natural gas, as well as CO_2 emissions rights and futures on coal (EEX, 2010b). In the following, those contracts which are relevant in the framework of this thesis are introduced. The information based on which spark spread option and swaption pricing is carried out in the later chapters is contained in natural gas and power futures contracts. Table 1 summarizes and categorizes the natural gas and power futures contracts available for trading at the EEX.

Natural gas futures are tradable as monthly, quarterly, yearly and since May 2009 also as seasonal contracts. As is mentioned again in chapter 5 in the context of the description of the data, not all tradable contracts are actually liquid. A major characteristic of the natural gas contracts is that they are settled through physical delivery. If traders want to circumvent this, they have to close out the position early. This stands in contrast to Phelix power contracts which are settled financially. This is due to the fact that Phelix is a power price index and does not imply actual delivery. The name Phelix stands for Physical Electricity Index. Phelix Day Base is the calculated average price of all traded hours of a day on the spot market of the market area Germany-Austria. Phelix Day Peak is calculated analogously but only for trading hours 9 to 20 (EEX, 2009). There are in fact also power futures contracts which are settled via physical delivery. These are the German and French Baseload and Peakload contracts. Baseload usually refers to a delivery of electricity throughout all the hours of the day while peakload indicates the delivery between 8am and 8pm. Off-Peak hours are between midnight and 8am, or between 8pm and midnight. In contrast to these varying delivery hours for power, the natural gas day generally involves delivery from 6am to 6am of the following day (EEX, 2010a).

<table>
<tr><th colspan="3">EEX futures contracts
Natural Gas</th><th></th><th colspan="3">Power</th></tr>
<tr><td colspan="7">cascading of contracts
1MW per traded hour</td></tr>
<tr><td colspan="3">physical delivery
market areas NCG and GPL</td><td rowspan="7"></td><td colspan="3">financial settlement</td></tr>
<tr><th>Contract</th><th>Delivery</th><th>Availability</th><th>Contract</th><th>Delivery</th><th>Availability</th></tr>
<tr><td>monthly</td><td rowspan="5">6am – 6am on every day subject to the contract</td><td>6 months</td><td>Phelix Base</td><td>00am - 12pm Mon.-Sun.</td><td rowspan="3">4 weeks
9 months
11 quarters
6 years</td></tr>
<tr><td>quarterly</td><td>7 quarters</td><td>Phelix Peak</td><td>8am - 8pm Mon.-Fr.</td></tr>
<tr><td>seasonal (NCG)</td><td>4 seasons</td><td>Phelix Off-Peak</td><td>00am - 8am/ 8pm - 12pm Mon.-Fr.</td></tr>
<tr><td rowspan="2">yearly</td><td rowspan="2">6 years</td><td>Baseload</td><td rowspan="2">physical delivery</td><td rowspan="2">Germany
France</td></tr>
<tr><td>Peakload</td></tr>
</table>

Table 1: Overview of EEX spot and futures contracts on natural gas and power (EEX, 2008), (EEX, 2010a)

For both natural gas and power, contract volumes are usually 1MW per hour which is subject to the contract. The minimum order size for contracts varies per product. A specificity of the EEX natural gas and power futures contracts is that they are generally cascaded over time. For a yearly futures contract for example this means that shortly before the start of the delivery period, a yearly contract for natural gas delivery is sliced into three monthly contracts (for the first quarter of the year) and three quarterly ones. This is done under the final settlement price of the yearly contract (EEX, 2010a).

The two market areas (also referred to as virtual hubs) for which natural gas contracts are traded are NetConnect Germany (NCG) and Gaspool (GPL). In the run of the energy market liberalization the number of natural gas hubs in Germany has decreased substantially. While at the start of 2007 there were still 21 market areas, only eight were left by October 2008 (IEA, 2008). Because of the consolidations, Gaspool contracts have only been traded since October 2009. Together NCG and GPL now cover 95% of the German h-gas[1] market (Beidatsch, 2009). As can be seen in figure 1, Gaspool covers much of the eastern and northern parts of Germany while NetConnect Germany spreads mostly across the west and south. The companies belonging to the hubs are the network operators which are coordinated in the respective market areas.

[1] H-gas indicates the quality of the natural gas delivered (EEX, 2010a).

Figure 1: German natural gas market areas (EEX, 2010c)

As far as trading volumes and the number of market participants are concerned, increases for both natural gas and power have been observed. By December 2009, 191 market participants were registered at the EEX. They include energy suppliers and distributors, industrial companies, power traders, but also banks and financial services companies. For natural gas the number of registered members was above 50 for the futures market and greater than 60 for the spot market (EEX, 2010c). As can be seen in figure 2, there has been much more trading volume in the natural gas futures market than in the spot market. With respect to power trading the same difference between the volumes of spot and futures markets can be observed. This fact highlights the importance of futures markets in the industry.

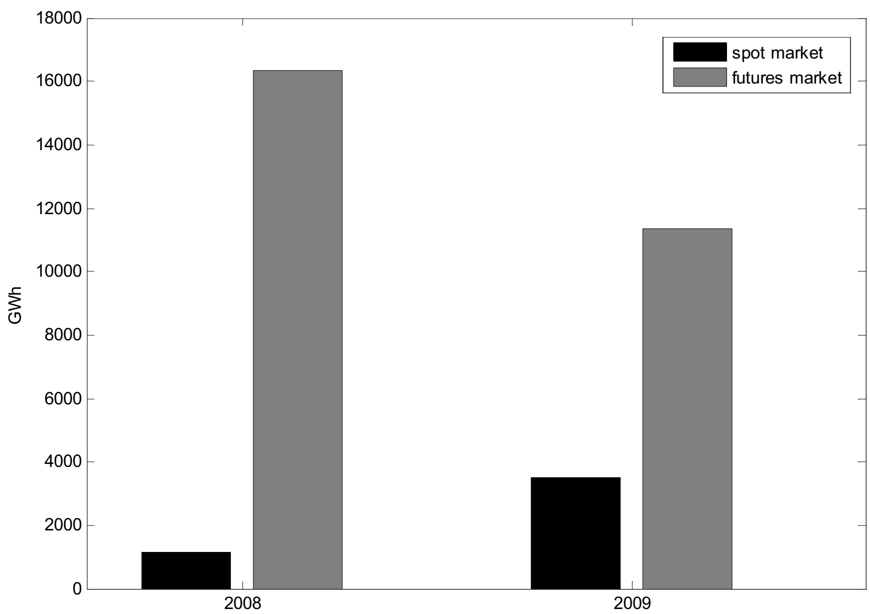

Figure 2: Development of natural gas trading volumes in the spot and futures markets (EEX, 2010c)

Proof for the substantial growth the power spot market has experienced is shown in figure 3. From 2003 to 2009 the spot trading volume in the market areas Germany/Austria, France and Switzerland has more than quadrupled.

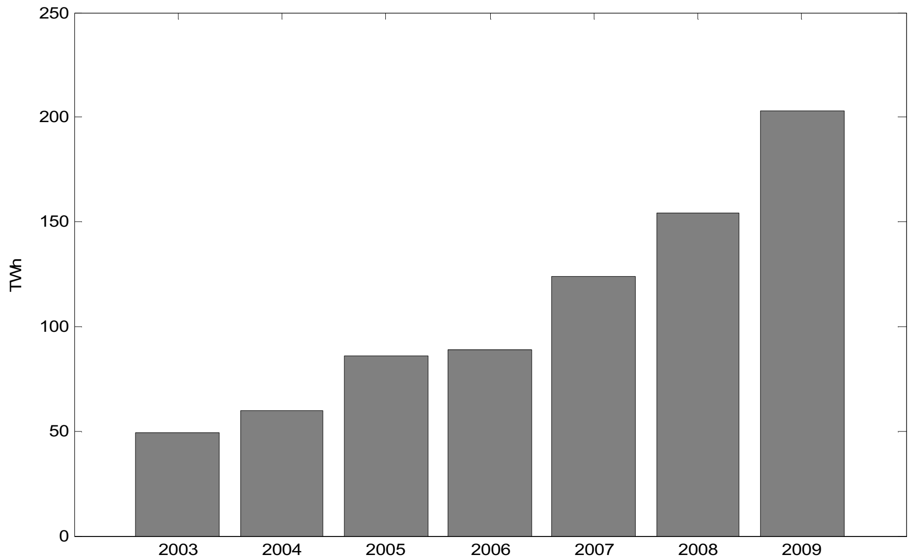

Figure 3: Development of trading volumes in the power spot markets for market areas Germany/Austria, France and Switzerland (EEX, 2010c)

Growth in the power futures market has also been observed albeit of a more moderate degree. Most of the non-spot market activity is still concentrated in the over-the-counter (OTC) market (EEX, 2010c). However, with increasing liquidity in the spot market, which creates trust for the futures market, the observed growth should continue in the futures market (Beidatsch, 2009).

The trading liquidity mentioned is a necessary condition for the emergence and trading of reliable and efficient risk management products. A special kind of these instruments are tailored to meet risk management needs of energy producers who have an exposure to an input energy on the one hand, and are exposed to the fluctuations in the price of the energy they sell on the other hand. Their margin is commonly referred to as the *spark spread* (Carmona & Durrleman, 2003). The relevance of the spread between natural gas and power in the German energy sector is explained in the following paragraph.

As was stated in the very beginning, the spread is a highly relevant concept and product in today’s energy markets (Eydeland & Wolyniec, 2003). In Germany it also plays an important role. While nuclear and lignite fired power generation is of high importance for baseload production in Germany, natural gas and heating oil fired plants are commonly used for peak production (BDEW, 2009). In the following the spark spread between natural gas and power is the center of attention because these are the energy commodities the subsequent analysis is based on.

The main uses of natural gas are traditionally for industrial and household purposes. Its use in energy generation has, however, increased over the past years. While at the beginning of the energy market liberalization, electricity generation played only a minor role in the natural gas sales portfolio

(Lohmann, 2006), the percentage of natural gas sold for electricity generation of the overall natural gas sales increased from 8% in 1998 to 14% in 2008. The percentage of electricity produced by burning natural gas increased as well. It rose from 9% in 1998 to 14% in 2008. These 14% are more than proportional to the capacity percentage natural gas fired power plants represent of the maximum producible amount of power with the installed yearly production capacity in Germany, which is 11% (BDEW, 2009). This reflects the importance of natural gas as an input fuel to power generation and there is reason to believe that its relevance (and thus the relevance of the spark spread) might even increase in the future. One argument for an increased importance of natural gas is the increased awareness for ecological issues and the introduction of a carbon emission trading scheme. By introducing this trading scheme the EU took a step which gives incentives for the choice of natural gas over coal in power production. Coal, which has a heavier carbon footprint, is made more expensive through the obligation to buy emission rights (Lohmann, 2006). It can thus be expected that the importance of natural gas in power production will prevail.

In this section it was shown that the natural gas and power trading infrastructure has developed substantially over the past decade, that currently a multitude of spot and derivatives contracts can be traded at the EEX, and that natural gas plays a key role for power production in Germany. The next section focuses on the hedging of the spark spread and the use of spark spread derivatives.

2.2 HEDGING WITH SPREAD OPTIONS AND SWAPTIONS

The management and hedging of the spark spread can be considered as important as the spark spread itself is. Examples for spark spread hedging instruments are spark spread options, spark spread swaps, and spark spread swaptions. In the following, these instruments are introduced and their uses and benefits presented. The goal of this section is to explain the necessary basics in derivative instruments uses for the analysis of spark spread options and swaptions in the following chapters.

To start out, the concept of the spark spread is explained in more detail. As was mentioned, the spark spread corresponds to the margin earned by energy producers who use natural gas as an input fuel to generate electricity. They are exposed to the price of natural gas in their purchasing activities and to the price of electricity in their sales activities. How big the margin is depends on the prices of natural gas and power, respectively. The other factor influencing the margin is the efficiency of the power plant. The less input fuel a power plant requires for the production of a given amount of power the more profitable the business will be. The characteristic number describing the efficiency with which power is generated by burning fuels is known as the *heat rate*. It is used to calculate the fuel input necessary to produce a certain amount of electricity. The spark spread thus depends on the respective commodity prices and the heat rate in the following way (Carmona & Durrleman, 2003, p. 363):

$$spark\ spread = power\ price - heat\ rate * fuel\ price. \qquad (2.1)$$

Usually the heat rate does not only reflect the power plant efficiency but it also serves as a conversion factor. The latter function is necessary to convert the units in which the fuel (here: natural gas) is quoted to those in which electricity is quoted. While electricity is generally measured and quoted in megawatt hours (MWh) natural gas is rather measured in cubic meters (m^3) or Btus (British thermal units). The first step in the calculation of the spark spread is thus to express both energies in terms of the same units. The heat rate incorporates the factor which yields the MWh equivalent of the total energy contained in a certain amount of natural gas. The second factor incorporated in the heat rate is the efficiency of the power plant. It usually ranges between 20% and 55%, where the lower end is achieved with Single Cycle Gas Turbines and the top end can be achieved with Combined Cycle Combustion Turbines (Eydeland & Wolyniec, 2003). The latter are more efficient for they use additional steam turbines to recover the waste heat of the process of burning gas to generate power (Northwest Power Planning Council, 2002). The level of the heat rate is inversely related to the power plant efficiency percentage. The higher the efficiency of the power plant, the less input fuel is necessary to produce a given amount of power, and the lower the heat rate is. Besides the type of the power plant, the efficiency rate depends on further factors such as the current temperature and capacity utilization of the plant. Because of this dependency on various factors, the heat rate is usually not a constant number (Eydeland & Wolyniec, 2003). For the exemplary analysis in this context, a constant approximation to the heat rate is used.

The necessity of hedging the above described spark spread is illustrated in the following. Figure 4 shows the spot spark spread of NCG natural gas and Phelix base prices with an implied power plant efficiency of 40% from June 1st 2008 to May 31st 2010. It can be seen that there are substantial variations in the margin the operator of a natural gas fueled power plant can make by buying the input fuel and selling the generated power. It is well possible that the margin becomes negative at times. Given that the heat rate in figure 4 is assumed to be constant the picture would again look differently if it was drawn for the actual heat rates of a power plant. Obviously, based on this illustration a strong argument in favor of spark spread management and hedging can be made. One way to reduce or to limit the spark spread price risk would be for an energy supplier to vertically integrate its business model. By trying to control the entire supply chain from natural gas production to the final sale of power, energy providers such as Eon, RWE or Electricité de France are attempting to spread their risk (Hoppe & Flauger, 2010). Besides these longer term strategic approaches, day to day spread risk hedging is necessary to ensure profitability and reliable business planning. To show how this kind of risk management can be done the use of futures contracts, spread options, and spread swaptions are described in the following.

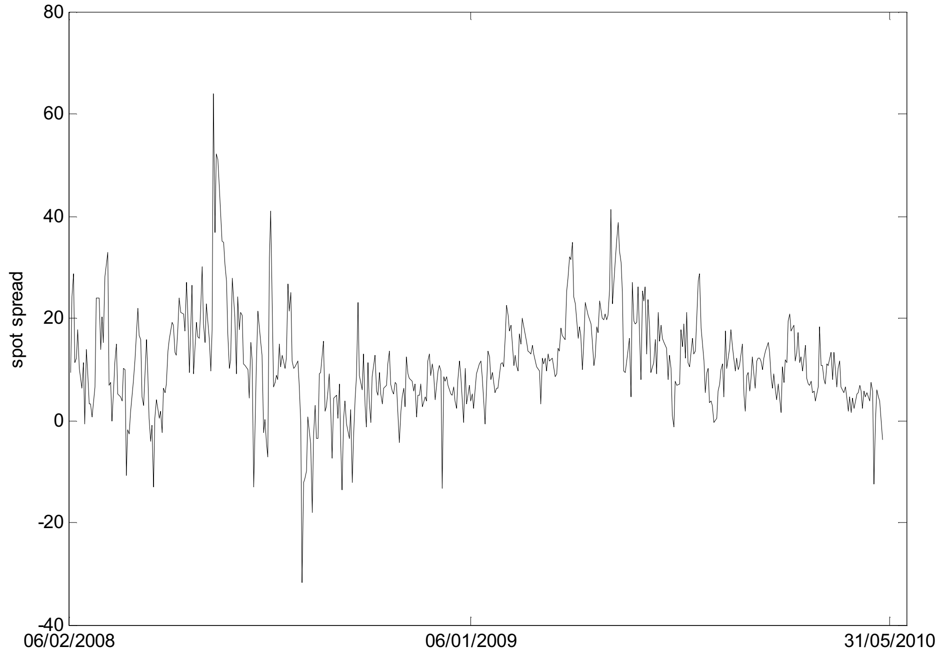

Figure 4: Spot spread between EEX NCG natural gas and Phelix base prices between 06/02/2008 and 05/31/2010 with an implied power plant efficiency of 40%[2]

One of the simplest ways to hedge a future commodity price exposure is to enter into a futures contract. To make a simple example, the operator of a power plant could enter into a forward or futures contract to sell power over a fixed period in the future for a fixed price. Note that in contrast to other markets futures and forwards in the commodity and especially energy markets usually involve delivery not only at a certain maturity date but over a pre-specified delivery period, which is a logical consequence of the limited storability of most energy commodities. In case of physical delivery under the futures contract the commodity buyer will take delivery of the power for the fixed period of time starting at the expiration date. The price paid for the power delivery will be the price fixed in the contract. If the contract is financially settled the only transaction will be the payment of the difference between a certain (usually spot price) index and the agreed fixed price. In case the spot price is higher than the fixed price the power plant operator will have to pay the difference, if the fixed price is higher than the spot price the electricity buyer has to pay. This way the power plant operator has locked in a fixed price no matter what the future market price of power. A peculiarity of futures contracts is that they are standardized exchange traded products. The counterparties of a contract usually do not know each other. Furthermore, futures contracts are usually market-to-marked on a daily basis via a margin account of each counterparty at the exchange. These two properties differentiate futures contracts from forward contracts which are traded over-the-counter and are usually only settled at expiration (Saunders & Cornett, 2008). As far as the hedging result is concerned, however, futures and forward contracts can be considered equivalents.

[2] All prices referred to in this thesis are in Euro.

The hedging of the spark spread and not just a future delivery of a single commodity can be done accordingly. Additionally to entering into a futures contract for the sale of power the plant operator needs to enter into futures purchasing contracts of natural gas. How many of these are needed is determined by the heat rate. In the case of hedging with EEX contracts only the power plant efficiency needs to be known for the calculation of the heat rate because natural gas futures contracts are already quoted in terms of MWh. In this case the number of natural gas contracts needed to hedge the margin with respect to one electricity futures contract corresponds to the inverse of the efficiency. For example with an efficiency of 50%, two gas contracts would be needed to hedge the spark spread from selling one electricity contract.

It was noted that by the use of futures the price (and in case of the spark spread the margin) is fully locked in no matter how the market moves. This means that the operator is secured against a deterioration of power prices, but it also means that he will not be able to profit in case power prices go up. A simple futures hedge is therefore more adequate the surer the operator is that prices will move into a certain direction. In case he is not sure where prices move, a different hedging instrument might fulfill the hedging needs more adequately. Options have become very popular hedging instruments for the option holder has the choice but not the obligation to buy (or sell) a certain underlying asset at maturity at a predefined fixed price, the strike. While call options give the holder the right to buy the underlying at maturity, put options embody the right to sell the underlying at maturity. Thus, if at option maturity the market price for the underlying is below the strike (i.e. the option is in-the-money), the put option holder will exercise his option to sell the underlying at maturity at the strike. In the situation in which the strike is lower than the market price (i.e. when the option is out-of-the-money) the put option holder will let the option expire and sell the underlying on the spot market. For call options the mechanics work vice versa. An option holder can thus profit from favorable market movements but secure protection in case of unfavorable market movements. This flexibility comes at the price of the option premium which needs to be paid at the point of entering into the options contract (Das S. , 2004).

Concerning the underlying of the option it should be noted that option contracts cannot only be written on spot underlyings but also on delivery dates or periods which lie farther in the future than the option maturity. Contracts with these characteristics are called futures options. It was shown above that the futures markets for natural gas and power in Germany are more voluminous than their spot counterparts. This is an observation which is generally made in commodity markets, and is the reason why futures options are traded more heavily in commodity markets (Geman, 2005). In the following analysis futures options are focused on.

A further issue to note in this context is that option contracts cannot only be bought and sold for single underlying assets. Also combinations of assets and spreads between assets are potential option underlyings. In the energy markets the spark spread can be such an option underlying. How a power plant operator can secure his margin by the use of a spread option is illustrated now. The initial situation is such that the energy producer earns the variable margin between the natural gas price and the power price. The exact relationship is given by the heat rate. The operator thus has varying net cash flows from the operation of buying natural gas, producing power by burning it, and selling the power. By purchasing a put option on the spark spread the operator could exchange this variable cash flow for a fixed one, which is set by the strike of the option. Again, since he would only exercise the option in case the strike is above the margin he is expecting to earn over the delivery period at option maturity, he will be able to participate from favorable market developments while still keeping downside protection. In this way spread options can be a useful tool in margin management. As usual, however, they also come at a cost. How spread options are priced is the subject of the coming chapters.

A special case of spread options are exchange options. Exchange options give the option holder the right to exchange a certain amount of one asset for a certain amount of another asset. Their valuation is also described later on. The reason why they are considered in this framework is that exchange options can be considered spread option with a strike price of zero (Deng, Johnson, & Sogomonian, 2001). This can be seen from the spark spread exchange option payoffs (Deng, Johnson, & Sogomonian, 2001, p. 385):

$$payoff_{exchange\ option,\ call} = \max\left(S_e - H_{fixed}S_g, 0\right) \tag{2.2}$$

$$payoff_{exchange\ option,\ put} = \max\left(H_{fixed}S_g - S_e, 0\right). \tag{2.3}$$

A spark spread exchange option thus allows the holder to buy or sell the variable spark spread against a price of zero. The goal of its use in the price risk management operations of a power marketer consists again in bottoming the margin in the power generation operations. The use of a spark spread exchange put is in fact analogous to that of its non-zero strike counterpart. As is well known, an option will only be exercised if it yields a positive payoff. From equation (2.3) it is clear that the payoff is positive only if the negative of the spark spread is positive, i.e. if the spark spread itself is negative. The use of a spark spread exchange option therefore consists in providing loss protection. The negative spark spread can be exchanged for a strike of zero and thus the power plant operator must not take a loss into his books. Further uses of exchange options are in power plant valuation. These are not described here since the focus of this thesis is on risk management applications of options.

During the rest of this chapter further hedging instruments on the spark spread, namely swaps and swaptions, are introduced. Before this is done, swap basics are explained. In its essence a swap corresponds to the exchange of cash flows between two parties. One party pays a fixed sum (the swap price) and the other party pays a floating or variable sum which depends on the movement of an underlying index or spot price (OECD/IEA, 1998). In this way swaps actually work according to the same mechanism as forward and futures contracts, which in turn can be viewed as one-period swaps

(Eydeland & Wolyniec, 2003). In general, however, swaps imply an exchange of cash flows not only once but at various dates over an agreed period of time. Like futures contracts they allow consumers to fix the purchase price of a commodity relative to a certain price index or benchmark. Analogously a producer or seller of a commodity can fix his exposure to sales price variations by the use of a swap. The swap payments are usually not exchanged in their full amounts but only that party which has the net obligation at a given payment date will pay the difference of the owed cash flows (Das S. , 2004). The periodicity of these payment dates can be agreed upon in the individual contract and can vary from annual, semiannual, quarterly, monthly, or even shorter frequencies such as weekly or even daily ones. On each of the so called reset dates the amounts owed between the counterparties are determined. Often the floating payment is based on the average of a price index over a certain time, e.g. the past month. This is where commodity swaps are different from interest rate or other swaps which are usually settled against the price of the underlying at a specific point in time (Das S. , 2004). This fact helps commodities users and sellers to better manage their exposure to the spot price because they usually use and sell, respectively, the commodity constantly and not only on single dates (The Globecon Group Ltd, 1995). In this context swaps allow for tailored and efficient solutions to price risks, and it is the reason why they are highly popular risk management instruments in commodities and energy markets. In fact, according to the Globecon Group they constitute “the largest portion of the commodities derivatives market” in general (The Globecon Group Ltd, 1995, p. 184).

The key advantage of swaps, especially compared to the exchange traded futures, is that swaps can be structured to very closely match the price risk exposure of the company or energy producer (Eydeland & Wolyniec, 2003). This has already been hinted at above. Swaps can vary with respect to the underlying commodity, the number of commodities, the volume of the underlying, the fixed rate, the maturity, the periodicity and number of payments, the floating index. In futures markets these advantages cannot be exploited to such a degree because futures only cover a small number of products. A further advantage of swaps consists in the fact that they are not limited in terms of liquidity and maturity. This is again in contrast to futures contracts whose liquidity often does not stretch very wide into the future. (This topic is also an issue in chapter 5 for the description of the data.) Swaps themselves are written as well for short maturities such as a few months as well as for longer maturities spanning various years. In terms of trading liquidity and maturity, swaps can thus be considered complementary to futures contracts (Fusaro, 1998b). Last but not least it should be mentioned that swaps are easy to handle risk management instruments since they do not require an upfront payment (such as options), they do not need to be rolled over or managed actively, and they are off-balance-sheet transactions which makes accounting issues easy (Eydeland & Wolyniec, 2003). Finally swaps are financially settled and therefore risk management and “real” commodity or energy transactions can be carried out separately. This fact is also helpful in that it provides the advantage of the “ability to time pricing decisions to exploit market cycles in a manner which is both flexible and retains confidentiality from physical suppliers” (Das S. , 1994, p. 494).

Although there are many advantages swaps have, they are not unconditionally beneficial in the sense of eliminating any kind of risk. It always depends on the structure of the swap how accurately risk is contained or managed. If the settlement mechanism does not exactly match the exposure of the counterparty then some substantial risks can be left unhedged (Jarrow & Turnbull, 2000). Furthermore, since entering into a swap agreement means fixing a purchasing or selling price to a predetermined level, swaps carry the risk of significant losses if prices move into the wrong direction and one counterparty repeatedly has to pay high net differences (Kaminski). Swaps can thus imply similar downsides as futures and forward contracts. As explained above, to keep a certain upside potential, options would have to be used. To hedge longer time periods using options, series of call or put options, like caps and floors, could be entered into.

Because of the great variability in swap construction, all kinds of variations and deviations from "plain vanilla" swaps are possible. One type of these more "exotic" swap variations are differential swaps. The main characteristic of a differential swap is that the underlying is not a single commodity price but consists in the differential between two prices. In this way the counterparties exchange the difference between two floating indexes or prices (the differential) for a fixed amount (Clewlow & Strickland, 2000). The spark spreads which are the focus of this thesis can be the underlying to such a differential swap agreement. Spark spread swaps combine the advantages of swaps as well as those of spark spread risk management instruments.

The last hedging instrument to be presented is a swaption. A swaption is an option to enter into a swap at a certain future date. In practice swaptions are bought by companies who know that they will need the protection of a swap in the future but are not sure about swap price developments. The latter depend on the development of the forward curve. In this case a swaption is superior to a forward start swap (an agreement fixed today with the swap payments starting only at some point in the future (Das S. , 2004)) because a forward start swap would lock in the current swap market price for the swap in the future. In the case the market swap price moves in the "wrong" direction this can entail substantial losses. Purchasing a swaption leaves the company the flexibility to exercise the option in case it is in the money or to abandon it and enter into a spot swap if it is out of the money (Clewlow & Strickland, 2000). Further uses of swaptions consist in the context of structured swaps. Here the swaptions are embedded into certain structures such as extendible commodity swaps or variable volume commodity swaps. Under an extendible swap one party has the right to extend the swap at the price of the original term. This corresponds to a combination of a swap and an additional swaption. In the case of a variable volume swap, which is sometimes also called "double-up" swap, the party holding the option has the right to increase (e.g. double) the volume of the underlying of the swap. Again, the additional volume will be swapped at the same conditions as the original swap, and the structure corresponds to a portfolio of a swap and a swaption (Das S. , 2004). Obviously swaptions are derivative instruments which can be very useful in a lot of risk management structures, which gives another justification for the pricing analysis which is carried out.

In this chapter it was shown that spark spread derivatives can be useful hedging instruments for they allow to either fix the margin or to provide downside protection while keeping the upside margin potential for a power plant operator. The next chapter is concerned with the methods used to price the described futures, exchange options, spread options and spread swaption contracts.

3 VALUATION TECHNIQUES FOR SPARK SPREAD CONTINGENT CLAIMS

After an introduction to energy trading in Germany and the establishment of the uses and relevance of spark spread derivatives, this chapter goes into the details of the methodology used for pricing these instruments. The description of the valuation techniques is organized according to the products to be priced. In section 3.1 some option pricing basics as well as various approaches to pricing spread options are introduced. The approaches include two closed form approximations, as well as the Bachelier method, and Monte Carlo simulation. In section 3.2 the introduction of swap pricing basics and the explanation of the pricing of spark spread swaps and swaptions follows.

3.1 SPREAD OPTIONS

Option valuation is one of the most researched and discussed topics in the field of financial derivatives. This statement does not only refer to “exotic” option types but also to “plain vanilla” products, which are challenging to handle and to capture in convenient closed-form formulas. To lay the foundations for the following presentation of pricing approaches, the basics in option pricing are repeated in a short summary.

The essentials of the way in which options contracts function are captured in their payoffs at maturity (Das S. , 2004, p. 329):

$$payoff_{call} = maximum\ (S_T - K, 0) \tag{3.1}$$

$$payoff_{put} = maximum\ (K - S_T, 0). \tag{3.2}$$

S_T stands for the value of the underlying commodity S at option maturity T, and K is the strike price. The payoff at maturity to the holder of a call option will be the difference between the spot price at maturity and the strike price in case the further is higher than the latter, and zero if the spot is lower than the strike. Put payoffs are explained analogously.

The fair price of the option at valuation time t corresponds to the present value of the expectation of this payoff. In the risk neutral framework the discount factor is given by the riskless interest rate r (Hull, 2006). It can thus be written:

$$option\,price_t = e^{-r(T-t)}E_t\{payoff_T\}. \tag{3.3}$$

In order to derive the expectation of the price of the underlying at option maturity T a model for its dynamics needs to be specified. Most often the underlying price is assumed to be lognormally distributed. Its dynamics are described by the following equation (Hull, 2006, p. 270):

$$dS = \mu S dt + \sigma S dz, \tag{3.4}$$

where μ and σ are the drift and volatility terms, respectively, and dz is random variable that is normally distributed with mean zero and variance dt. These dynamics are termed Geometric Brownian motions (Hull, 2006). By the use of Itô's lemma the dynamics for the log of the price, $ln(S)$, can be derived (Hull, 2006, p. 275):

$$dln(S) = \left(\frac{\mu - \sigma^2}{2}\right) dt + \sigma dz. \tag{3.5}$$

Equation (3.5) shows that $dln(S)$ is normally distributed. The advantages of the lognormal specification are that it can easily be incorporated into closed-form solutions, and the property that the modeled prices cannot become negative (Hull, 2006).

Once the dynamics of the underlying are specified, option valuation can be carried out. A popular method is Monte Carlo simulation. This is, however, a computationally demanding and time consuming procedure. Consequently, closed form formulas for option valuation are much more popular. Their main advantage is that they can be implemented more easily and faster than long simulation methods, which is decisive for real-time trading. Furthermore, given the same input parameters, a closed form formula will always yield the same result while the outcome of Monte Carlo simulations depend on random numbers and can therefore change each time it is calculated. Besides the fact that closed form formulas are often able to quickly provide the user with the sensitivities of the option price to changes in certain input factors, the Greeks, they can usually also be inverted to yield "implied" values through a comparison with market quotes. These variables are important in hedging and pricing. Lastly, in spite of the inaccuracies they entail, closed form formulas are very popular because they are usually simple, relatively robust, and well understood by all practitioners (Carmona & Durrleman, 2003).

The most popular closed form formula for option valuation was found by Fischer Black and Myron Scholes in 1973. They build a portfolio consisting of a long position in an asset and a short position in an option on that asset. The amount invested in the option is determined by the inverse of the sensitivity of the option with respect to changes in the stock price. The so constructed portfolio has a deterministic payoff and is thus riskless. By setting its value equal to the riskless rate and solving for the option value, Black & Scholes derived their well known formula the value of a European call option C on a lognormally distributed spot asset at valuation time $t = 0$ (Black & Scholes, 1973, p. 644):

$$C = SN(d_1) - Ke^{-rT}N(d_2) \tag{3.6}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

where $N(.)$ represents the cumulative standard normal distribution function and σ the constant volatility of the underlying. Besides lognormality, as well as constant volatility and interest rates, further assumptions for the derivation of this formula are frictionless capital markets without transaction costs (Black & Scholes, 1973).

Prices for put options with the same characteristics are easily derived through the well-known put-call-parity (Black & Scholes, 1973, p. 647)

$$C + \mathrm{e}^{-r(T-t)}K = P + S. \tag{3.7}$$

The Black Scholes formula and the insights it builds on are cited here because some of the next models to be described are alterations of the Black-Scholes formula, or at least follow the same basic approach. First the Black-Scholes formula is used to illustrate the difference between the valuation of options on spot assets and options on futures contracts. It was mentioned before that the focus is on futures options. The derivation of a closed form formula for options on futures was introduced by Fischer Black (1976) and is based on the Black-Scholes formula. Black assumes that the futures returns are distributed lognormally and have a certain known and constant variance σ^2. All other assumptions are the same as under the Black-Scholes model. His futures option pricing formula is the following (Black, 1976, p. 177):

$$C_t = e^{-r(T-t)}\left[F_t^{T^*}N(d_1) - KN(d_2)\right] \tag{3.8}$$

$$d_1 = \frac{ln\left(\frac{F_t^{T^*}}{K}\right) + 0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

The underlying is the time t price of a futures contract with a maturity of T^*, $F_t^{T^*}$. As can be seen, the difference between the two formulas lies in the discount factor which now also refers to the underlying, and the riskless rate which dropped out of d_1. This is the consequence from the fact that the Black (1976) formula was obtained by substituting $F_t^T e^{-r(T-t)}$ for S_t in the original Black-Scholes formula. A detailed explanation of this spot-forward relationship is given in chapter 4. For now the only further statement to be made is that in its essence this option value is the same as the value that would be obtained if it were calculated for a stock that pays a continuous dividend of the same amount as the interest rate, und thus a stock with a drift of zero. Such a stock has the same characteristics as a futures contract in the risk-neutral environment, which also has a drift of zero (Black, 1976). The latter fact is explained through the characteristic of futures to not necessitate any upfront investments (Clewlow & Strickland, 1999).

In general the futures option formula can be applied to futures contracts of any maturity date T^* greater or equal to T. Its application to a futures contract with an expiration date equal to the option maturity will yield the same value as an option written on the spot price, which emphasizes again the relationship between the spot and futures pricing formulas.

The put-call-parity in the case of futures options is again derived through the substitution of the discounted futures price for the stock price, as it was explained above. It is written as follows (Hull, 2006, p. 329):

$$C + Ke^{-r(T-t)} = F_t^{T^*} e^{-r(T-t)} + P. \quad (3.9)$$

With the presentation of option pricing basics and valuation formulas for plain vanilla spot and futures options the foundations are laid for the discussion of the more complex spread options. The valuation of spread options is an even trickier and also much different task from “regular” option valuation. Shimko (1994) explains the reasons for this. The first reason consists in the fact that the spread is determined by the behavior of two underlying assets and can therefore not be modeled as a single asset. Furthermore the spread’s property to potentially become negative is in conflict with many known option-pricing frameworks where the incorporated asset prices cannot become negative. Third, the covariance between the two assets which make up the spread is crucial in the determination of the option premium. Because of these difficulties often simulation techniques such as Monte Carlo simulation are considered to be the more accurate procedure for the valuation of more complex types of options such as spread options or swaptions. In practice, however, there are many reasons why closed form formulas are often preferred. These were described above. Concerning spread options, some closed form formulas are available. Most of them, however, only apply for special cases or under extensive restrictions. An example for a closed form solution to a special type of option is the one introduced by Margrabe (1978) for exchange options. The formula yields values for options under which one asset can be exchanged for another at maturity. It can therefore be viewed as representing the special case of a zero strike price for a spread option. A closed form solution for a non-zero spread option was found by Bachelier who models the spread itself as an arithmetic Brownian motion (Carmona & Durrleman, 2003). For the case in which the two underlying assets are modeled as geometric Brownian motions, and an exchange option with a non-zero strike price is to be calculated, no closed form solution exists. Therefore, approximation formulas are used, which are analytically tractable and provide results quickly. Since they are approximations, they are usually not as accurate as the results obtained from the use of numerical methods. However, for the reasons given above they are preferred in practice (Bjerksund & Stensland, 2006). One such approximation which has been cited much (see e.g. Bjerksund & Stensland (2006) and Carmona & Durrleman (2003)) was introduced by Kirk (1995). It was improved recently by Bjerksund & Stensland (2006).

All of the methods mentioned above represent important contributions to the spread option pricing theory and are presented in the following. Then the application of Monte Carlo simulation for futures spread option valuation is illustrated. This serves as basis for the comparison of the performance of the various formulas and methods in chapter 6. Note that the following comments present the general case of two assets. The transfer from the general spread option case to the specific spark spread option case can be done easily given the spark spread formula shown in equation (2.1).

As mentioned above, the first spread options which could be captured analytically were exchange options. Their payoff reads as follows (Margrabe, 1978, p. 178):

$$payoff_{exchange\ option} = \max(0, S_1 - S_2). \tag{3.10}$$

Margrabe (1978, p. 178) further notes that "this option is simultaneously a call option on asset one with exercise price S_2 and a put option on asset two with exercise price S_1". Given that the option holder owes one of the assets he will end up with the asset that is worth more at maturity of the option. If the asset he has is worth more than the exchange asset he will not exercise the exchange option. If the exchange asset is worth more he will exercise the option. Already in 1978 Margrabe provided a pricing formula for an exchange option along the lines of Black & Scholes (Margrabe, 1978, p. 179):

$$C = S_{1.t}N(d_1) - S_{2.t}N(d_2) \tag{3.11}$$

$$d_1 = \frac{ln\left(\frac{S_{1.t}}{S_{2.t}}\right) + 0.5\Sigma^2(T-t)}{\Sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \Sigma\sqrt{T-t}$$

$$\Sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}.$$

The volatility is calculated as the volatility of a portfolio comprising two assets with a certain correlation. Thus, the single volatilities as well as their correlation need to be known. This version of the formula refers to the exchange of the assets on the spot. The adaptation to futures contracts is easily made, analogously to the transfer from the Black-Scholes to the Black formula. Deng, Johnson,

& Sogomonian (2001) provide such a closed form formula for exchange options based on futures contracts. As usual, the futures prices follow geometric Brownian motion processes. Their option formula is given in equation (3.12). It is already specifically designed for the spark spread, i.e. it refers to an option to exchange a certain amount of electricity for a certain amount of natural gas. The exact ratio is given through the heat rate H (Deng, Johnson, & Sogomonian, 2001, p. 387):

$$C_{exchange\ option} = e^{-r(T-t)}\left[F_{e,t}^{T} N(d_1) - HF_{g,t}^{T} N(d_2)\right] \tag{3.12}$$

$$d_1 = \frac{ln\left(F_{e,t}^{T}/\left(HF_{g,t}^{T}\right)\right) + v^2(T-t)/2}{v\sqrt{T-t}}$$

$$d_2 = d_1 - v\sqrt{T-t}$$

$$v^2 = \frac{\int_t^T\left[\sigma_e^2(s) - 2\rho\sigma_e(s)\sigma_g(s) + \sigma_g^2(s)\right]ds}{T-t}$$

Note how this formula is identical to the Black (1976) formula with the strike price being replaced by the second asset, for which asset one is exchanged. The indexes e and g refer to electricity and natural gas, respectively. Deng, Johnson, & Sogomonian (2001) calculate the volatility by taking an integral over the time-to-expiry. Further details on this are given in chapter 4. The put-call-parity equation for this exchange option is (Deng, Johnson, & Sogomonian, 2001, p. 385):

$$C = P + e^{-r(T-t)}\left(F_e^T - HF_g^T\right) \tag{3.13}$$

Obviously, exchange options can be priced quickly in this framework. Its performance is evaluated in chapter 6. For the valuation of spark spread options with a non-zero strike price, however, other methods need to be considered. One idea was to model the spread between two assets directly. Treating the spread itself like a specific underlying good to an option, i.e. as if it were traded itself, can be justified in cases where there exists an active market for the spread. According to Kaminski, Gibner, & Pinnamaneni (2004) the US natural gas market is an example for this situation. Theoretically, if the spread was modeled directly and as a geometric Brownian motion one could apply the Black-Scholes formula (or in case of futures spreads the Black (1976) formula) to value a spread option with non-zero strike price. The problem with this approach is that under a geometric Brownian motion the spread will never become negative, but in reality the spread can very well become negative (Kaminski, 2004). This was already emphasized and shown in figure 4. Therefore, this approach will not yield accurate spread option values. A different approach to the direct modeling of the spread was taken by Bachelier. In his framework the spread is treated as an arithmetic Brownian motion and thus can also take on negative values. The Gaussian nature of the framework further facilitates the derivation of closed form formulas. A description of this approach for the futures spread can be found in Poitras (1998) and for the spot spread in Carmona & Durrleman (2003). In the following, the more detailed explanation by Carmona & Durrleman (2003) is summarized and transferred to a futures framework. In their formulation the two underlying asset prices are still modeled as geometric Brownian motions, and the dynamics of the spread, which is modeled as arithmetic Brownian motion, are deduced from them. The dynamics of the spread S^* are (Carmona & Durrleman, 2003, p. 649)

$$dS_t^* = \mu S_t^* dt + \sigma dW_t, \tag{3.14}$$

and the dynamics of the underlying spot assets (Carmona & Durrleman, 2003, p. 649)

$$dS_{1,t} = S_{1,t}\left[\mu dt + \sigma_1 dz_{1,t}\right], \tag{3.15}$$

$$dS_{2,t} = S_{2,t}\left[\mu dt + \sigma_2 dz_{2,t}\right]. \tag{3.16}$$

Note that all three equations share the same drift parameter μ. Comparing these dynamics to the futures spread dynamics described by Poitras (1998), one notes that in the futures case the drift term μ is zero. This is again in line with the observations made above concerning the futures drift. In the futures case the closed form formula for the arithmetic spread model looks as follows:

$$C = \left(m_t^T - Ke^{-r(T-t)}\right)\Phi\left(\frac{m_t^T - Ke^{-r(T-t)}}{s_t^T}\right) + s(T,t)\varphi\left(\frac{m_t^T - Ke^{-r(T-t)}}{s_t^T}\right), \tag{3.17}$$

where the functions m_t^T and s_t^T are defined by

$$m_t^T = (F_{2,t}^{T^*} - F_{1,t_1}^{T^*})e^{-r(T-t)}$$

and

$$(s_t^T)^2 = e^{2(-r)(T-t)}\left[(F_{1,t}^{T^*})^2\left(e^{\sigma_1^2(T-t)} - 1\right) - 2F_{1,t}^{T^*}F_{2,t}^{T^*}\left(e^{\rho\sigma_1\sigma_2(T-t)} - 1\right) + (F_{2,t}^{T^*})^2\left(e^{\sigma_2^2(T-t)} - 1\right)\right].$$

The volatility for the arithmetic dynamics are calculated from the volatilities of the two separate geometric dynamics of the two underlyings. The model is therefore still in line with the condition of nonnegative commodity prices[3].

How good this approach performs is evaluated later in the empirical part. In their comparison Carmona & Durrleman (2003, p. 651) find that the formula can be "surprisingly accurate for specific ranges of the parameters". Obviously, the advantage of this model is the easy and fast implementation, its disadvantage, however, is that the actual spread might not be symmetric, a fact which is not taken into account in the model (Kaminski, Managing Energy Price Risk: The New Challenges and Solutions, 2004, p. 121).

[3] Note that power prices actually can at times become negative (Sewalt & De Jong, 2003). This fact is not reflected in the analysis and the common modeling assumptions are kept.

Although there is at present actually no closed form formula to price spread options with a strike price different from zero (Eydeland & Wolyniec, 2003), an attempt by Kirk (1995) to capture the spread dynamics in an easy-to-use formula has been much cited and used as a benchmark for other, more complex approaches (see Carmona & Durrleman (2003) or Bjerksund & Stensland (2006)). According to Bjerksund & Stensland (2006, p. 10) this formula "is the current market standard in practice". It reads as follows:

$$C = e^{-r(T-t)}\left(F_{1,t}^{T^*}N(d_1) - \left(F_{2,t}^{T^*} + K\right)N(d_2)\right) \tag{3.18}$$

$$d_1 = \frac{\ln\left(\frac{F_{1,t}^{T^*}}{(F_{1,t}^{T^*} + K)}\right) + \frac{\sigma^2(T-t)}{2}}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$$\sigma = \sqrt{\sigma_1^2 + \left(\sigma_2 \frac{F_{1,t}^{T^*}}{F_{2,t}^{T^*} + K}\right)^2 - 2\rho\sigma_1\sigma_2 \frac{F_{1,t}^{T^*}}{F_{2,t}^{T^*} + K}}$$

As Eydeland & Wolyniec (2003) note, this corresponds to an adjusted Margrabe formula where the forward price $F_{2,t}^{T^*}$ is replaced by $F_{2,t}^{T^*} + K$ and the volatility factor σ_2 multiplied by $F_{1,t}^{T^*}/(F_{2,t}^{T^*} + K)$. Their conclusion is that this formula actually yields values very close to the "true" values.

The put-call-parity for futures spread options is the following (Bjerksund & Stensland, 2006, p. 3):

$$P = C - \mathrm{e}^{-r(T-t)}\left(F_{1,t}^{T^*} - F_{2,t}^{T^*} - K\right). \tag{3.19}$$

The Kirk formula, however, does not describe the latest innovation in the research in the field. Many approaches have been taken (see also e.g. Carmona & Durrleman (2003)), and to introduce another, newer closed form formula which departs from Kirk's contribution, the formula by Bjerksund & Stensland (2006) is presented in the following. The representation, which is also along the line of the Black-Scholes, Black, and Margrabe formulas presented above, reads as follows (Bjerksund & Stensland, 2006, p. 6):

$$C = e^{-r(T-t)}\left\{F_{1,t}^{T^*}N(d_1) - F_{2,t}^{T^*}N(d_2) - KN(d_3)\right\} \tag{3.20}$$

where d_1, d_2, d_3 are defined by

$$d_1 = \frac{ln\left(\frac{F_{1,t}^{T^*}}{a}\right) + \left(\frac{1}{2}\sigma_1^2 - b\rho\sigma_1\sigma_2 + \frac{1}{2}b^2\sigma_2^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{ln\left(\frac{F_{1,t}^{T^*}}{a}\right) + \left(-\frac{1}{2}\sigma_1^2 + \rho\sigma_1\sigma_2 + \frac{1}{2}b^2\sigma_2^2 - b\sigma_2^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_3 = \frac{ln\left(\frac{F_{1,t}^{T^*}}{a}\right) + \left(-\frac{1}{2}\sigma_1^2 + \frac{1}{2}b^2\sigma_2^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$\sigma = \sqrt{\sigma_1^2 - 2b\rho\sigma_1\sigma_1 + b^2\sigma_2^2}$$

and where the constants a and b are

$$a = F_{2,t}^{T^*} + K$$

$$b = \frac{F_{2,t}^{T^*}}{F_{2,t}^{T^*} + K}$$

The applicable put-call-parity was given in equation (3.19).

The link between the approaches described is given by the fact that both the Kirk and the Bjerksund & Stensland formulas degenerate to the Margrabe formula in the case of a strike-zero spread option (Bjerksund & Stensland, 2006). However, for non-zero strike prices there are some differences in the performance of the Kirk and the Bjerksund & Stensland formulas. Bjerksund & Stensland (2006) find that the Kirk formula tends to under-price the option for strikes close to zero, and to overprice it for strikes which are further away. Their numerical example suggests that practitioners should prefer their formula instead of Kirk's. As with the other approaches, the performance of their formula is evaluated and compared in the empirical part of this paper.

In order to be able to evaluate the relative performance of the formulas introduced above, a benchmark for the "true" futures spread option value needs to be defined. As done by many authors (e.g. Bjerksund & Stensland (2006)) the results are compared to the outcome of a Monte Carlo simulation where the two underlying commodities are modeled to follow geometric Brownian motions. In the case of a futures spread option the underlyings are the futures prices of two commodities for the same maturity. For option pricing via Monte Carlo simulation first the values of the underlyings at the time of option maturity need to be simulated. In a lognormal one-asset setting the futures dynamics can be derived applying Itô's lemma (Hull, 2006, p. 275):

$$dlnF = \mu dt - \frac{\sigma^2}{2}dt + \sigma dz. \tag{3.21}$$

Since μ in equation (3.21) is zero in the futures framework it actually drops out.

For the Monte Carlo simulation equation (3.21) is then discretized as follows

$$dlnF = -\frac{\sigma^2}{2}\Delta t + \sigma\Delta z, \tag{3.22}$$

with

$$\Delta z = \varepsilon\sqrt{\Delta t}$$

and ε being a standard normally distributed random variable (Clewlow & Strickland, 2000).

In a setting with two correlated assets (indicated by indexes 1 and 2) the joint evolution over time period Δt can be modeled analogously:

$$\begin{bmatrix} dln(F_1) \\ dln(F_2) \end{bmatrix} = -0.5\begin{bmatrix} {L_{11}^T}^2 & 0 \\ {L_{21}^T}^2 & {L_{22}^T}^2 \end{bmatrix} * \begin{bmatrix} \Delta t \\ \Delta t \end{bmatrix} + \begin{bmatrix} L_{11}^T & 0 \\ L_{21}^T & L_{22}^T \end{bmatrix} * \begin{bmatrix} \Delta z_1 \\ \Delta z_2 \end{bmatrix}, \tag{3.23}$$

where L^T stands for the transpose of the upper-triangular matrix that results from a Cholesky decomposition (L^T is thus lower-triangular). This transformation decomposes the joint covariance matrix Σ into an upper triangular matrix whose product with its transpose yields again the covariance matrix:

$$\Sigma = LL^T. \tag{3.24}$$

The Wiener processes in equation (3.23) are independent since all correlation is contained in the Cholesky decomposition of the covariance matrix.

Once this calculation is carried out the exponential of the simulated log-return is taken and the result multiplied with the initial value of the forward price $F_0^{T^*}$ to yield $F_T^{T^*}$. Using $F_T^{T^*}$ the call option payoff is then evaluated as described:

$$payoff_{call\ spread\ option} = max\left(F_{1,t}^{T^*} - aF_{2,t}^{T^*} - K, 0\right), \tag{3.25}$$

where a can be the heat rate in the spark spread setting. This procedure has to be carried out various times. The accuracy depends on the number of runs in such a way that the number of runs needs to be quadrupled in order to double the accuracy. Having calculated many payoffs in this way the average of the payoffs is taken over the number of the simulation turns and the result is discounted back to the time of valuation using the riskless rate. The result obtained from the simulation should give a fairly accurate result given the underlying dynamics of the model. This model is therefore used as our benchmark result for futures spread option pricing (Hull, 2006). The procedure can also easily be extended to price claims contingent on various contracts of the forward curve, such as swaptions. This is the topic of the next section.

3.2 SPREAD SWAPTIONS

It is generally agreed that for pricing purposes swaps can be decomposed into and represented as a portfolio of forward contracts (Das S. , 2004). The payoffs of the individual swap legs correspond to the payoff under the corresponding number of forward contracts. The value of a swap at a certain point in time corresponds to the discounted sum of the volume-weighted payoffs from the single swap legs, and thus to the present value of the forward portfolio (Jarrow & Turnbull, 2000, p. 446):

$$PV_{swap,t} = \sum_{i=1}^{n} V_i \left(F_t^{T_i^*} - X \right) e^{-r(T_i^* - T)}, \tag{3.26}$$

where X is the fixed swap price. As concerns forward prices it should be noted that for the valuation of swaps and swaptions in this paper, futures prices are taken as proxies for the prices of forward contracts because only futures price data is available. According to Geman (2005) this is an appropriate approximation to make in commodities and energy markets, where traders speak indifferently of the two contracts. The reason for this lies in the fact that, as Cox, Ingersoll & Ross (1981) state, under a constant interest rate, futures and forward contracts are equivalent. And even if interest rates are considered stochastic Geman (2005, p. 44) notes that "the exact equality holds as long as the covariance [calculated under the risk neutral measure] between changes in commodity price and interest rates is zero, the situation most encountered in practice". A further argument for the use of futures prices in the valuation of swaps is the fact that many financial institutions use futures contracts in hedging commodity swaps (Das S. , 1994). Given these justifications only futures prices are used in the following analysis.

The formulas derived above can be used to calculate the present value of a swap, in which case the amount of the fixed payment X already needs to be known. The actual notion of *swap pricing,* however, corresponds to the determination of the fixed price under the given swap terms. This is done under the imposition that swaps need to have a zero present value at the date of initiation, i.e. that the present value at initiation is the same for both parties and no upfront payments need to be made (Chance, 1998). The calculated value for X is also called the swap price, it is calculated by solving equation (3.26) for X (Eydeland & Wolyniec, 2003, p. 37):

$$X = \frac{\sum_{i=1}^{n} V_i F_t^{T_i^*} e^{-r(T_i^* - T)}}{\sum_{i=1}^{n} V_i e^{-r(T_i^* - T)}}. \tag{3.27}$$

As was described in section 2.2 a differential swap corresponds to a swap written on the difference between two floating indexes. The adjustment of the plain vanilla pricing formula for a differential swap is straightforward. The floating payment is substituted by the differential, i.e. the difference of the two floating payments in the corresponding proportions. As was also explained above, the relationship between the numbers of the two floating contracts will in the case of the spark spread depend on the heat rate. Essentially the pricing of a spark spread differential swap thus corresponds to a combination of the spark spread equation (2.1) for futures prices and the swap price equation (3.27):

$$X = \frac{\sum_{i=1}^{n} V_i (F_{e,t}^{T_i^*} - HF_{g,t}^{T_i^*}) e^{-r(T_i^* - T)}}{\sum_{i=1}^{n} V_i e^{-r(T_i^* - T)}}. \tag{3.28}$$

Given this differential swap pricing formula and combining it with the basics of option and swaption pricing, the step to obtain the equation for valuing differential swaptions can be made quickly using the option basics introduced above. Although swaps have a present value of zero at inception, this does not necessarily have to apply to the present value of the swap at a certain point in the future (e.g. at the maturity of the option). The swap price was already fixed in advance, i.e. at the purchase/sale of the option and thus the value can be different from zero (Geman, 2005). The value of a swaption can therefore be written as:

$$C_{swaption} = e^{-r(T-t)} max\left(\sum_{i=1}^{n} V_i \left(F_t^{T_i^*} - X\right) e^{-r(T_i^* - T)}, 0\right) \tag{3.29}$$

$$P_{swaption} = e^{-r(T-t)} max\left(\sum_{i=1}^{n} V_i \left(X - F_t^{T_i^*}\right) e^{-r(T_i^* - T)}, 0\right) \tag{3.30}$$

Note that while swap pricing refers to finding the fixed payment amount X, for swaption pricing this quantity is already fixed and the premium which needs to be paid to obtain the option is calculated as the swaption price. In the special case of spark spread swaptions it reads as follows:

$$C_{spark\ spread\ swaption} = e^{-r(T-t)} max\left(\sum_{i=1}^{n} V_i \left(\left(F_{e,}^{T_i^*} - HF_{g,t}^{T_i^*}\right) - K\right) e^{-r(T_i^* - T)}, 0\right) \tag{3.31}$$

$$P_{spark\ spread\ swaption} = e^{-r(T-t)} max\left(\sum_{i=1}^{n} V_i \left(K - \left(F_{e,}^{T_i^*} - HF_{g,t}^{T_i^*}\right)\right) e^{-r(T_i^* - T)}, 0\right) \tag{3.32}$$

According to (Blanco, Gray, & Hazzard, n.a., p. 1) “options on swaps (or swaptions) are one of the most difficult derivative instruments to value. The main reason is that their prices are heavily influenced by the properties of the forward contracts with maturities over the term of the underlying swap.” Amongst these properties are the term structure of the volatility for each single forward price, and the term structure of the co-movement between the various maturities. With respect to differential swaps the task of pricing options on these instruments becomes even more complex. Not only the co-movement between the various maturities are important, but also the co-movements between the two commodities with respect to all maturities. The only alternative to valuing such instruments using simulation techniques is according to Blanco, Gray, & Hazzard (n.a.) the estimation of the swap volatility itself and consequent incorporation into the Black option pricing framework such that the formula is applied directly to the swap. The necessary condition for this is that there exist market quotes for the swaps on which options are to be priced. Since this kind of data hardly exists, especially in Germany where energy trading is very young, this is no viable alternative here. Option values on spark spread swaps are therefore calculated in this thesis by carrying out Monte Carlo simulation. It is carried out analogously to the procedure described for the one-payment case. The only difference is that the dimensions of the covariance matrix depend on the number of swap payments. The operations are therefore more extensive and the simulation takes even longer. The advantage is that under this approach actually all correlations between commodities and maturities can be taken into account appropriately and the results are very close to the true value (of course, always given the assumptions made).

A further topic to discuss in this context concerns volatility. As mentioned above, the volatility in the Black formula refers to the futures returns. It is shown in chapter 5 in the description of the natural gas and power price data that the various maturities of futures usually exhibit different levels of volatilities. Since the time-to-maturity of a futures contract with a fixed expiration time will change over the life of the contract, so will therefore its level of volatility. It is thus necessary to find a way to incorporate the various stages the volatility of a futures contract passes throughout its life into the valuation scheme. It could be seen that for all of the option valuation models some volatility parameter is needed, and in fact the volatility is probably the most discussed and crucial input parameter in option pricing. Another way of putting this issue is to state that for futures option and swaption pricing the dynamics of the underlying, i.e. the forward curve, need to be modeled. Since in a risk-neutral environment the expected return and thus the drift of a specified return process is zero, this actually corresponds to specifying the volatility of the process.

To handle this issue, in the next chapter a multifactor forward curve is introduced and its incorporation into the option valuation methods described above is shown.

4 THE MULTIFACTOR FORWARD CURVE MODEL

Modeling the evolution of an entire forward curve – as compared to modeling only one spot price – is a venture of great complexity. A model which has been applied in the framework of energy price modeling is the multifactor forward curve model presented by Clewlow & Strickland (1999). It is on the one hand relatively easy to understand and to implement but still represents a modeling approach consistent with the observed forward curve in the market, and can also reflect such important issues as seasonality and dependencies between various commodities. Next to the option and swaption valuation formulas this model is the key ingredient to the pricing framework for spark spread contingent claims, and is thus explained in detail in this chapter.

When trying to specify a model for the forward curve dynamics it should be mentioned that probably the most prominent approach to modeling commodity forwards is to first model the spot price dynamics. These are then used to conclude to the forward price dynamics. The most basic of the stochastic differential equations describing commodity price dynamics is the following model (Clewlow & Strickland, 2000, p. 89):

$$dS = (r - \delta)Sdt + \sigma Sdz. \tag{4.1}$$

In this framework δ stands for the net convenience yield, which describes the net of the benefits the holder gets from holding the commodity and the cost this entails. Its functioning is analogous to that of the dividend rate in the dynamics of a dividend paying stock (Geman, 2005). To conclude from the spot dynamics to the forward dynamics a link between the two commodity prices needs to be made. This is in theory achieved through the so called “cash and carry” argumentation. To derive it the following case construction is necessary. One buys a certain amount of a commodity in the spot market and at the same time sells a forward contract on it. At the maturity of the forward contract the commodity, which has been held or ‘carried’ until then, is used to honor the forward contract. Based on the payoffs from these proceedings the spot-forward relationship is established (Geman, 2005, p. 270):

$$F_t^T = Se^{(r-\delta)(T-t)}. \tag{4.2}$$

Given this link between spot and forward prices the above mentioned modeling of spot prices and consequent conclusion to forward prices can be reasoned. The problematic side of the “cash-and-carry” argument in the energies environment arises for non-storable commodities. These cannot be bought and “carried” until maturity of the forward contract and thus there is actually “a collapse of the spot-forward relationship for electricity, which is one of the unique difficulties presented by electricity markets” (Geman, 2005, p. 37). However, since equation (4.2) can still be solved for δ when inserting spot and forward prices, Geman (2005) suggests to term the so derived value “risk premium”, instead of calling it convenience yield. The rejection of the above described arguments in the framework of modeling commodity and especially energy prices can thus be avoided.

The model described in equation (4.1) only incorporates one source of risk and is therefore an example of so called one-factor models. Due to its very basic nature it has been improved and adapted various times, amongst others by Schwartz (1997), who introduced an improved version of this one-factor model by introducing mean reversion in the spot price and thus also imposing a more realistic volatility structure. He extended this model to a two-factor model in which dynamics of the convenience yield are described in an additional stochastic equation. His three-factor model additionally incorporates stochastic interest rates. These are just examples for the manifold adaptations and kinds of spot price models which are used to implicitly model the dynamics of futures and forward contracts. However, the well known problem with these models is the fact that the forward curves which are derived from them are only very seldom consistent with the actually observed ones. There are measures which can be taken to improve consistency, but they are generally not fully satisfactory (Carmona & Durrleman, 2003). Eydeland & Wolyniec (2003, p. 198) also note that "energy models based on spot processes – no matter how powerful, elegant, or efficient-have several fundamental drawbacks". These consist mainly in the fact that the convenience yield particularly in energy markets is very hard to observe, and that not enough historical data is available to estimate a reliable and stable model for its dynamics. They therefore suggest to work with pure forward curve models, in which the convenience yield is not necessary for risk-neutral valuation (Eydeland & Wolyniec, 2003). In contrast to the spot price models described above forward price models use as input data past and current forward curves and model the dynamics based on the information contained in it (Clewlow & Strickland, 2000). The main difference consists in the fact that in a forward curve model the entire forward curve evolution is modeled, and not only the evolution of its "foremost point" (Carmona & Durrleman, 2003, p. 674). These forward curve models do not have their origin in commodity markets but in interest rate markets. The prototype was introduced in 1992 by Heath, Jarrow & Morton who define a continuous time stochastic process for the evolution of an initially given forward rate curve across time. It has been applied to commodity markets amongst others by Cortazar & Schwartz (1994) who price copper interest-indexed notes, and to the energy markets by Koekebakker & Ollmar (2005) who specify a forward price model for electricity, as well as Clewlow & Strickland (1999) who use the same framework to price energy derivatives. The evolution of the forward curve is generally specified as follows (Clewlow & Strickland, 1999, p. 4):

$$\frac{dF_t^T}{F_t^T} = \sum_{i=1}^{n} \sigma_{i,t}^T dz_{i,t}. \tag{4.3}$$

In this framework $\sigma_{i,t}^T$ represent so called volatility functions. Each of these is associated with one of n independent Brownian motions and therefore represents an independent source of uncertainty. To calculate the movement of a given forward contract F_t^T over time, the products of the volatility function values evaluated at t and maturity T with their corresponding random impulses need to be summed up. Notice also that equation (4.3) does not have a drift term. This is again due to the fact that we are working in the risk-neutral framework where forward and futures contracts have an expected return of zero because they do not require an upfront investment (Clewlow & Strickland, 2000). Equation (4.3) can be integrated to yield the value of forward contracts at arbitrary points in time, depending on the volatility functions and the initial price of the contract (Clewlow & Strickland, 1999, p. 4):

$$F_t^{T^*} = F_0^{T^*} \exp\left[\sum_{i=1}^{n} \left\{-\frac{1}{2}\int_0^T \left(\sigma_{i,t}^{T^*}\right)^2 du + \int_0^T \sigma_{i,t}^{T^*} dz_{i,u}\right\}\right] \tag{4.4}$$

From this equation it becomes obvious that the forward returns are assumed to be jointly lognormally distributed. This is a key assumption of the model and facilitates its easy application.

The advantage of this model consists in the fact that the user has great flexibility with respect to the choice of the form and the number of the volatility functions. In general, the user can decide whether to estimate the volatility functions based on historical data by applying time series analysis or from current market prices of options (Clewlow & Strickland, 2000). The approach used for the estimation of the volatility functions in this thesis is the application of Principal Component Analysis. It is a common approach which was also used by the authors mentioned above who applied the multifactor forward curve model in the energy and commodity markets. Applying Principal Component Analysis corresponds to determining the eigenvectors and eigenvalues of the covariance matrix of a set of historical forward prices which represent the forward curve.[4] Assume a data set of past forward prices for each of one of the forward maturities τ_j $(j = 1, \dots, m)$. The covariance matrix of the forward log returns calculated from the dataset will be a symmetric positive semi-definite matrix of dimension m, and is denoted by Σ. The eigen-decomposition will yield a matrix Γ, which contains the eigenvectors v_i $(i = 1, \dots, m)$ of length m each, and a diagonal matrix Λ which contains the m corresponding eigenvalues λ_i (Clewlow & Strickland, 2000, p. 144):

$$\Sigma = \Gamma \Lambda \Gamma^T \tag{4.5}$$

$$\Gamma = \begin{bmatrix} v_{11} & \cdots & v_{1m} \\ \vdots & \ddots & \vdots \\ v_{m1} & \cdots & v_{mm} \end{bmatrix} \tag{4.6}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_m \end{bmatrix} \tag{4.7}$$

The eigenvectors are orthogonal and will represent the independent volatility functions introduced above. Each is multiplied by the square root of its corresponding eigenvalue, which corresponds to "loading" each vector according to its importance in explaining the (co)variances. This operation will yield the volatility functions used for the simulation of the forward curve dynamics (Clewlow & Strickland, 2000, p. 145):

$$\sigma_{i,t}^{t+\tau_j} = v_{ji}\sqrt{\lambda_i}. \tag{4.8}$$

Applying this procedure has various advantages. First, it provides a set of independent factors, which permits the easy implementation of the model using independent Wiener processes. Second, along with the components or eigenvectors themselves, the eigenvalues are provided. Their value gives

[4] If not indicated otherwise the following description of the approach is taken from Clewlow & Strickland, "Energy Derivatives - Pricing and Risk Management" (2000).

information on how important an eigenvector is in explaining the variations. This fact facilitates in turn the reduction of the dimension of the simulation of the forward dynamics because it permits the selection of just a couple of volatility functions to model the dynamics. How many factors are needed to explain a certain percentage of the variation can be calculated from the eigenvalues. This characteristic is key because it truncates the integration space of equation (4.4) and therefore makes the entire modeling procedure faster while only inducing a controllable amount of loss of accuracy (Blanco, Soronow, & Stefiszyn, 2002a). In their application to the Nordic electricity market for example, Koekebakker & Ollmar (2005) find that two factors explain 75% of the variation in forward prices. Cortazar & Schwartz (1994) use PCA to obtain a three-factor model which explains more than 98% of the variation in copper forward prices. There are, however, also some drawbacks of the model. One is that the identified factors cannot be interpreted in an economical way. “For example, using PCA on electricity prices would not ‘discover’ weather and coal prices as important factors in electricity price evolution” (Blanco, Soronow, & Stefiszyn, 2002a, p. 77). Furthermore, the use of Principal Component Analysis does not guarantee that the concentration on only a couple of factors, say two or three, will explain a sufficient amount of variation. As suggests the work by Koekebakker & Ollmar (2005), especially in electricity markets usually relatively many factors are needed to reach a certain threshold of explanation power. The topic is touched upon again in chapter 6 where the results of the analysis on the EEX dataset are presented.

A very important topic which has not been addressed yet is seasonality. The importance of the incorporation of seasonality is stressed by Blanco, Soronow and Stefiszyn (2002a, p. 77): “Without taking into account the seasonal dependence […] the resulting curve may not be realistic. For example, the results may characterize price behavior in a ‘typical month’, but that month will never exist in reality because it is a composite average of 12 real months, none of which exhibit the ‘typical’ behavior”. Their approach to taking seasonality into account in the framework introduced above is one of two approaches which are mainly discussed in that context. They advocate carrying out a “seasonal PCA” which consists in sorting the historical data into twelve monthly groups such that e.g. the “January group” would contain all past forward returns the contracts of the various maturities have had in the month of January. Then PCA is performed twelve times, i.e. on each of the twelve monthly covariance matrices such that the volatility functions can be chosen according to time-to-maturity and time-of-valuation. Carmona & Durrleman (2003) however doubt the robustness of this approach and propose a different method, which is also used by Clewlow & Strickland (2000). Their approach corresponds to splitting the volatility functions into two components, one which depends on time-of-valuation, and one which depends only on time-to-maturity $\tau\ (\tau = T - t)$. The forward curve dynamics are then described by (Clewlow & Strickland, 2000, p. 147)

$$\frac{dF_t^T}{F_t^T} = \sigma_t \sum_{i=1}^{n} \sigma_i^\tau \, dz_{i,t}. \tag{4.9}$$

To achieve this isolation of the two effects, the set of historical log returns needs to be standardized by the spot volatility which can for example be estimated in a 30-day rolling window of the historical data set (Carmona & Durrleman, 2003, p. 681):

$$dlnF_{standardized,\tau_j} = \left(\frac{ln\,F_{t+1,\tau_j} - ln\,F_{t,\tau_j}}{\sigma_t}\right)_{j=1,\ldots,N} \tag{4.10}$$

Carrying out the PCA on this standardized data will yield volatility functions σ_i^τ which depend only on time-to-maturity. The time-dependent components σ_t are then calculated again as the spot price standard deviation in a rolling window (Carmona & Durrleman, 2003). This latter approach is chosen in this thesis for the incorporation of seasonality in the model for the forward curve dynamics.

The greatest advantage of using the multifactor forward curve model in the way it was presented lies in its flexibility and easy application. In its simplest application the estimated volatility functions can be used to calculate the volatility integral which describes the dynamics of a forward contract from a starting date to a certain date in the future. This computation is needed to for example value options on futures. Due to the joint modeling of the entire forward curve also claims contingent on various contracts of the forward curve, such as swaps and swaptions can be priced. Since forward prices of different maturities are correlated but do not move together in perfect unison (Clewlow & Strickland, 2000), the important feature of this model is that the dependencies between the different forward maturities are taken into account in the model and are therefore reflected adequately in the valuation results (Blanco, Gray, & Hazzard, n.a.). A third use and advantage of the model is the easy extension to a multi-commodity setting. If the covariance matrix is estimated for various forward contracts not only of one commodity but of e.g. two commodities the framework is still as easy to implement and correlations and cross-correlations between the various maturities *and* the various commodities can be captured according to the given assumptions underlying the model. In our case a multivariate lognormal model would describe the joint evolution of the multiple commodity forward curves. For this joint estimation using PCA, data sets for various maturities of the two commodities are necessary. Out of these a joint covariance matrix is calculated and PCA performed. The dimension of the joint covariance matrix is equal to the number of commodities times the number of input maturities. Additionally, in the case where the covariance matrices are estimated separately for the different commodities, the model allows for great flexibility in modeling the dependency structure between the commodities. This can go as far as incorporating copula dependence structures into the Monte Carlo simulation framework. The model is furthermore easily amendable to the use in closed-form approximations. Last but not least this forward curve model can also be linked to spot price models to build a unified framework for the joint modeling of spot and forward dynamics. Two approaches are illustrated by Clewlow & Strickland (2000) and Blanco, Soronow & Stefiszyn (2002b). Since spot price processes are not subject to this thesis the interested reader is referred to the two mentioned works.

A disadvantage of the model consists in the restriction by the input data. Only contingent claims with maturities equal to the longest maturity of the input data contracts can be valued. This is a natural restriction because without information on the market price of a certain maturity, generalized valuation statements are not possible. The second limitation with respect to the input data is the liquidity of the futures or forward contracts. Only liquid contracts can serve as reliable input data for valuation. This topic is discussed again in chapter 5 which describes the input data.

After the description of the multifactor forward curve model, its estimation using PCA, and its advantages of great flexibility as well as the disadvantages concerning the input data, one last question is left to be answered: how can the results from the use of this model be incorporated into the simulation and closed-form option valuation methods described above? The answer is given now. As described, the volatility of a forward contract with a given maturity over a certain period of time depends on time and time-to-maturity and is calculated as an integral. Geman (2005, p. 94), Amin, Ng, & Pirrong (2004, p. 523), and Clewlow & Strickland (1999, p. 8) mention that the constant variances in closed form formulas can be easily replaced by the average over the integrated time-dependent variance:

$$\sigma_{const.}^2 = \frac{\int_t^T Var\left[\frac{dF_t^{T^*}}{F_t^{T^*}}\right]}{T-t}. \tag{4.11}$$

This is exactly the way in which the result from the implementation of the multifactor forward curve model is incorporated into the closed form formulas and approximations. It was shown in equation (3.12) that Deng, Johnson, & Sogomonian (2001) explicitly use this method.

In the case of Monte Carlo simulation for spread options and swaptions the covariance matrix is calculated by taking an integral over the volatility functions. For two futures contracts with different maturities s_k and s_l this can be done in the following way (Clewlow & Strickland, 2000, p. 154):

$$\Sigma_{kl} = Cov\left[\ln\left(F_t^{s_k}\right), \ln\left(F_t^{s_l}\right)\right] = \sum_{i=1}^{n} \left\{\int_t^T \sigma_{i,u}^{s_k} \sigma_{i,u}^{s_l} du\right\}. \tag{4.12}$$

In the case where the volatility functions were derived from a joint covariance matrix representing the variances and covariances of and between both commodities, it is not only possible to calculate the variances of the single commodities by integrating over the volatility functions, but also the covariances between the commodities for the given time horizon can be derived. Thus the correlation structure is also reflected in the result.

To conclude, it can be stated that the described model is a useful and at the same time easy to handle framework for valuing contingent claims on the forward curve. Its key benefit lies in the reflection of the various inter-temporal and inter-asset dependencies, as well as of seasonality. The results from its application in the various valuation frameworks are presented in chapter 6 after chapter 5 introduces the data which is used for its estimation.

5 ENERGY PRICE BEHAVIOR AND DESCRIPTION OF THE DATA

This chapter is dedicated to a short summary about natural gas and power prices, and the introduction of the data set which is used for the analysis. The factors influencing natural gas and power prices are enumerated and their peculiarities described. Seasonality and correlation are discussed, as well as the properties of the energy forward curves. Then some facts and figures are presented to characterize the data set used for the application of option pricing methods in the next chapter.

5.1 ENERGY PRICE BEHAVIOR

Energy prices are generally very volatile commodity prices. “Their spikes are more violent, and the non-stationary behavior is much more pronounced” than for other products (Eydeland & Wolyniec, 2003, p. 85). By far the most volatile commodity is electricity, but natural gas prices also display continuously high levels of volatility (Geman, 2005). There are various factors which drive energy prices and their volatility. Factors which influence both natural gas and power include weather and climatic influences (Eydeland & Wolyniec, 2003), as well as the economic conditions (Das S. , 2004). Considering that natural gas is used much for heating purposes, and that air-conditioning usage in the summer requires high volumes of electricity, it is obvious how the weather and especially temperatures can have an influence on energy prices. The economic conditions are a factor in the determination of prices in that industrial consumption of electricity but also of fuels, such as natural gas, rises and falls with the business cycle and thus affects the demand side of the market.

Apart from these two factors there are also some idiosyncratic factors which influence each single energy commodity. Natural gas prices can for example be influenced by the availability of storage capacity and pipeline capacity. Natural gas storage is actually a complex topic itself, and various factors such as temperature, available storage, and storage utilization determine the degree to which natural gas can be inserted into and taken out of the storage facility. These factors determine how fast sudden changes in demand can be met by stored gas, and thus affect prices (Das S. , 2004). The cost of extracting natural gas from the source is a further key factor for the price. In this context also technology and technological development play a major role, and expected improvements can very well influence price levels in the natural gas forward curve (Pilipovic, 2007). Naturally, price competition with other fuels is also of importance in the determination of the natural gas price (Das S. , 2004). A last factor which affects natural gas demand and thus prices are the relatively low capital costs of gas-fired power plants, as well as the lower emissions of natural gas compared to fossil fuels (Das S. , 2004).

Also electricity prices have some particular influencing factors. The factor which represents the main idiosyncratic property of electricity and makes the biggest difference compared to other commodities and energy commodities, is its non-storability. This characteristic is the reason why the well-known and common arbitrage arguments do generally not hold for electricity. This means that forward prices cannot be concluded to from spot prices on a cost-of-storage base, and other linkages have to be found and modeled. In fact most of the other factors which determine electricity prices are linked in one way or the other to this special property (Nagarajan, 1999). Another issue are overcapacities. They can have long term influences and the degree of influence depends on how long the situation is expected to last (Pilipovic, 2007). Last but not least the input fuels which are used to generate electricity are of crucial importance on the supply side of the electricity market. As mentioned, natural gas is not used for baseload production but rather for demand peaks because it is more expensive than some other fuels, such as coal. On the other hand it can be used very flexibly. In fact, natural gas and also oil are said to play a key role in setting power prices since they are likely "to be the marginal units at this intersection" of the supply and the demand curve which determine the price (Nagarajan, 1999, p. 249).

These explanations lead to the discussion of another important topic in the energy markets, which are the dependencies between the various types of energy commodities. In more developed markets such as the US, a convergence movement between electricity and natural gas prices has been observed as deregulation has progressed. Due to the fact that natural gas is an input fuel in electricity production the two energy commodities have a strong relationship which is emphasized the more freely they can be traded and hedged (Hemsworth, 1998a). Consequently, with the energy market liberalization also greater convergence can probably be expected in Europe. Not only in the long term is it necessary to observe the relationship, but also in the short term it is necessary to be informed about it. Especially when modeling energy prices and pricing derivative instruments which depend on various energy prices, correlation is a key variable. And while at times correlations between gas and power are weak, they can also exhibit seasonal patterns (Nagarajan, 1999) and it is thus necessary to take them into account. Seasonality is another issue which cannot be left unaddressed when discussing energy prices.

"Unlike forward curves in the financial markets, power and natural gas curves have a pronounced seasonal character. The particular seasonal shape differs from region and depends on factors such as regional weather conditions, current and future supply characteristics, demand growth, and demand structure" (Eydeland & Wolyniec, 2003, p. 79). Natural gas prices often exhibit a pronounced seasonal pattern with high price levels in the winter because natural gas is amongst others used for heating purposes. It is, however, relatively simple compared to electricity which has the most complicated seasonality structure. Additionally to the peaks in the winter because of heating and in the summer because of air-conditioning, power prices exhibit significant daily and hourly patterns. (Clewlow & Strickland, 2000).

Since this thesis centers on the forward curve as basis for valuation models, the characteristics of energy forward curves are laid out also. How far a forward curve reaches into the future depends on how liquid the energy market is. The greater the volume and maturity of longer-dated traded instruments the more reliably and further in to the future a forward curve can be built. In the case where this needed degree of liquidity is not given, forward curves are sometimes built based on an approach which is partly market-based and partly model-based. Concerning the model based part it should be mentioned again that the common arbitrage arguments in commodity markets cannot solely be applied to the power forward curve, "at best, they can be viewed as setting fairly loose bounds for forward prices" (Nagarajan, 1999, p. 250). The above mentioned limits determining how far into the future a forward curve reaches could be extended if forward price quotes from OTC markets were used in building the curve. Unfortunately, this kind of information is not publicly available and therefore one has to rely on futures prices as input data for the forward curve (Nagarajan, 1999). A further issue in constructing a forward curve is the granularity of the forward prices. Usually for the short end of the curve forward prices with a high resolution (e.g. monthly) are available while for the longer end the contracts are written on longer delivery periods (e.g. a year) and the granularity at the longer end is thus not as high. Especially the seasonal behavior of energy prices then necessitates an adaptation to the curve to reflect this behavior. This aim is usually reached through the application of interpolation methods which yields a forward curve with a more uniform granularity (e.g. monthly granularity for the entire curve) (Eydeland & Wolyniec, 2003). Concerning the actual behavior of forward prices it should be noted that energy forward prices are in general less volatile than spot prices, although they are affected by the latter. Forward prices are driven by longer term supply and demand factors. For natural gas these include changes in storage and pipeline infrastructure, regulatory issues and production capacity, as well as seasonality (Das S. , 2004). The latter three can of course also influence power forward curves. However, also differences within the forward curves can be found. While the shorter end energy prices are more driven "by the short-term conditions of storage", the longer end depends more on the "long-term conditions of future potential energy supply". Therefore the shorter and the longer end of the curve can exhibit quite different behavior, which has led some to say that energy prices exhibit a "split personality" (Pilipovic, 2007, p. 28). The last issue to cover with respect to energy forward curves is correlation. In spite of the above mentioned "split personality" of energy prices "inter-temporal correlation, that is, correlation among movements of different parts of a forward curve, is very strong, particularly in the far segments of the curve, for fuel curves. For power curves the correlation in the early parts of the curve is quite weak" (Eydeland & Wolyniec, 2003, p. 80). The latter fact is again a result of the non-storability of electricity and the fact that this issue has a higher impact the more the shorter end of the curve is approached. In general, however, forward "correlations are much more stable than those of spot products" (Eydeland & Wolyniec, 2003, p. 80).

Based on this introduction to energy price drivers and the characteristics of the energy forward curves, the presentation of the forward curve characteristics in the data set used for the coming analysis is the subject of the next section.

5.2 DATA

The valuation methods described in chapter 3, as well as the multifactor forward model introduced in chapter 4, are applied to a set of past natural gas and power price data. The goal is to illustrate the application of the methods and to compare their performance. The data set used is described in this section.

In the following analysis EEX natural gas and power futures prices which were kindly provided by the Institute for Operations Research and Computational Finance – HSG are used. The contracts to which the multifactor forward curve model introduced in chapter 4 is applied were chosen as follows. For natural gas only contracts for the market area NCG were chosen since trading volumes are higher there than in the market area Gaspool. Concerning power, Phelix base contracts were chosen. Phelix base contracts involve the delivery of power during all 24 hours of a day. Given that natural gas contracts also include the delivery during an entire day, the data choice was subject to this matching condition for the delivery time periods. Since the contracts for both commodities are written on the delivery of 1MW per hour of the delivery period, a monthly contract with thirty delivery days contains 720 (i.e. 1 x 24 x 30) MWh of delivered energy, a quarterly contract 2 160 MWh, and a yearly contract 8 640 MWh. This counts for natural gas as well as for power.

With respect to the delivery periods the contracts were chosen to cover a continuous time period into the future. Thus, the nearest three monthly contracts, the nearest three quarterly contracts, as well as the nearest two yearly contracts were chosen.

In this way, for example in mid December of a given year the complete period of the next three years can be covered with the chosen contracts. This choice was also influenced by liquidity issues. Longer dated contracts are available at the EEX (see chapter 2), however, many of them are not very liquid yet and thus the described choice was made. The availability and liquidity of certain contract maturities is a common problem in the kind of analysis done here. When only contracts with heterogeneous delivery periods (i.e. one month, one quarter etc) are available, an overlap of contracts in the simulation occurs for certain dates. As an example, in mid January of a given year our selection of contracts covers delivery in the month of April twice. The three-month futures contract will be for delivery in April, but also the next quarterly futures contract includes the April delivery. In the calculation of the volatility integrals, given such an overlap, always the contract with the shorter delivery period will be chosen. The intuition behind it is that in the underlying data the liquidity increases with decreasing time-to-maturity and length of the delivery period.

All valuations in the following analysis are made for the date of the 31st May 2010. From that date backwards, two years of price information for the mentioned contracts are used. Figures 5 and 6 show the prices of the futures contracts on natural gas and power, respectively, with delivery periods in the next month month, the next quarter, and the next year. The time period includes all natural gas and Phelix base trading days between 1st June 2008 and 31st May 2010. This corresponds to 504 data points per maturity and per commodity.

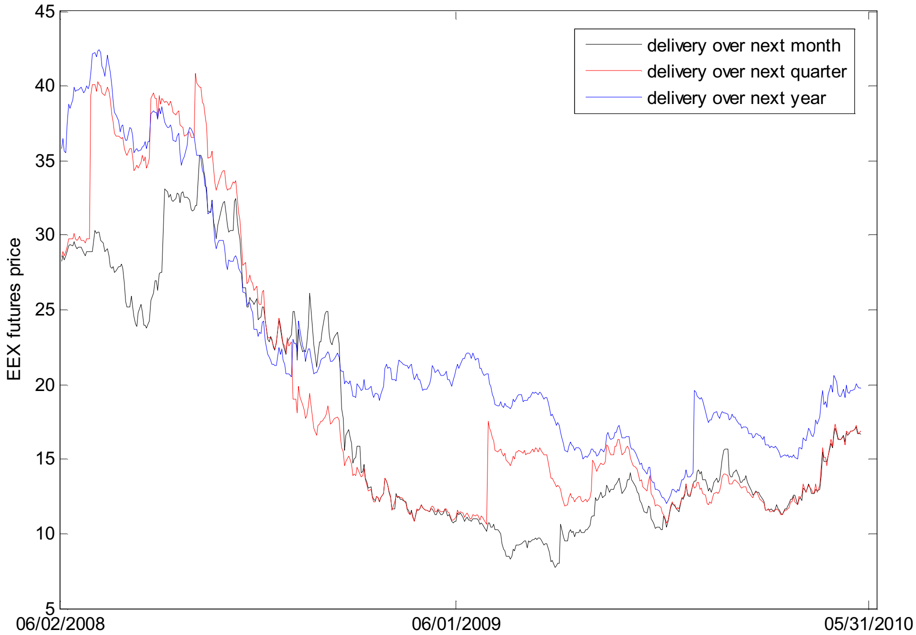

Figure 5: EEX natural gas futures prices from June 2008 to May 2010 for contracts covering delivery during the next month, the next quarter, and the next year

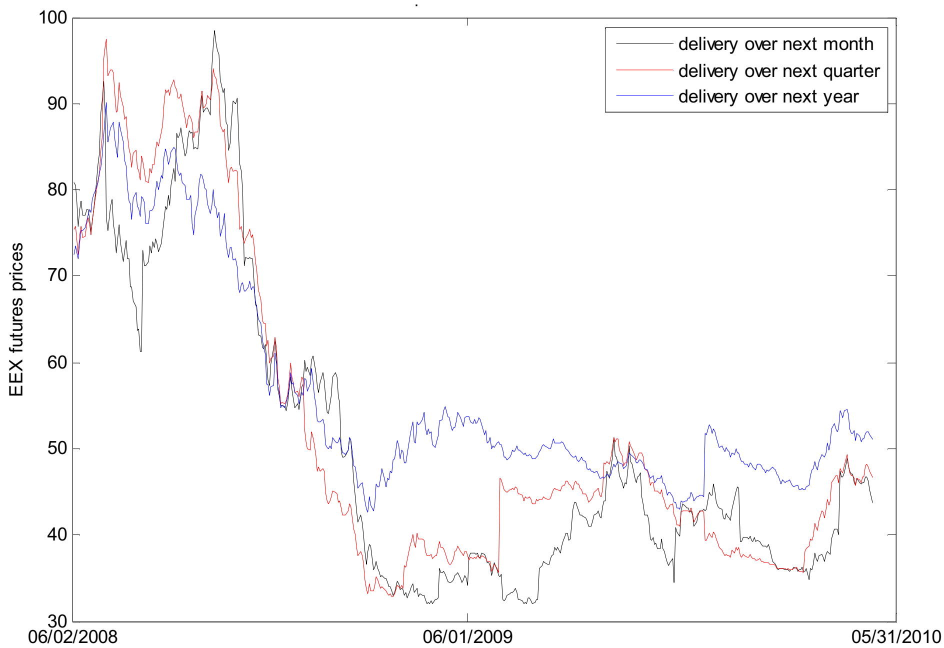

Figure 6: EEX Phelix base futures prices from June 2008 to May 2010 for contracts covering delivery during the next month, the next quarter, and the next year

It is obvious from the figures that natural gas and electricity prices fell substantially during the winter of 2008/2009 and have remained at about the level they reached in the spring of 2009. The prices for the two energy commodities move very much alike, which is the expected behavior.
Note also that many of the jumps are due to changes in the contract period. Since the monthly futures contract refers to delivery in the next month, it will be the February contract in January, and the March contract in February. At the end of January, when the February future is settled, the price for the February future drops out and the March contract is used as next-month future. This is why certain jumps at the switch of the contracts are observed.

As input to the formulas and simulations the prevailing forward curve at the valuation date is needed. Since the price data that can be obtained from the EEX is not of a constant and high granularity but becomes less precise the longer the time horizon, its use would reduce the accuracy of the calculated derivatives prices. For example using the two-year futures price as the underlying forward price for the entire second year into the future would correspond to assuming no existence of seasonal patterns. To prevent such inaccuracies, an underlying forward curve which is based on market data but is set up to reflect seasonal patterns, was kindly provided by the Institute of Operations Research and Computational Finance – HSG. This curve is of monthly granularity and reaches as far into the future as the multifactor forward curve model can be used to price spark spread contingent claims. The underlying term structures for natural gas and power for the 31st May 2010 are shown in figures 7 and 8.

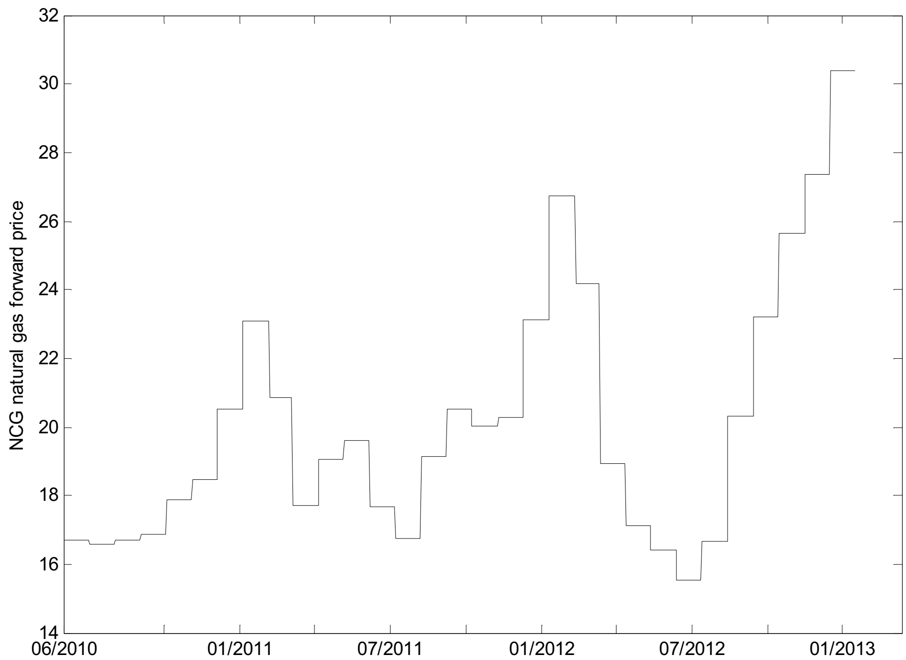

Figure 7: NCG natural gas forward curve with monthly granularity at 05/31/2010

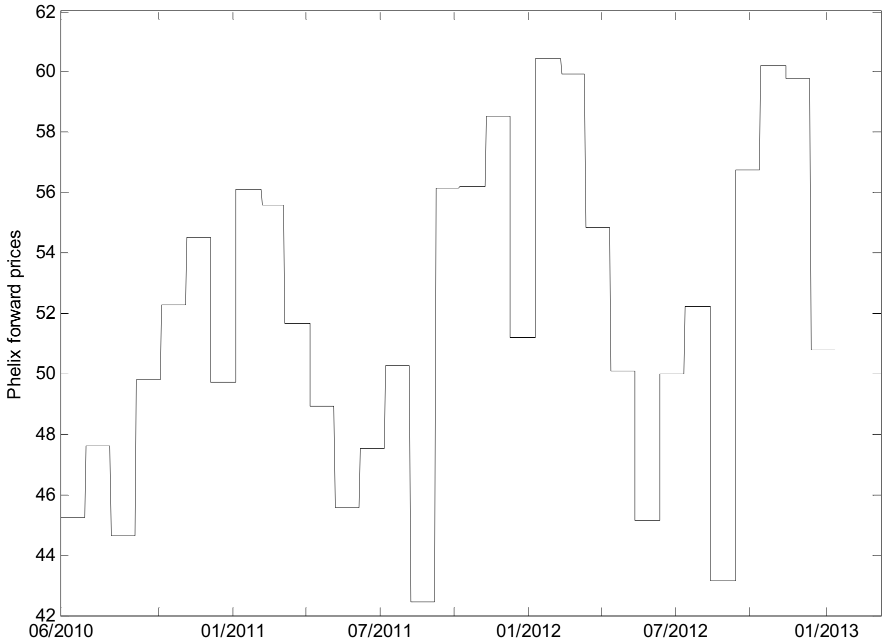

Figure 8: Phelix forward curve with monthly granularity at 05/31/2010

Both natural gas and power exhibit substantial seasonal variations in the term structure. They also both have an increasing overall slope. Figure 9 combines the two term structures to show the term structure of the spark spread with an implied power plant efficiency of 50% until the end of the year 2012. Its overall level obviously decreases over that time horizon and in the winter of 2012 even becomes negative. This further backs the potential benefits of the use of spark spread contingent claims hedging this price risk.

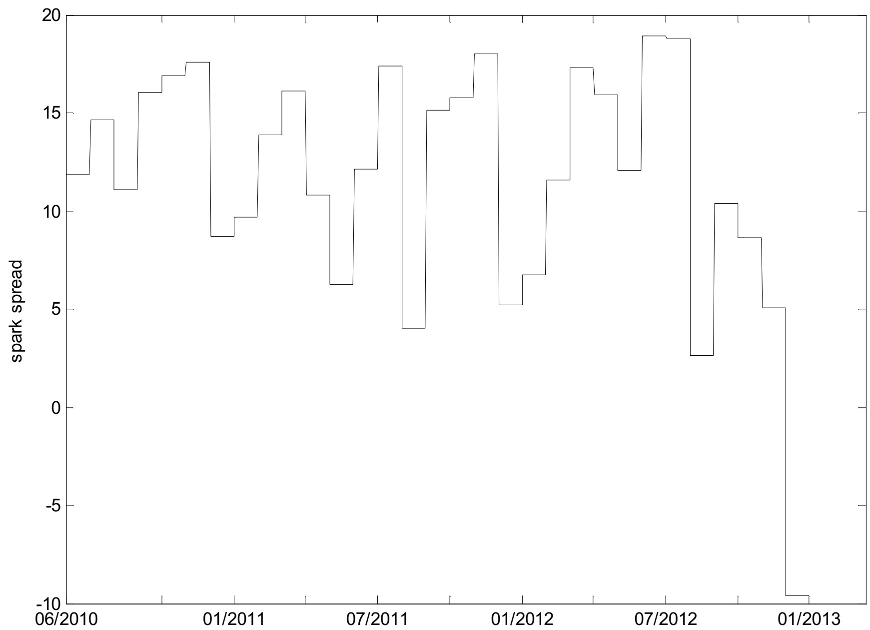

Figure 9: Spark spread term structure with monthly granularity at 05/31/2010 (implied power plant efficiency is 50%)

The existence of seasonality in energy commodity prices as well as the importance to take seasonality into account when modeling these prices has been made clear. Figures 10 and 11 show standard deviations of the natural gas and power futures returns of the next-month, next-quarter, and neyt-year contracts. They were calculated as 30-trading-day rolling windows and clearly reflect seasonal influences and therefore emphasize the importance in modeling the further.

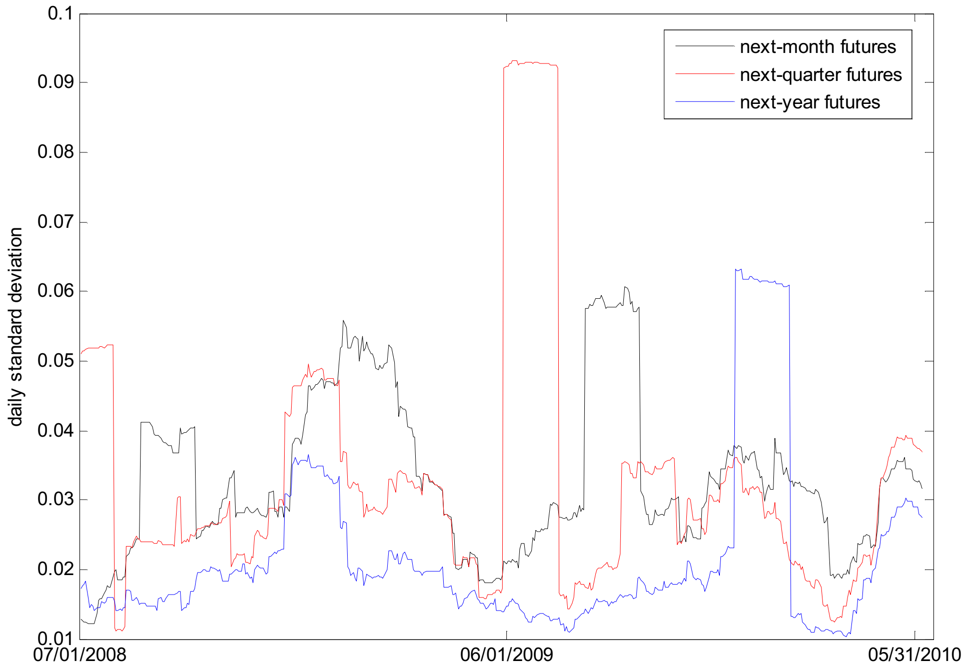

Figure 10: Daily standard deviation of EEX NCG natural gas next-month, next-quarter, and next-year futures calculated as a 30-day rolling window

Note that the jumps in the standard deviations are due to the rolling window technique. If one very high value is in the 30-day-sample the standard deviation goes up, and as soon as it drops out the standard deviation also drops again. The pattern of high volatilities in the winter season and lower volatilities in the summer months is clearly distinguishable for the natural gas contracts. Typically, in the winter natural gas demand increases in Germany due to the fact that much natural gas is used for heating purposes.

Furthermore, it can be seen that in general longer-dated contracts are less volatile. This is again a common property of commodity prices since shorter term futures contracts move more in line with spot prices and are often more heavily traded (Spargoli & Zagaglia, 2008).

The power volatilities presented in figure 11 show a less pronounced seasonal pattern compared to their natural gas counterparts. A reason could be that electricity use in Germany is not as seasonal as that of natural gas.

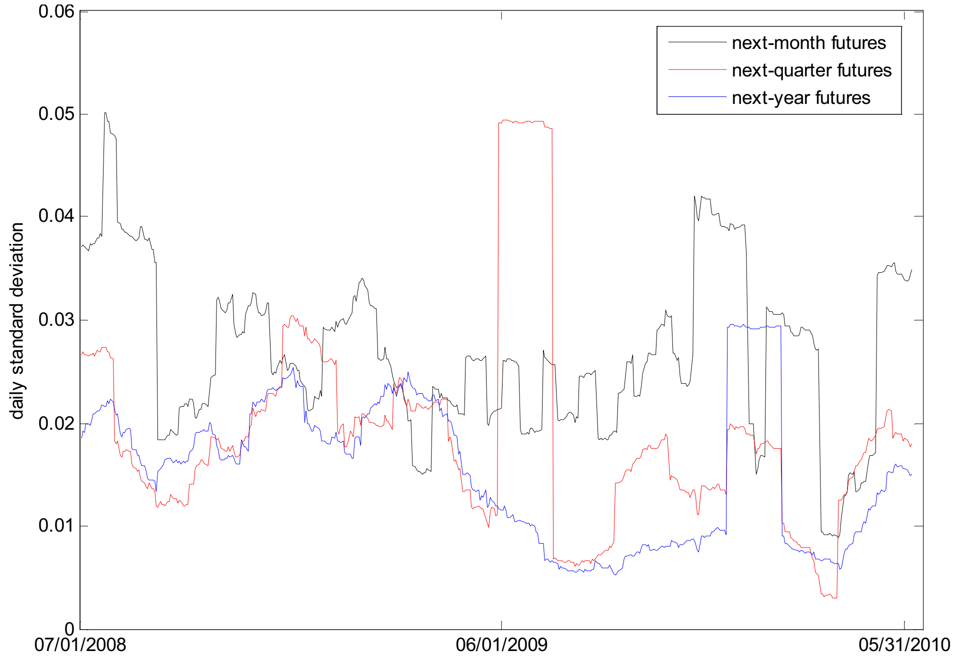

Figure 11: Daily standard deviation of EEX Phelix base next-month, next-quarter, and next-year futures calculated as a 30-day rolling window

As was mentioned above, the correlation between the contracts of the two commodities is a key determinant for spread option prices. Figure 12 shows the correlation between the natural gas and power next-month, next-quarter, and next-year contracts calculated as a 30-trading-day rolling window.

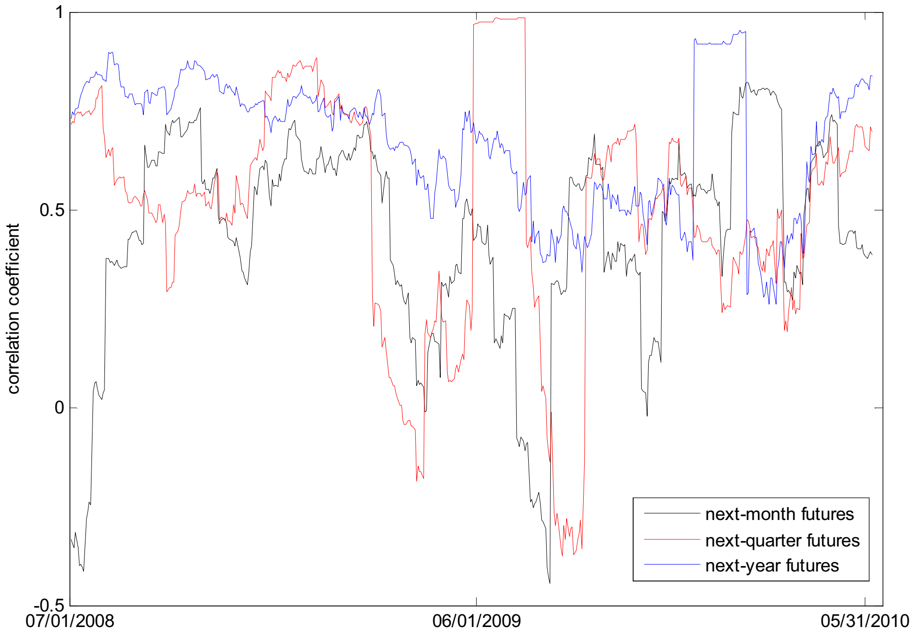

Figure 12: Correlation coefficient between EEX NCG natural gas and Phelix base next-month, next-quarter, and next-year futures calculated as a 30-day rolling window

It has already been stated above that “correlations between gas and power curves can be seasonal and weak” (Nagarajan, 1999, p. 247). Looking at the front-month correlation between natural gas and power in figure 12, a certain seasonal pattern can be identified. While correlation rises to levels above 0.5 in the winter months, correlations in the summer are rather low. Furthermore it can be noted that the longer-dated the contract, the higher is the correlation between the natural gas and power contracts. Given the above statement that the longer maturities are also the least volatile ones, it can be stated that the correlation is highest between the least volatile contracts. These insights support our approach for modeling not only the volatilities of the single commodities but also the covariances which include the correlation between the commodities.

After this introduction of the data set, the next section explains how it is used in the multifactor forward curve model. It was described above that for the use in the model the raw futures price data needs to undergo some transformations. The first step is to take the log-returns of the prices. This procedure is well-known:

$$r^* = ln\left(\frac{F_t^T}{F_{t-1}^T}\right). \tag{5.1}$$

The resulting returns series for natural gas and power front-month futures are depicted in figures 13 and 14.

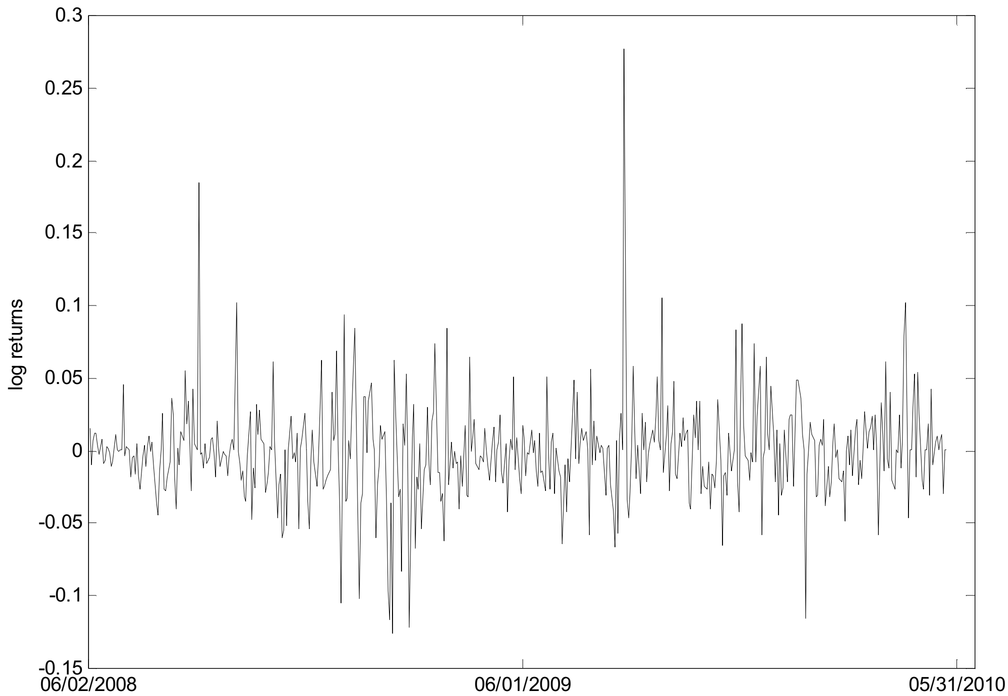

Figure 13: Log returns of EEX NCG natural gas front-month futures

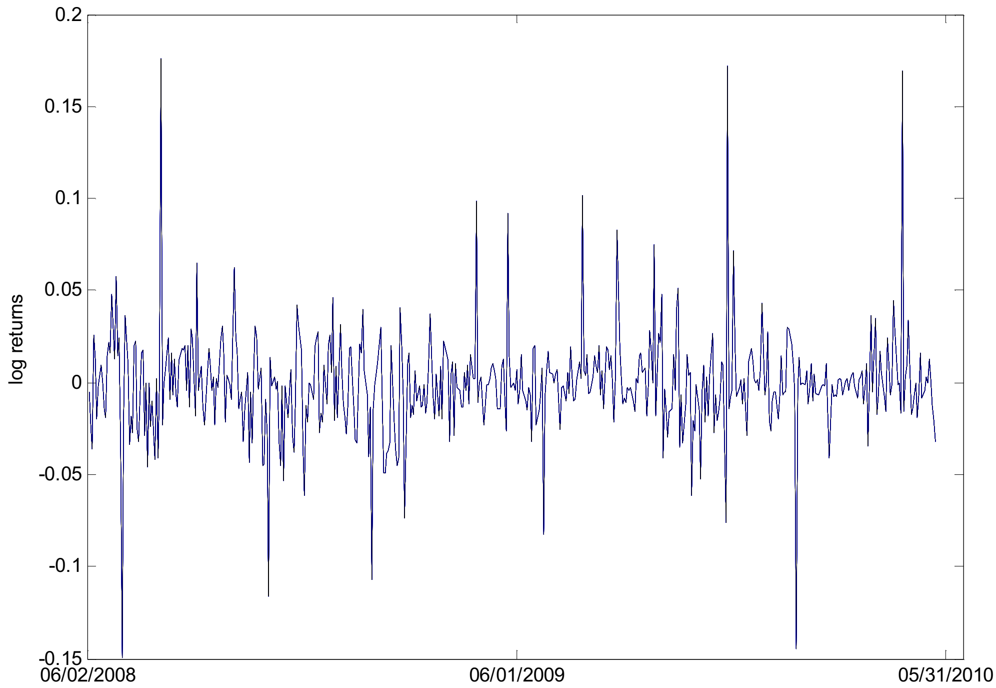

Figure 14: Log returns of EEX Phelix base front-month futures

The next step in the preparation of the data is to standardize the log returns by the spot volatility. The information about the spot volatility will find its way back into the model through the later multiplication with the volatility functions obtained from Principal Component Analysis. Figures 15 and 16 show the rolling window of natural gas and power spot volatilities taken over 30 trading days. For natural gas again a clear seasonal pattern can be observed. For power it is harder to distinguish an explicit seasonal pattern.

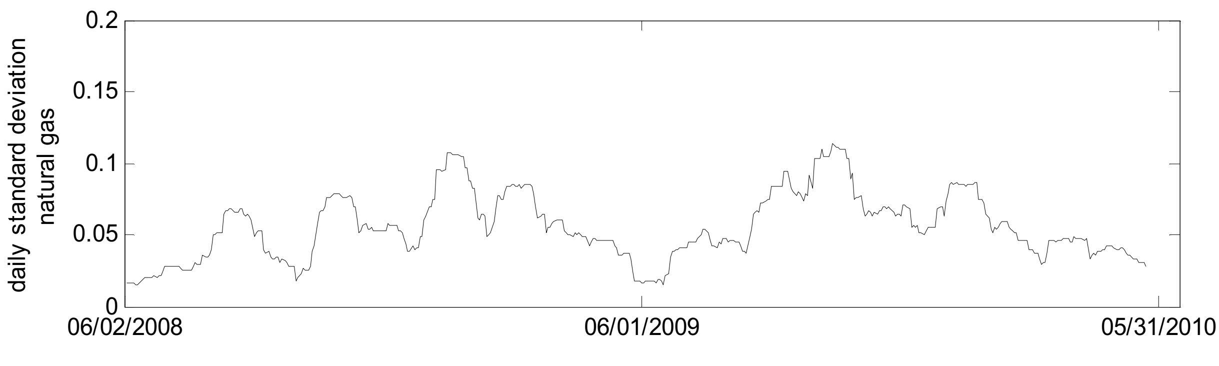

Figure 15: EEX NCG natural gas spot daily standard deviations calculated as a 30-day rolling window

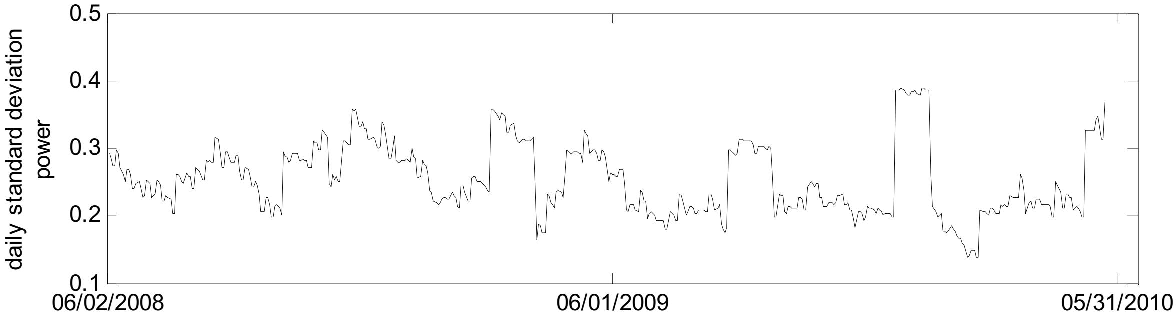

Figure 16: EEX Phelix Day base daily standard deviations calculated as a 30-day rolling window

The second half of the spot volatility data per commodity is used again in the model. It is taken as a factor to the volatility functions and represents the time-dependent component of the volatility. This is done by indexing the spot volatilities to the number of the day in the year on which they occurred. In the calculation of the volatility integral the respective volatility is then matched to each day which is part of the integration period. Having divided the data by the spot volatility and multiplying it again at this point brings the volatilities to the original levels again. This transformation permits to also incorporate seasonal information in the volatility functions.

After the description of the data set and its preparation for the use in the multifactor forward curve model, the next chapter shows the results obtained through the use of the model and its application in spread option and swaption pricing.

6 IMPLEMENTATION AND RESULTS

This chapter presents the results from the use of the multifactor forward curve model in spread option and swaption pricing. First the results from Principal Component Analysis are shown. Then the various methods to price futures options, spark spread options with zero and non-zero strike prices, as well as differential swaptions are applied using the volatility functions which result from the first part of the analysis. The performances of the various closed-form solutions are compared to the fully lognormal model evaluated through Monte Carlo simulation. Concerning swaptions some issues with respect to correlation are discussed and illustrated.

6.1 PRINCIPAL COMPONENT ANALYSIS

How Principal Component Analysis is carried out and which data is used in the analysis was described in the above chapters. In the presentation of the results first the calculated volatility functions are presented and then the truncation of the integration space based on the height of the eigenvalues is discussed.

Principal Component Analysis on energy price data has been performed by various researchers. Usually the first three factors (which are often also the main factors used to model the forward curve dynamics) have certain characteristics. The loadings, i.e. the individual value points, of a certain factor indicate how the prices of the original contracts are affected by a change in that factor. The first factor usually represents a parallel shift. This means that the factor loadings generally have the same sign and roughly have the same level. The factor loadings of the second factor usually make up a more or less straight line with a certain slope, and therefore determine the overall inclination of the volatilities term structure. The third factor is associated with the curvature of the term structure. It can induce one or various bends in the curve (Blanco, Soronow, & Stefiszyn, 2002a). For the data set described in chapter 5 the same characteristics of the first three components for both natural gas and electricity can be observed. Figures 17 and 18 show the three main volatility functions for the two energy commodities.

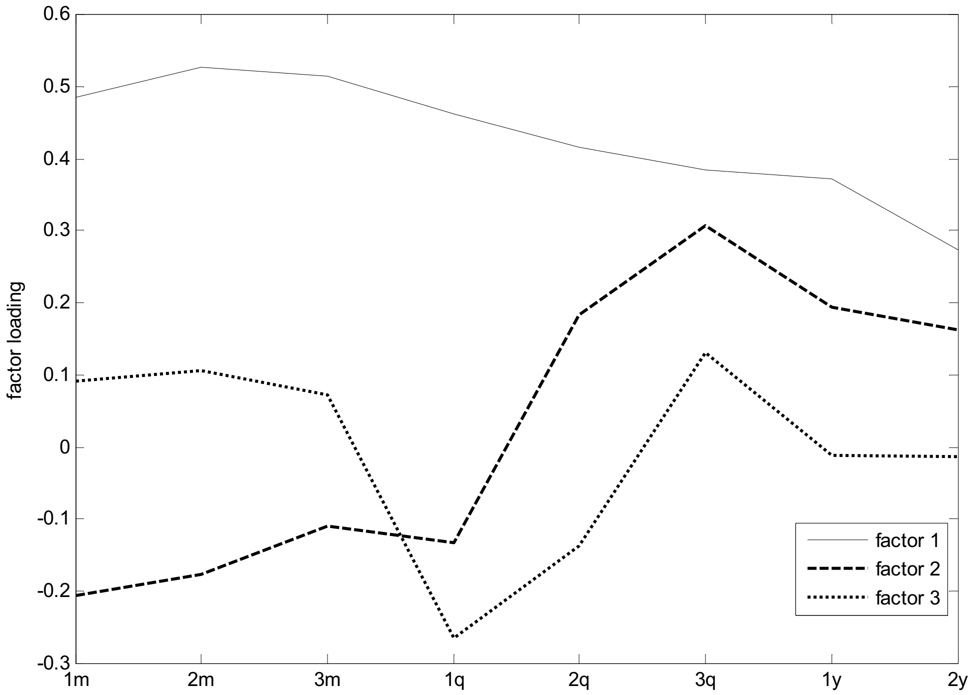

Figure 17: First three volatility functions for natural gas

For natural gas, it can be seen that the first factor is positive for all maturities and the various points are of comparable magnitude. It thus influences the volatility term structure on a level-determining basis. The second factor includes both negative and positive loadings, where the further are rather found for the shorter maturities and the latter for the longer maturities. This factor thus induces a roughly positive slope. The third factor has also negative and positive loadings but in this case the sign of the loadings changes more than once over the increasing maturities. Therefore, it can be interpreted as the bending factor described above.

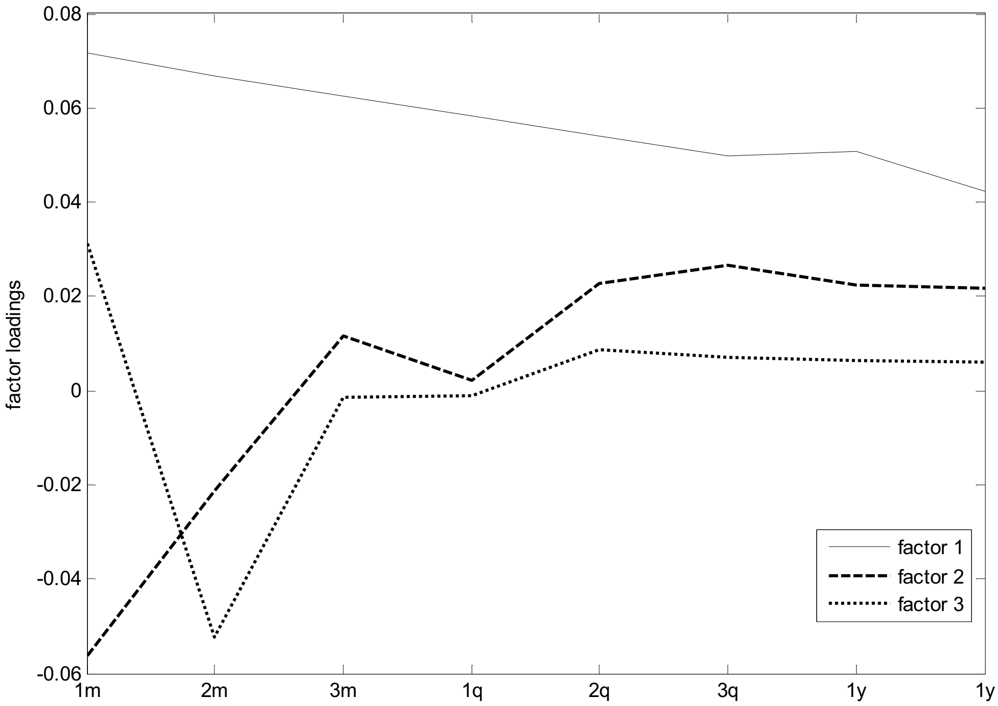

Figure 18: First three volatility functions for power

A similar structure as that described for the natural gas volatility functions can be observed for the power volatility functions. The shift, slope and bend components can again be identified as such.

Another issue to be mentioned concerning the volatility functions is the importance of truncating some outliers in the data set. The patterns in the three main components are only distinguishable as such when the Principal Component Analysis is applied to the data set from which outliers, which are farther away from the mean than three standard deviations, are truncated. This means that a certain continuity or regularity in the data is necessary for PCA to deliver usable results. The method is used to describe the general behavior of the underlying data and cannot incorporate more delicate issues such as jumps or exceptionally high returns. Thus the truncation of outliers to a certain degree is necessary to obtain reasonable results from PCA.

The next issue to discuss is the importance of the various factors in describing the dynamics of the forward curve. It has been explained above that the importance of each eigenvector calculated via PCA is indicated by its corresponding eigenvalue. To calculate this relative importance each eigenvalue is divided by the sum of all eigenvalues. The resulting percentages indicate how much each factor contributes to the overall dynamics. Ideally only a small number of factors together explains a fairly big amount of the dynamics. Often in commodity dynamics modeling three factors are enough to explain a sufficiently big part of the underlying data. It has been experienced, however, that in the energy markets rather more factors are needed. Koekebakker & Ollmar (2005), as mentioned above, use ten factors to explain the main dynamics of the Nordpool electricity forward curve. However, it needs to be mentioned that the number of factors which describes a certain percentage of the dynamics can only be compared from one case to the next if the same number of input data contracts is used. This is so because the overall number of factors identified will always equal the initial number of maturities. Thus comparing the results of estimations with different numbers of input contracts is not necessarily appropriate. The results from the PCA on the described EEX data are illustrated in figure 19. Using eight input maturities per commodity, 95% of the dynamics can be explained by the five biggest factors, both for natural gas and electricity.

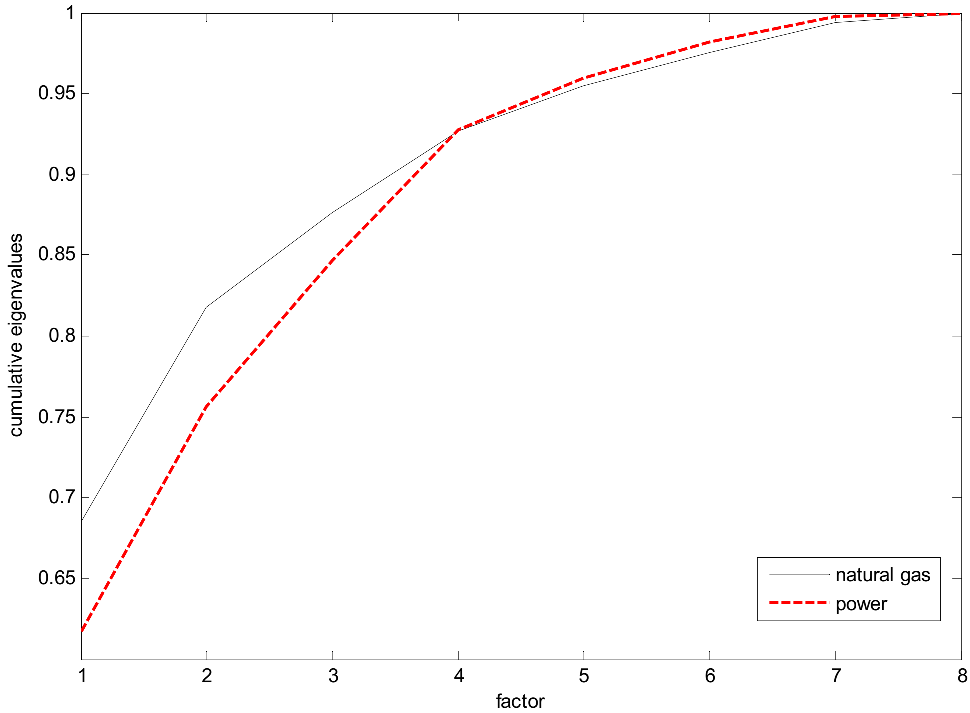

Figure 19: Cumulative eigenvalues for natural gas and power

The above results refer to the case where PCA is performed separately on the natural gas and electricity data sets, respectively. Since in the pricing efforts later on in this chapter the dynamics derived from PCA on the joint covariance matrix of natural gas and power are used, these are also presented now. It can be seen in table 2 that as many as six factors are sufficient to describe more than 96% of the joint evolution of the natural gas and electricity forward curves. This information is important for the decision concerning the number of factors to be used in the forward curve model and the truncation of the integration space for the volatility integral. The goal is to make the calculations faster while still yielding sufficiently accurate results.

factor number	eigenvalue	explanation percentage	cumulative explanation percentage
1	1,5381	0,6764	0,6764
2	0,2965	0,1304	0,8068
3	0,1321	0,0581	0,8649
4	0,1126	0,0495	0,9144
5	0,0639	0,0281	0,9425
6	0,0460	0,0202	0,9628
7	0,0403	0,0177	0,9805
8	0,0172	0,0076	0,9880
9	0,0139	0,0061	0,9942
10	0,0047	0,0021	0,9962
11	0,0037	0,0016	0,9978
12	0,0028	0,0012	0,9991
13	0,0009	0,0004	0,9995
14	0,0007	0,0003	0,9998
15	0,0004	0,0002	1,0000
16	0,0001	0,0000	1,0000

Table 2: Eigenvalues and explanation percentages for all 16 factors (natural gas and power)

This discussion raises the question concerning which criterion is used to decide how many factors should be chosen for the model. The most popular criterion consists in calculating the ratio between each eigenvalue and the sum of all eigenvalues. The number of factors is then chosen such that a certain percentage benchmark of explanation power is reached. This approach is chosen by Cortazar & Schwartz (1994) as well as by Koekebakker & Ollmar (2005). There are, however, also other approaches which have been proposed. According to the Kaiser-criterion for example, only those eigenvalues which are greater than one should be used (Kaiser & Dickman, 1959). In this case this would correspond to using only one factor, which seems rather little. In the further analysis the first approach is chosen and therefore six factors are used for option pricing.

6.2 RISK-NEUTRAL OPTION AND SWAPTION VALUATION

In the following the results from the implementation of the option and swaption valuation methods introduced above are shown, compared and discussed. First is the presentation of some results from the Black futures option formula, then the performance of the Margrabe exchange option formula is compared to values obtained through Monte Carlo simulation. This analysis is then repeated for the Bachelier method, as well as for the Kirk and Bjerksund & Stensland spread option formulas. Lastly, spark spread swaption prices obtained from Monte Carlo simulation are presented and certain issues concerning correlation are discussed.

As a start some results from the use of the Black (1976) futures option formula are presented in figure 20. Shown are prices for put options on power front-month futures. The maturities are chosen in thirty-day-intervals over the next two years from 05/31/2010. The strike is chosen as the initial forward price such that the option at valuation is at-the-money.

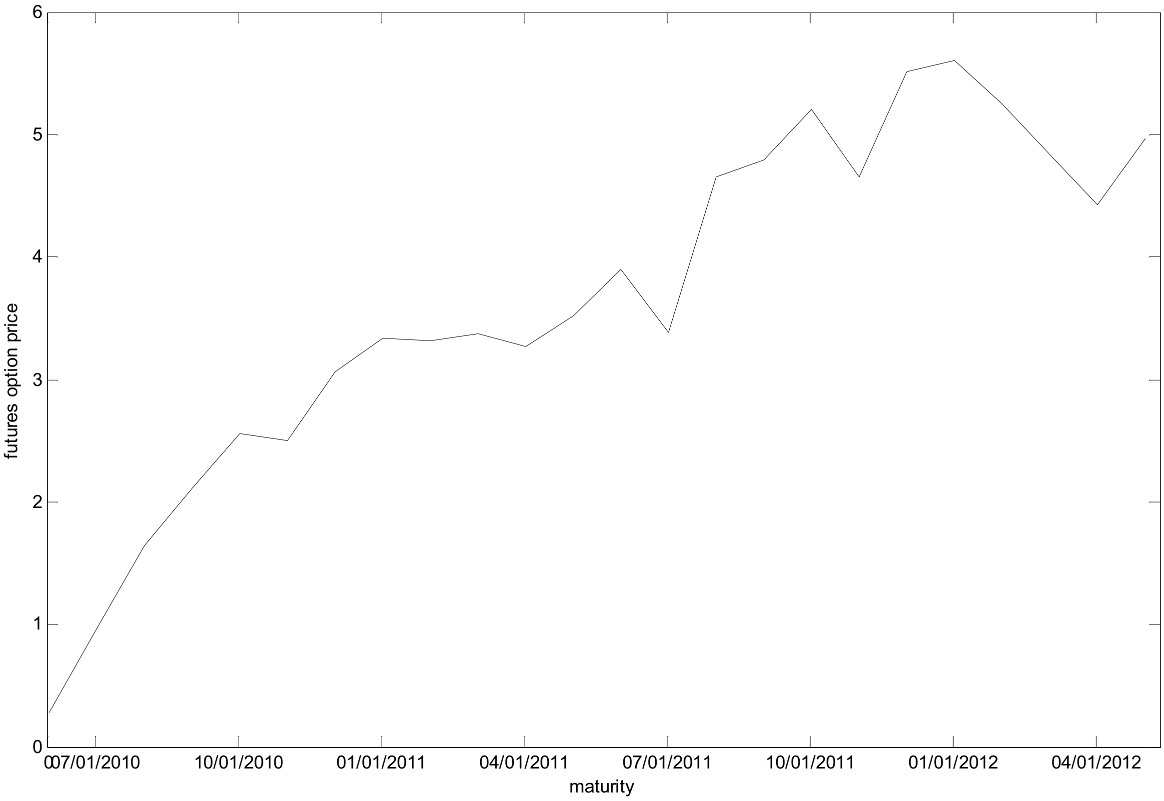

Figure 20: Front-month power futures put option prices calculated by the use of the Black (1976) formula for maturities from 1 to 720 days

An important observation to make is the impact of seasonality. The option prices do generally increase with time-to-maturity but the slope of the increase varies. For maturities in the spring and summer months of 2010 for example, the slope is much flatter than for the fall and winter months. For some points the option prices even decline compared to their value the month before. This is due to the time-dependence which is reflected in the volatility function model. This supports the stated importance of taking the seasonal and time dependencies into account when modeling the forward curve dynamics.

The next section treats the first performance comparison between the fully lognormal simulation model and a closed form formula. For the zero-strike option the Margrabe formula's performance is evaluated. It has been mentioned before that other formulas, such as Kirk's and Bjerksund & Stensland's, reduce to the Margrabe formula in the case of a strike price of zero. The analysis confirms that in the zero-strike case the results of these three formulas are the same. How the Margrabe formula performs in comparison to a Monte Carlo simulation with a zero-strike incorporated is shown in figure 21. The result is that the formula is fairly accurate and only minor differences occur. It can thus be concluded that if exchange options are evaluated the Margrabe formula can well be used to obtain fairly accurate but much faster results than by the use of a full Monte Carlo simulation.

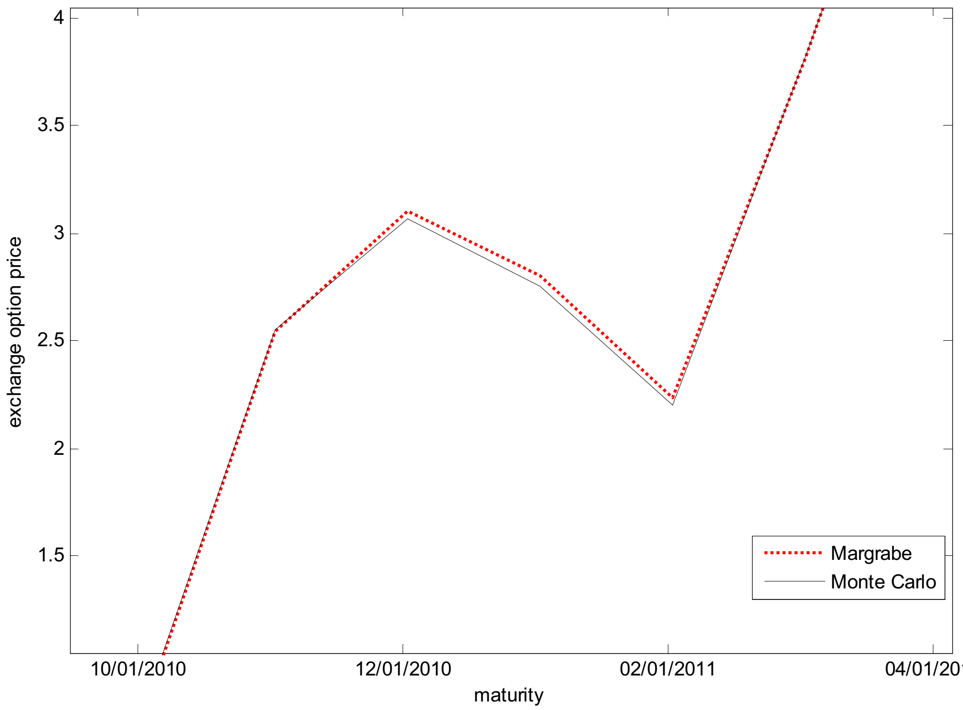

Figure 21: Margrabe and Monte Carlo exchange option prices for maturities of two to 360 days with an implied heat rate of 2 (underlyings are the respective front month futures)

In practice, options with non-zero strike prices might be even more relevant and interesting to study. The first approach to deriving these prices which was introduced is Bachelier's model. It involves the direct modeling of the spread as an arithmetic Brownian motion. A performance test was done by Carmona & Durrleman (2003) who find that the Bachelier method can be quite accurate for small times-to-maturity but the error increases by much for longer times-to-maturity. They also test the impact of the correlation between the two underlying asset prices and find that the error compared to values derived through Monte Carlo simulation decreases as the correlation coefficient decreases. Lastly, they find that the error increases as the strike increases.

Given a specific underlying data set the effect of various correlations was not tested but the result was obtained for those correlations which are implied in the volatilities calculated under the multifactor forward curve model. The effects of time-to-maturity and the strike price were tested. Figure 22 shows the difference between the Monte Carlo simulation and the Bachelier method as a surface plot with varying strike price and time-to-maturity.

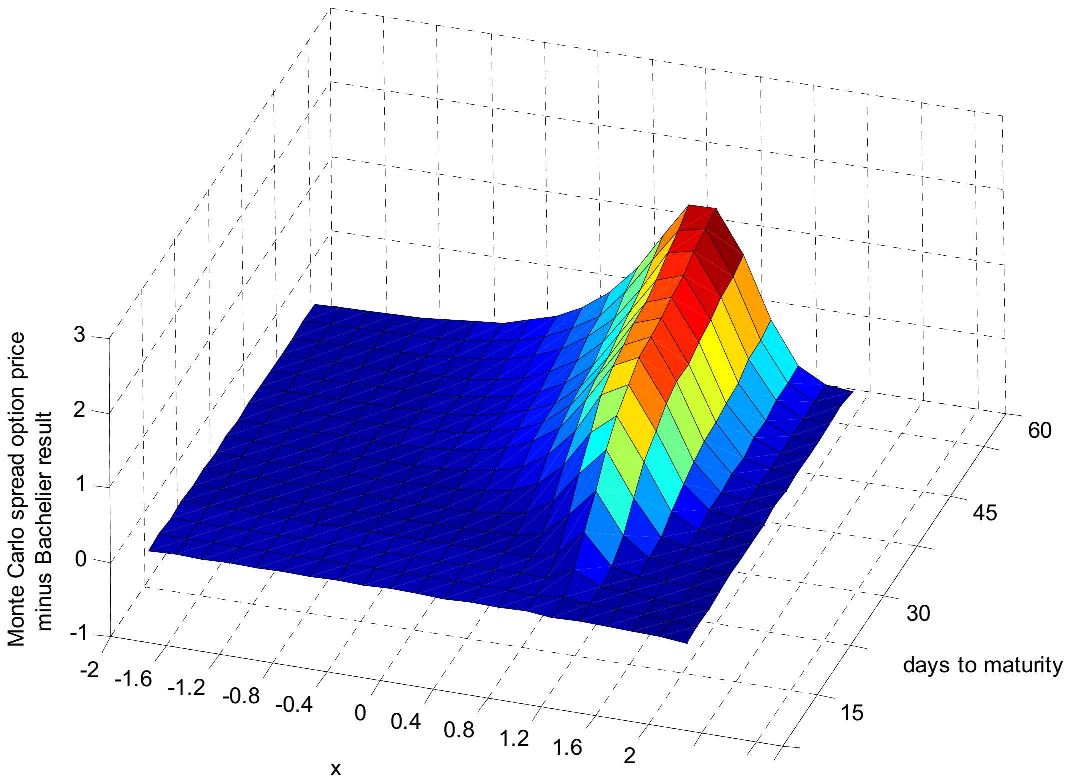

Figure 22: Difference between Monte Carlo and Bachelier spread put option values for various strike prices and times-to-maturity

The factor x in figure 22 gives the strike price as a multiple of the at-the-money strike. $x = 1$ therefore corresponds to the at-the-money strike, $x = 0$ to a strike price of zero:

$$K = x * K_{at-the-money}. \tag{6.1}$$

The at-the-money strike corresponds to the futures spread with maturity T at the time of valuation t:

$$K_{at-the-money} = Spread_t = F_{p,t}^T - HF_{g,t}^T. \tag{6.2}$$

The observations which can be made from figure 23 are that the error increases with time-to-maturity. This is in line with Carmona & Durrleman's (2003) observations, as well as the finding that the Bachelier method tends to underestimate the value of the option: all calculated differences are positive. With respect to the strike price the figure however suggests that the error does not generally increase with the strike price but it is highest for a strike that lies between the at-the-money and a zero strike and decreases as the strike increases or decreases from that point. From there it approaches the Monte Carlo value. This can also be seen in figure 23.

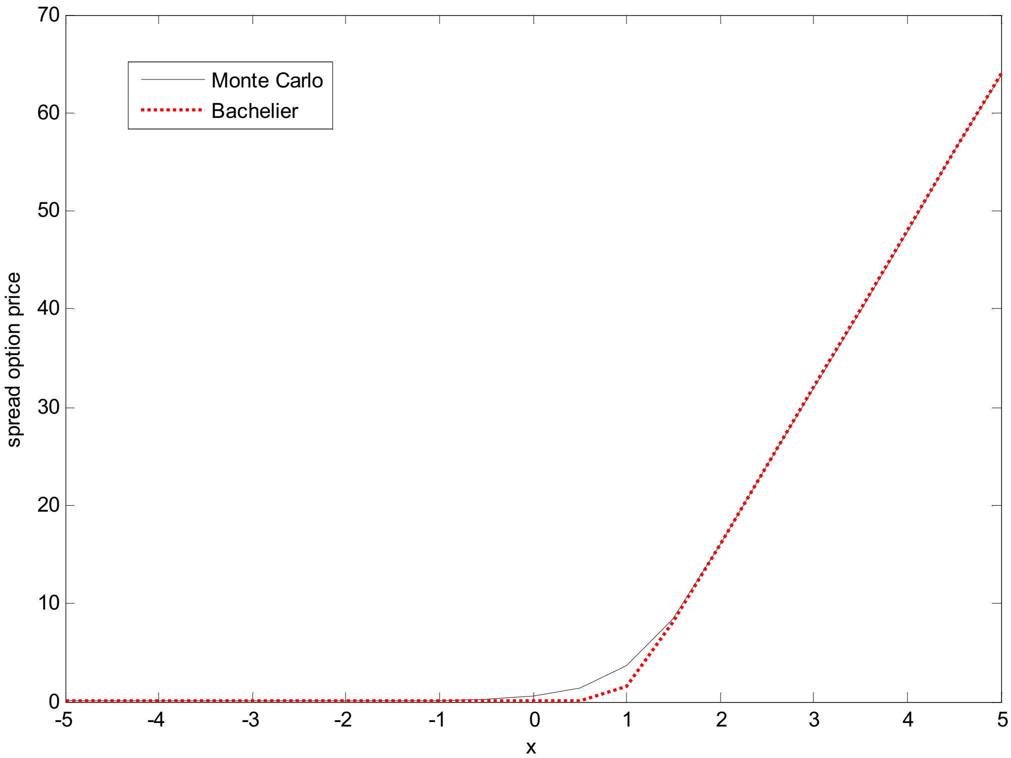

Figure 23: Monte Carlo and Bachelier spread put option values for various strike prices and option maturity of 60 days

Given theses observations the conclusion is that the Bachelier method only performs well for certain ranges of the parameters. Its reliable application is therefore very limited and other methods should be considered. Potentially better results could be delivered by the two other methods introduced, namely the Kirk (1995) and the Bjerksund & Stensland (2006) approaches.

It is found that both these formulas deliver results relatively close to the lognormal model. The Bjerksund & Stensland formula, however, performs somewhat better than Kirk's. This can be told from the comparison of the two surface plots shown in figures 24 and 25. Figure 24 shows the difference between the Monte Carlo simulation spark spread put option values and the Kirk formula values for maturities between two and 60 days and for multiples of the at-the-money strike price between -2 and 2. Figure 25 shows the results for the difference between the Monte Carlo and the Bjerksund & Stensland values given the same parameters as in figure 24. From the magnitude of the error it is clear that the Kirk formula performs somewhat worse.

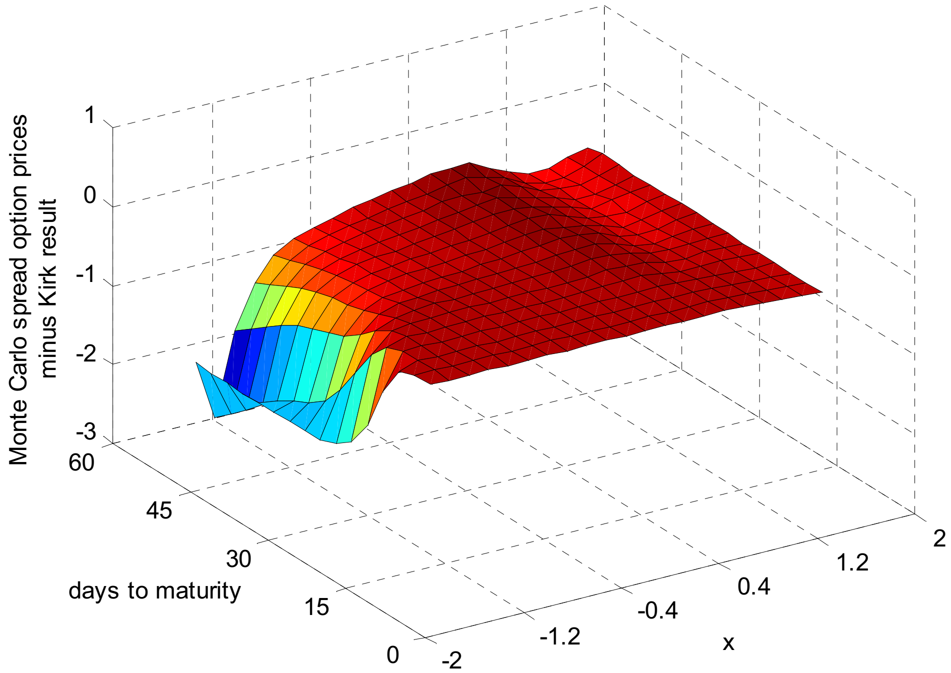

Figure 24: Difference between Monte Carlo and Kirk spread put option values for various degrees of moneyness and time-to-maturity

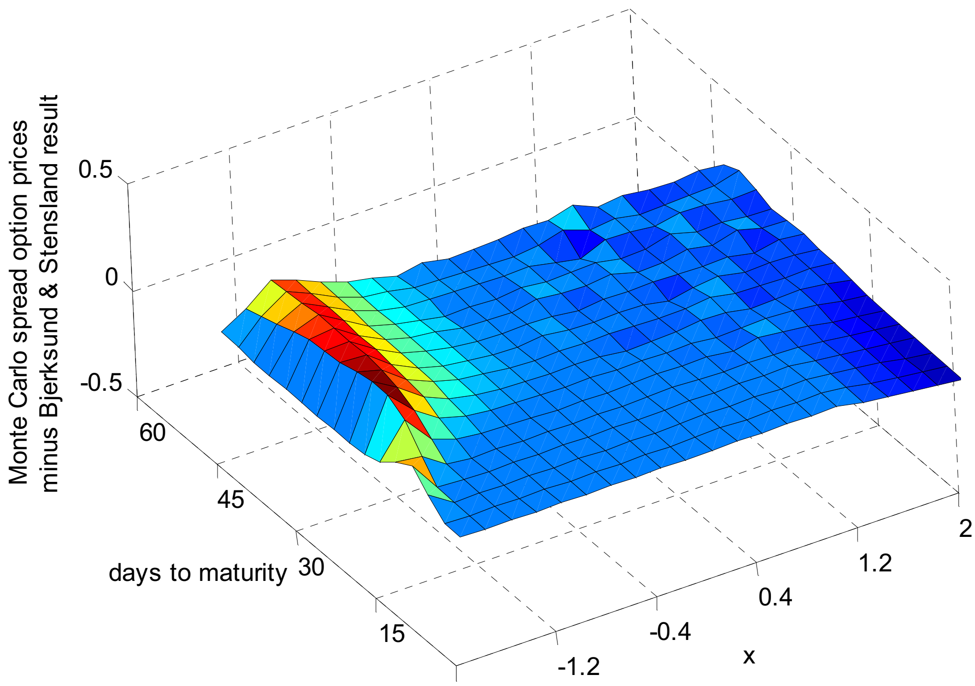

Figure 25: Difference between Monte Carlo and Bjerksund & Stensland spread put option values for various degrees of moneyness and time-to-maturity

With respect to the varying degree of moneyness the Kirk formula delivers higher values for way out-of-the money options than the Bjerksund & Stensland and the Monte Carlo methods do. While Bjerksund & Stensland's results also deviate somewhat from the Monte Carlo results with decreasing moneyness, the absolute error is much smaller for their method than for Kirk's. This is in line with their own findings. They therefore argue that their model is the better choice for the valuation of spread options (Bjerksund & Stensland, 2006).

After the presentation of the spark spread futures option pricing results, the next section is concerned with the pricing of options on spark spread differential swaps. Figure 26 shows swaption premiums with varying time-to-maturity. The underlying swaps start 30 days after option maturity and include the floating spark spread swapped against a fixed amount over three consecutive months with monthly resetting. Again a seasonal pattern can be distinguished. In the case of swaptions, it is however less pronounced than in the single-payment case. Since swaptions include multiple payment periods compared to a commodity futures option which includes delivery over only one short period, the seasonal effect is washed out to a certain degree and the swaption price mirrors rather the average seasonal influence over the swap period.

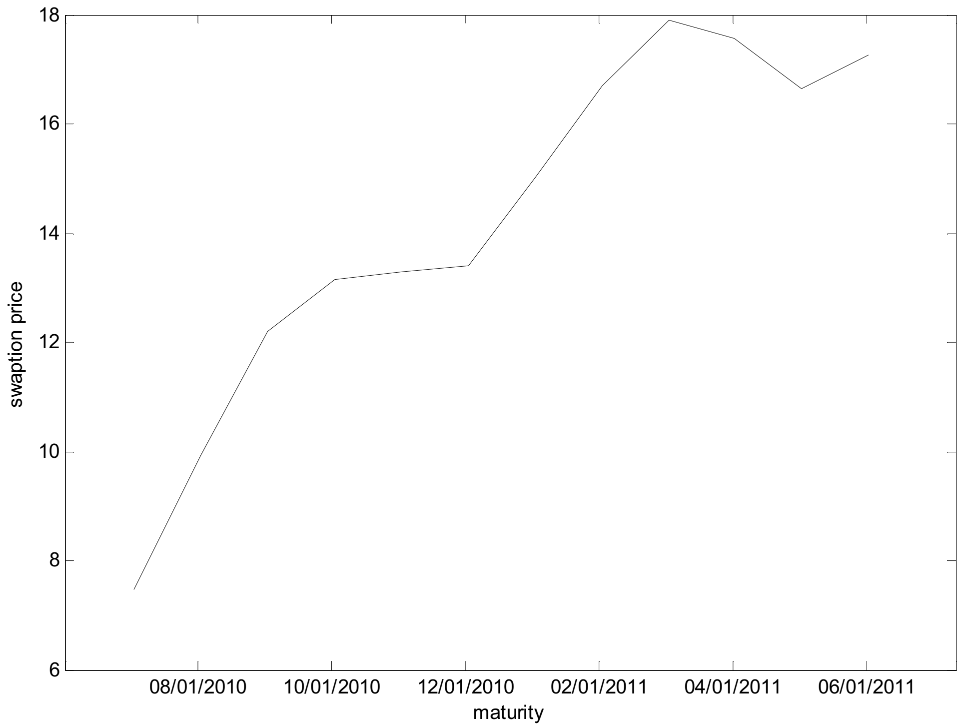

Figure 26: Spark spread swaption prices for maturities from 30 to 360 days, three swap legs, and implied heat rate of 2

The strike price dependence of spark spread swaptions is analogous to that of futures and spark spread options. For a swaption expiring 90 days from valuation date prices with various degrees of moneyness, indicated by x as described above, are depicted in figure 27.

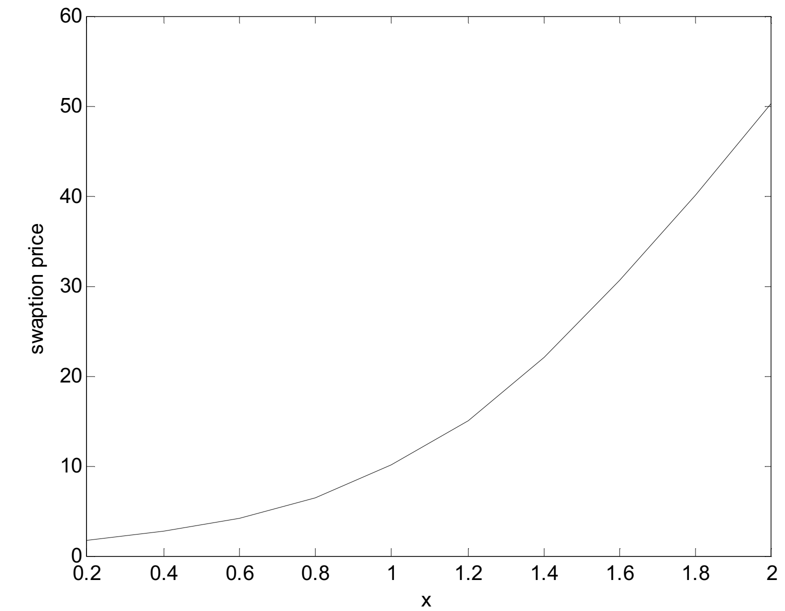

Figure 27: Strike price dependence of spark spread swaption values with three swap legs and heat rate 2

Another factor which influences spark spread swaption (and also option) values is the implied heat rate. With a given strike price the option to exchange a variable margin against a fixed margin is worth more the higher the power plant heat rate, i.e. the less efficient the power plant. This is illustrated in figure 28.

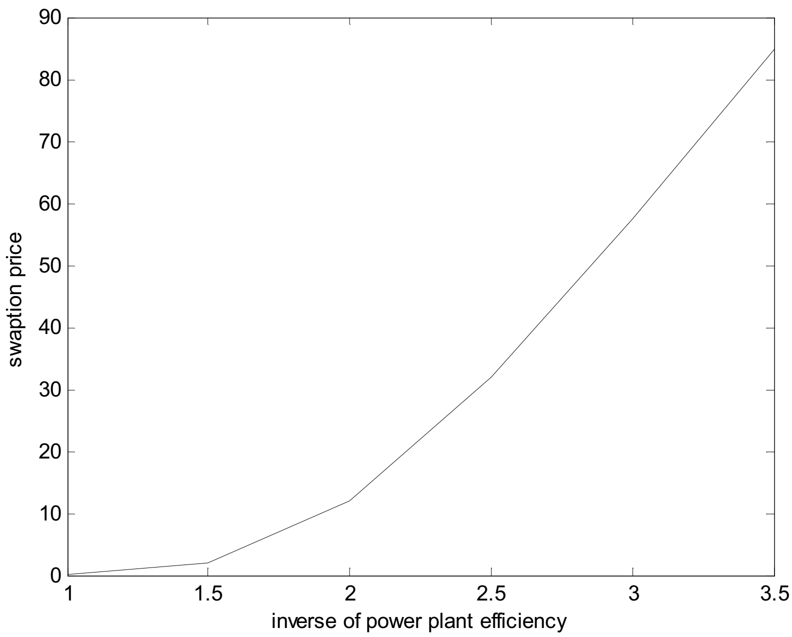

Figure 28: Heat rate dependence of spark spread swaption values (three swap legs)

The last topic to be emphasized in this analysis concerns the importance of correlation. While for spread option pricing only the correlation between the futures contracts of one certain maturity and two commodities is of importance, for differential swaptions the dependencies between the various futures maturities of both commodities are of importance. An example is shown here to illustrate these facts. It involves the comparison between swaption pricing in the fully lognormal model as it is known from the preceding analysis and a simplified version where only the correlations between the contracts for the two commodities of the same maturity are modeled. Within the single commodities *all* dependencies between maturities are modeled, i.e. the full covariance matrices of the single commodities are incorporated. Between the commodities only contracts with the same maturity are assumed to be correlated. Thus the correlation between a two-month futures contract for natural gas and a two-month futures contract for power is taken into account. These kinds of correlations are in general positive. However, the correlation between a two-month futures contract for natural gas and a three-month futures contract for power is implicitly assumed to be zero and thus not reflected in the model. Figure 29 shows the resulting differences when such a modeling approach is taken.

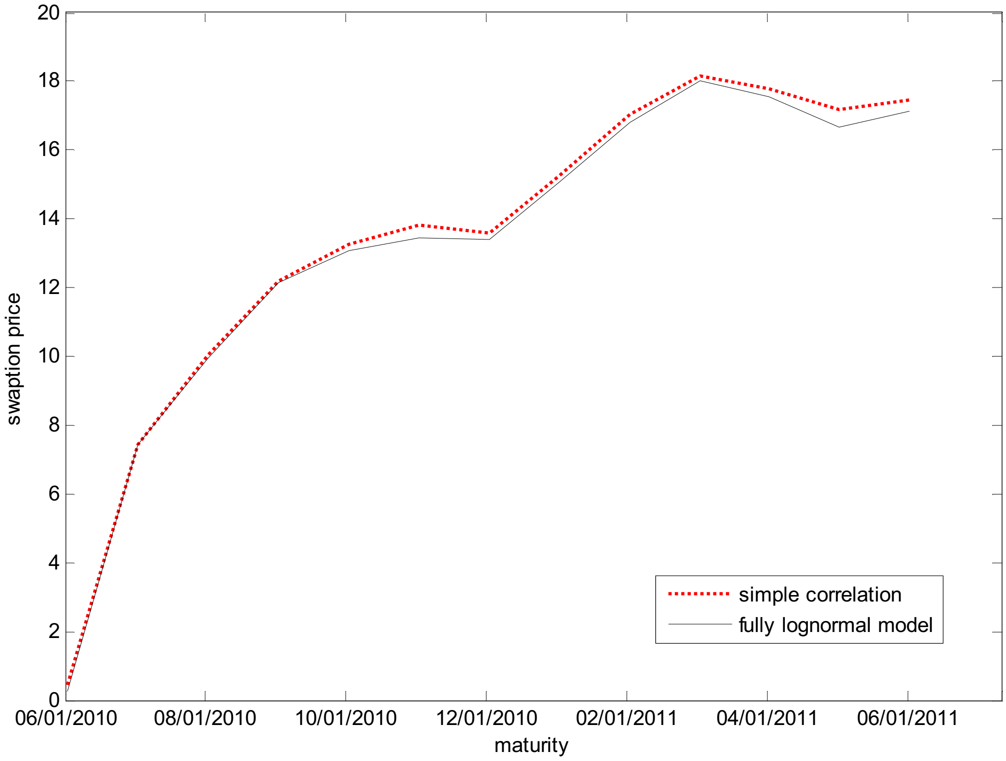

Figure 29: Swaption values for fully lognormal and simplified correlation structure

Obviously the fact that parts of the correlations are not modeled in the simplified simulation version is mirrored in higher swaption prices. This example thus emphasizes the importance to model all possible correlations between the contracts involved in option valuation. Reducing the simulation effort by only taking into account certain parts can only be done at the price of a lower degree of accuracy.

To summarize this chapter, which presented the results of our analysis, the following key statements can be made:

- The way the shown option prices reflect seasonality is a strong argument for the incorporation of seasonal influences in the forward curve model. Here, both the influences of time-to-maturity and time-of-valuation are critical.
- The Bachlier method is only accurate for certain ranges of the parameters and generally produces higher errors. In case a decision needs to be made a different method should be preferred.
- The closed form approximations to spread option pricing according to Kirk and Bjerksund & Stensland deliver results very close to those of Monte Carlo simulation and can therefore be considered accurate and fast alternatives. In a direct comparison Bjerksund & Stensland's approach performs somewhat better than Kirk's.
- Especially for spread option pricing the correlation between the two assets making up the spread is important and needs to be modeled accurately. This argument can be extended to the correlations between futures contracts of various maturities, which is a key input for spread swaption pricing.

Concerning the last issue, correlation, it should be stated that the joint lognormal evolution of the commodities returns might not be the model best fitting market observations. With the multifactor forward curve model in a Monte Carlos setting it is easy to incorporate different dependence structures between the two commodities and still retain the lognormal property of the marginal distributions. A concept which allows for this are *copulas*. Since copulas have enjoyed increasing popularity over the past years, in the next chapter an outlook is given on how they can be incorporated into spread option pricing. The goal is to present some starting points for further research.

7 OUTLOOK – THE INCORPORATION OF COPULAS

Given the above illustrated importance of modeling the dependence between the single commodities when pricing spread contingent claims, an outlook on dependence structures and how they are modeled is given in this chapter. A concept in the context of dependency structures between financial asset prices, which has gained substantial popularity, are copulas. In the field of energy, copulas have not been given much attention yet (Benth & Kettler, 2009) such that a peek at potential starting points for further research presents an even more interesting subject. After an introduction to their uses and some definitions, different types of copulas are presented. Then an example is made to illustrate the difference the incorporation of the copula concept can make in the above described pricing of spark spread contingent claims.

Explaining the root of the word copula indicates what the core of the concept is about. Sklar was the first one to use the term in 1959. He derived it from the latin word *copulare* which means to "connect" or to "join" (Schmidt, 2007). The "connections" described by copulas are those between distributions. In fact, "copulas are mainly used as tools for modeling the dependence of random variables and for describing their interrelation" (Schmidt, 2007, p. 3). The special way in which copulas are used for modeling dependencies is by disentangling the marginal distributions and their dependence structure (Schmidt, 2007). Joint distribution functions contain information both on the marginals and on their dependence. And copulas are a means to isolate the description of the two kinds of information (McNeil & Embrechts, 2005). In other words, copulas can be described as "functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions" (Nelsen, 2006, p. 7). Especially in the context of modeling and risk management the copula approach is very popular for it facilitates the combination of the more developed marginal models with various different dependence structures, as well as the consequent investigation of the sensitivity of risk to the certain model specification. A further advantageous property of copulas is that various copula types can be simulated easily and can be applied especially well in Monte Carlo simulations (McNeil & Embrechts, 2005). Before the incorporation into Monte Carlo models is shown the mathematical properties of copulas are described.

Firstly the basic properties of copulas are presented. A copula is in its essence a distribution function with standard uniformly distributed marginals. A d-dimensional copula is defined on the d-dimensional unit space $[0,1]^d$. In the following it is denoted (McNeil & Embrechts, 2005, p. 185)

$$C(\boldsymbol{u}) = C(u_1, \dots, u_d).$$

Copulas have the three following main properties (McNeil & Embrechts, 2005, p. 185):

- $C(u_1, \dots, u_d)$ is increasing in each component u_i
- $C(1, \dots, 1, u_i, 1, \dots, 1) = u_i$ for all $i \in \{1, \dots, d\}$, $u_i \in [0,1]$
- For all $(a_1, \dots, a_d), (b_1, \dots, b_d) \in [0,1]^d$ with $a_i \leq b_i$ we have
$$\sum_{i_1=1}^{2} \dots \sum_{i_d=1}^{2} (-1)^{i_1+\dots+i_d}\, C(u_{1i_1}, \dots, u_{di_d}) \geq 0,$$
where $u_{j1} = a_j$ and $u_{j2} = b_j$ for all $j \in \{1, \dots, d\}$.

In summary this corresponds to stating that a copula has the property of a distribution function (property 1), its marginals are uniform distributions (property 2), and sampling probabilities under the copula are nonnegative (property 3). If a function fulfills these criteria, it is a copula. The mechanics and core features of copulas are captured in Sklar's (1959) theorem (McNeil & Embrechts, 2005, p. 186):

" Let F be a joint distribution function with margins $F_1, \dots, F_d$. Then there exists a copula $C: [0,1]^d \rightarrow [0,1]$ such that, for all $x_1, \dots, x_d$ in $\overline{\mathbb{R}} = [-\infty, \infty]$,

$$F(x_1, \dots, x_d) = C\big(F_1(x_1), \dots, F_d(x_d)\big). \tag{7.1}$$

If the margins are continuous, then C is unique; otherwise C is uniquely determined on $Ran\ F_1 x Ran\ F_2 x \dots x Ran\ F_d$, where $Ran\ F_i = F_i(\overline{\mathbb{R}})$ denotes the range of F_i. Conversely, if C is a copula and $F_1, \dots, F_d$ are univariate distribution functions, then the function F defined in (7.1) is a joint distribution function with margins $F_1, \dots, F_d$."

This statement illustrates the relationship between multivariate distribution functions, univariate distribution functions, and the copula. It also shows how the information sets on the marginals and the dependence structure are separated in the copula framework. Furthermore it becomes clear why the copula is described above as a distribution function with uniformly distributed marginals: the marginals are the univariate distribution functions, which are characteristically uniformly distributed (Schmidt, 2007, p. 6):

$$F(x) \sim U[0,1]. \tag{7.2}$$

Consequently, the expression of the copula in terms of probabilities in the two-dimensional space is the following (Schmidt, 2007, p. 10):

$$C(u_1, u_2) = P(U_1 \leq u_1, U_2 \leq u_2). \tag{7.3}$$

McNeil & Embrechts (2005, p. 187) reformulate equation (7.1) to show that

$$C(u_1, \dots, u_d) = F\big(F_1^{\leftarrow}(u_1), \dots, F_d^{\leftarrow}(u_d)\big), \tag{7.4}$$

and that

$$P(X_1 \leq x_1, \dots, X_d \leq x_d) = C\big(F_1(x_1), \dots, F_d(x_d)\big), \tag{7.5}$$

and thus further illustrate the workings of copulas. Equation (7.4)[5] expresses the copula function explicitly in terms of the marginals and F, and illustrates how the copula information can be extracted. Equation (7.5) shows the probability interpretation of the multivariate distribution function expressed through the copula and the marginals.

A further key property of copula functions is the invariance property. A copula is invariant to strictly increasing transformations of the marginals (McNeil & Embrechts, 2005). This distinguishes the copula dependence structure from the correlation coefficient, which is only invariant under linear transformations (Schmidt, 2007).

[5] Note that for a distribution function G, $G^{\leftarrow}(y) = inf\{x: G(x) \geq y\}$ is the generalized inverse, and thus $G^{\leftarrow}(u) = x$ (McNeil & Embrechts, 2005, p. 186/187).

Analogously to the way in which Fréchet bounds, i.e. upper and lower bounds for a function, can be calculated for multivariate distribution functions in terms of the marginals (McNeil & Embrechts, 2005, p. 189)

$$max\left\{\sum_{i=1}^{d} F_i(x_i) + 1 - d, 0\right\} \leq F(x) \leq min\{F(x_1), \dots, F(x_d)\}, \tag{7.6}$$

upper and lower bounds can also be given for copulas (McNeil & Embrechts, 2005, p. 188):

$$max\left\{\sum_{i=1}^{d} u_i + 1 - d, 0\right\} \leq C(\boldsymbol{u}) \leq min\{u_1, \dots, u_d\}. \tag{7.7}$$

After this presentation of the main properties and definitions concerning copulas some copula types are presented by introducing representative copulas from the fundamental, implicit and explicit copula categories.

The first copula category is termed “fundamental” because it contains copulas which represent certain special dependence structures. These copulas are namely the independence, the comonotonicity, and the countermonotonicity copulas. The independence copula (McNeil & Embrechts, 2005, p. 189)

$$\Pi(u_1, \dots, u_d) = \prod_{i=1}^{d} u_i \tag{7.8}$$

describes the relationship between independent random variables and simply corresponds to the product of the uniform distributions. The comonotonicity and countermonotonicity copulas describe perfectly positive and negative dependence structures, respectively. They comonotonicity copula is given by the upper Fréchet bound given in equation (7.7), while the countermonotonicity copula is given by the lower bound (McNeil & Embrechts, 2005).

Implicit copulas have their name from the fact that they “are extracted from well-known multivariate distributions using Sklar’s Theorem, but do not necessarily possess simple closed-form expressions” (McNeil & Embrechts, 2005, p. 189). An example is the Gaussian copula, which for a Gaussian random vector $Y \sim N_d(\mu, \Sigma)$ with correlation matrix **P** is given by (McNeil & Embrechts, 2005, p. 191)

$$\begin{aligned} C_{\mathbf{P}}^{Ga}(u) &= P(\Phi(X_1) \leq u_1, \dots, \Phi(X_d) \leq u_d) \\ &= \Phi_{\mathbf{P}}\big(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_d)\big). \end{aligned} \tag{7.9}$$

It describes a Gaussian relationship between the random variables, and the combination of a Gaussian copula and Gaussian marginals therefore corresponds to a multivariate Gaussian distribution. In the two-dimensional case it corresponds to the correlation coefficient $\rho = \rho(X_1, X_2)$, and the Gaussian copula in that case “can be thought of as a dependence structure that interpolates between perfect positive and negative dependence, where the parameter ρ represents the strength of dependence”

(McNeil & Embrechts, 2005, p. 191). As was indicated above, there is no explicit closed form for the Gaussian copula, it can however be calculated as the integral over the underlying normally distributed random vector with mean vector zero and a given correlation matrix. Another example of an implicit copula is the t-copula which can be extracted in the same way as the Gaussian copula (McNeil & Embrechts, 2005).

For the third category - explicit copulas - closed form formulas do exist, like for example the bivariate Gumbel copula (McNeil & Embrechts, 2005, p. 192):

$$C_\theta^{Gu}(u_1, u_2) = exp\left\{-\left((-lnu_1)^\theta + (-lnu_2)^\theta\right)^{\frac{1}{\theta}}\right\} \quad (7.10)$$

with $1 \leq \theta \leq \infty.$

The Clayton copula, like the Gumbel copula, has a closed form. They both belong to the so-called Archimedean copula family (McNeil & Embrechts, 2005).

After the introduction of various copulas, the way in which they are distinct is illustrated in the following figures. Figures 30 to 32 show simulated point clouds from the Gaussian, Gumbel and Clayton copulas.

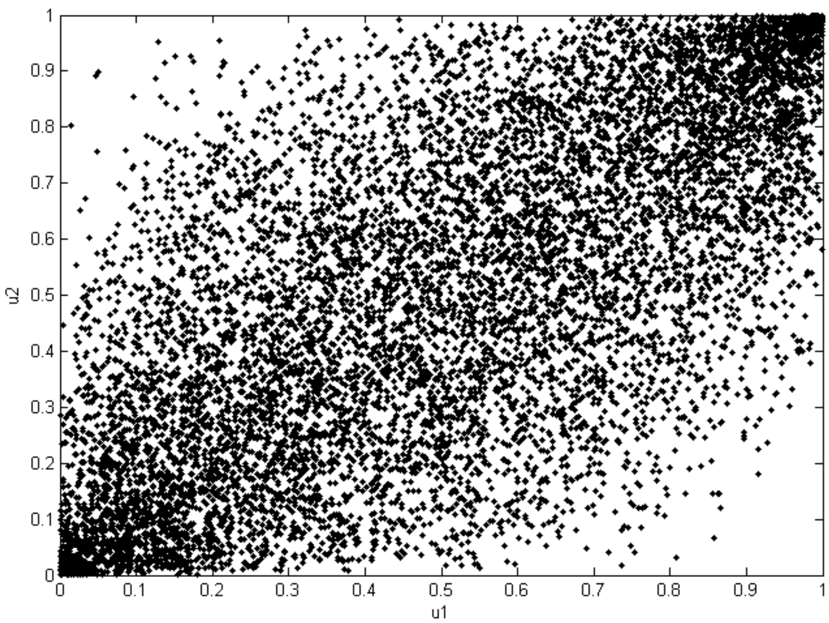

Figure 30: Gaussian copula point cloud corresponding to a correlation coefficient of 0.7

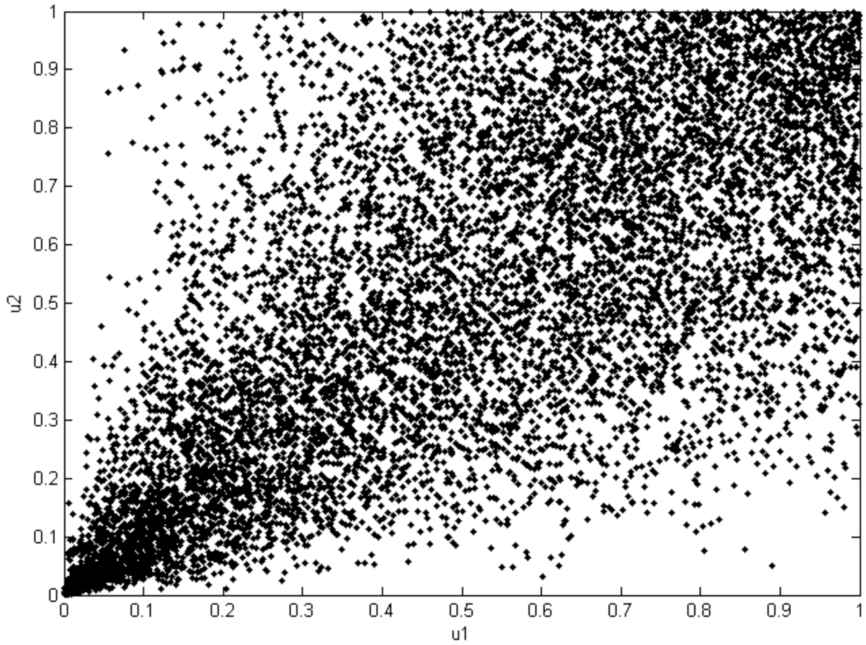

Figure 31: Clayton copula point cloud corresponding to a correlation coefficient of 0.7

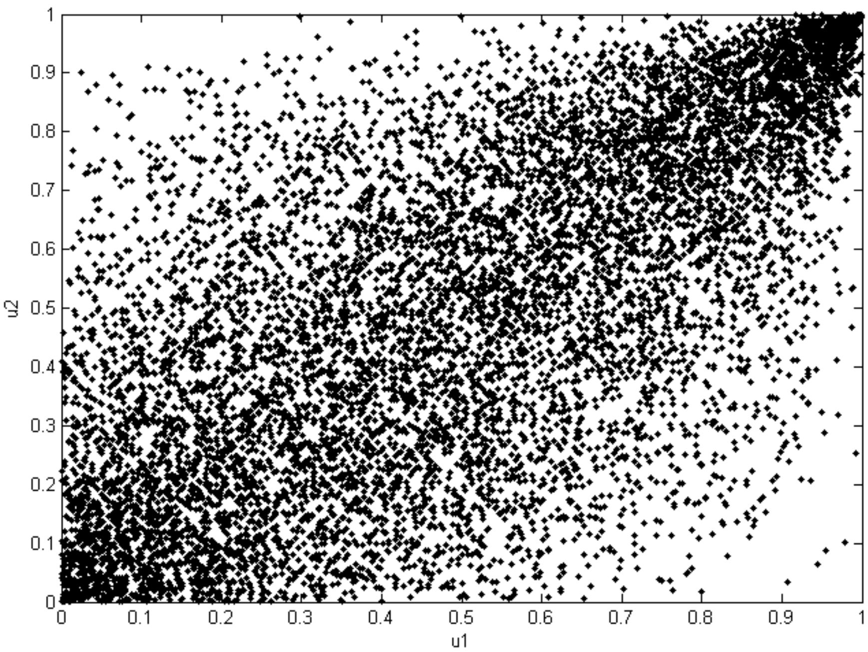

Figure 32: Gumbel copula point cloud corresponding to a correlation coefficient of 0.7

While the Gaussian copula forms a very symmetric picture, this is not the case for the Gumbel and Clayton copulas. The Gumbel copula has a high concentration of points at the top end while for the Clayton copula the points in the lower end are concentrated densely. This is why the Gumbel copula is said to exhibit "upper tail dependence" and the Clayton copula "lower tail dependence". Tail dependence is a concept which describes the *extremal dependence* between two random variables, i.e. the dependence in the tails of the bivariate distribution (McNeil & Embrechts, 2005). The concentrations in the Gumbel point cloud arises because a high realization of one random variable generally goes along with a high realization of the other random variable. The analogous interpretation applies to the Clayton case. It is this description of the various possible dependence structures that motivates the approach to incorporate copulas in the framework of pricing spark spread contingent claims. The dependence between natural gas and power prices is highly complex and it might very well be that these dependencies might be captured better through a different model dependence structure than the Gaussian one. In fact Benth & Kettler (2009) show that for spark spread spot options the incorporation of a copula dependence structure different from the Gaussian one does lead to price differences. To make a short illustration that it might also be worthwhile to further investigate the topic with respect to spark spread futures options, the Monte Carlo spark spread option valuation procedure presented above is repeated but with different dependence structures than the Gaussian one. This can be done because of the property of copulas to disentangle the dependence from the marginals. The underlying Gaussian model which was used for the spark spread option pricing above can be kept, and its advantage of easy handling can still be profited from. Additionally, new structures of dependence can be incorporated easily through the use of copulas, and thus more complex dependence structures can be reflected in the simulation. In order to carry out the described analysis in the Monte Carlo framework, the realizations of the two standard normally distributed random variables which are used for the simulation of the natural gas and electricity prices, respectively, need to be joined by the desired copula. According to the converse of Sklar's Theorem, the needed random numbers can be generated for arbitrary marginals. The first step is to generate realizations $\boldsymbol{U}$ from copula C. Then the random numbers with the desired marginal distributions and dependence structure given by C can be obtained according to McNeil & Embrechts's (2005, p. 193) elaborations:

"If $\boldsymbol{U}$ has df [distribution function] C, then we use quantile transformations to obtain

$$\boldsymbol{X} := \left(F_1^{\leftarrow}(U_1), \dots, F_d^{\leftarrow}(U_d)\right)',$$

which is a random vector with margins $F_1, \dots, F_d$ and multivariate df $C\left(F_1(x_1), \dots, F_d(x_d)\right)$."

To implement this procedure in Matlab the correlation information contained in the covariances calculated in the multifactor forward curve model is used to calculate the corresponding value of Kendall's Tau[6]. Kendall's Tau is then used to calculate the Clayton and Gumbel copula parameters. The latter are used as input to generate $\boldsymbol{U}$ before the inverse normal distribution is applied to it to yield the standard normally distributed random numbers for the Monte Carlo simulation. Those numbers then have Gaussian marginal distributions but are joined by the respective copula.

[6] Kendall's Tau is a *copula-based* dependece measure. It is a function "of the copula only and can thus be used in the parametrization of copulas" (McNeil & Embrechts, 2005, p. 201). It overcomes the inability of the correlation coefficient to distinctly determine the corresponding copula (Schmidt, 2007).

The results from the analysis are presented in figures 33 and 34. Spark spread put options were priced for various maturities and degrees of moneyness, and with the Gumbel and Clayton copulas imposed. The differences of the respective results to the Gaussian results are shown in the surface plots.

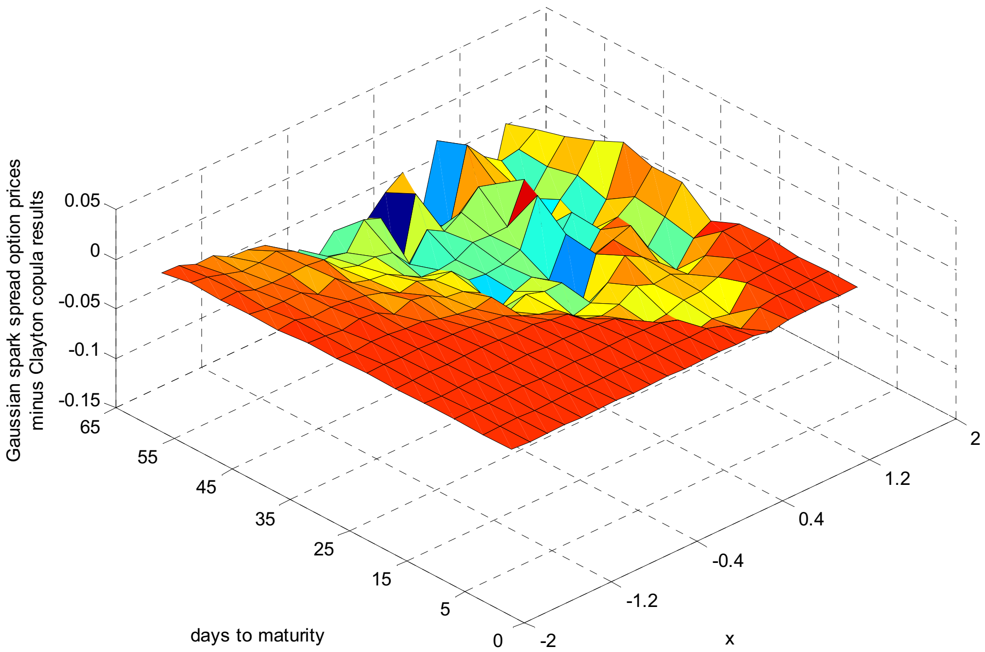

Figure 33: **Difference between spark spread put option prices under the Gaussian and the Clayton copula model for varying times-to-maturity and degrees of moneyness**

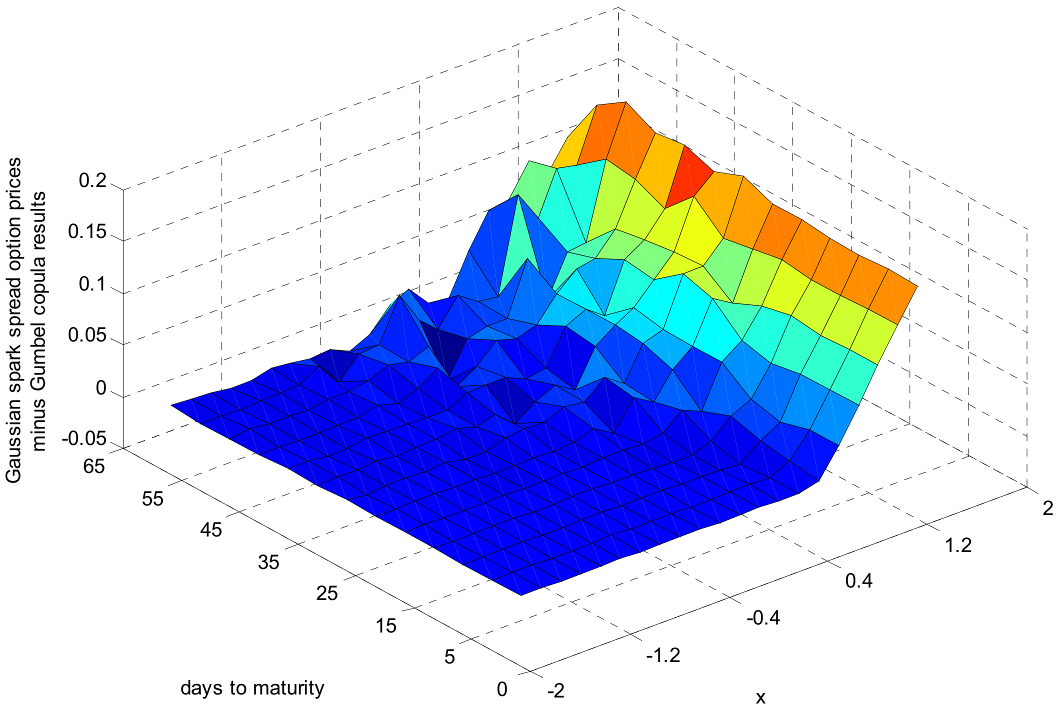

Figure 34: **Difference between spark spread put option prices under the Gaussian and the Gumbel copula model for varying times-to-maturity and degrees of moneyness**

It can be observed in figure 33 that the differences become negative around the at-the-money strike with increasing time-to-maturity. This means that option prices calculated under the Clayton copula version of the model yield higher prices around the at-the-money strike with increasing maturity. This is a first indication that different dependence structures than the Gaussian one can indeed make a difference in option pricing.

In the comparison of the Gaussian model with the Gumbel dependence structure illustrated in figure 34 again differences in the results can be observed. In this case the differences increase the further the option moves into the money. Also a slight increase of the effect can be told with increasing time-to-maturity. The spread option prices under the Gumbel regime are lower than under the Gaussian one.

To conclude these two short examples of how different dependence structures than the common Gaussian version can make a difference in spread option pricing, it should be stated that the use of copulas in the spread option and spark spread option framework could be a valuable starting point for further research and might very well yield results that can make the current valuation models more accurate. It should be examined which copula types best describe the dependence structure between power and natural gas before the results are again compared to the Gaussian model.

8 CONCLUSION

Having given an outlook on further potential research topics in the framework of pricing spark spread options and swaptions, this chapter is intended to give a brief summary of the topics presented, the issues discussed, and the results derived above.

It was stated that triggered by the European energy market liberalization natural gas and power trading volumes in Germany have grown substantially. The central exchange is the EEX where various spot and futures contracts have come to be are traded. Given this development it was stated that price risk hedging necessities as well as opportunities have arisen, and that spark spread contingent contracts could be helpful instruments for energy producers in the framework of margin hedging. Spark spread futures options allow producers to exchange their future varying margin against a fixed one in case the further is lower than the latter, and thus to hedge against a deterioration of the margin. Spark spread swaptions work analogously and refer to the swapping of the margins at prespecified dates over a given time period.

After laying the option pricing basics through the presentation of the Black & Scholes (1973) option formula and the Black (1976) futures option formula, various models for the pricing of spread options were introduced. While the Margrabe formula can only be used to calculate prices for zero-strike options, the Bachelier method, the Kirk (1995) formula, and the Bjerksund & Stensland (2006) approach can be used to price spread options with a non-zero strike price. In Bachelier's approach the spread is modeled directly as an arithmetic Brownian motion, while the latter two models rely on the modeling of the commodities as jointly lognormally distributed. Last but not least the more time consuming Monte Carlo simulation was introduced as the potentially most accurate method. For all the mentioned formulas a key input is the volatility of the forward prices over the time to option maturity, and through this the evolution of the forward curve. This is dealt with through the application of the multifactor forward curve model. It involves Principal Component Analysis on the past futures prices of various maturities, and allows for the calculation of volatility functions. The integral over the volatility functions yields the volatility of a futures contract over the time-to-maturity of a futures option contract. Seasonality is reflected in the model by standardizing the input data by the spot volatility before the PCA, and by multiplying the resulting volatility functions again by the spot volatility before the integration. This procedure was carried out using EEX NCG natural gas and Phelix base power futures prices.

The volatilities calculated were incorporated in the spread option pricing models and the performance compared. It was shown that the Margrabe formula represents a quite accurate alternative for the time consuming Monte Carlo simulation. With respect to the non-zero strike price formulas it was found that the Bachelier method is the least accurate one. The one which comes closest to the Monte Carlo results is the Bjerksund & Stensland model, followed by Kirk's formula.

Using spark spread swaption prices the importance of modeling the dependencies between the various maturities on the forward curve was emphasized. Swaption prices obtained from the fully lognormal Monte Carlo simulation were compared to a version in which only inter-commodity correlations of the same maturity were taken into account. It was shown that such a simplification (and reduction in the simulation effort) can only be achieved when accepting a loss of accuracy. This fact was used to illustrate the importance of modeling all inter-commodity and inter-temporal correlations when pricing swaptions.

Last but not least an outlook concerning the modeling of the dependence structure between the commodities was given. It was shown that the incorporation of copulas in the Monte Carlo spread option framework can be done easily, and that it might well be worthwhile to investigate further on modeling more complex dependence structures between energy commodities. The given comparison showed that in the exemplary case of the incorporation of the Gumbel and Clayton copulas, price differences compared to the Gaussian version do indeed occur.

Concluding, it should be stated that although spark spread contingent claims still play a minor role in Germany, this might change with the extension of energy trading, as it has in other countries such as the US. In this scenario the issues and approaches presented in this thesis will certainly be further discussed and developed.

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DECLARATION OF AUTHORSHIP

I hereby declare

- that I have written this thesis without any help from others and without the use of documents and aids other than those stated above,
- that I have mentioned all used sources and that I have cited them correctly according to established academic citation rules.