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International tests of a five-factor asset pricing model☆

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ABSTRACT

Average stock returns for North America, Europe, and Asia Pacific increase with the book-to-market ratio (*B/M*) and profitability and are negatively related to investment. For Japan, the relation between average returns and *B/M* is strong, but average returns show little relation to profitability or investment. A five-factor model that adds profitability and investment factors to the three-factor model of Fama and French (1993) largely absorbs the patterns in average returns. As in Fama and French (2015, 2016), the model's prime problem is failure to capture fully the low average returns of small stocks whose returns behave like those of low profitability firms that invest aggressively.

1. Introduction

Motivated by the dividend discount valuation model, Fama and French (FF) (2015) test a five-factor asset pricing model that adds profitability and investment factors to the market, *Size*, and value-growth factors of the Fama and French (1993) three-factor model. In FF (2015), the left-hand-side (LHS) assets used to test the five-factor model are portfolios formed using sorts on *Size* (market capitalization or market cap) and combinations of the book-to-market equity ratio (*B/M*), profitability (*OP*), and investment (*Inv*). The LHS portfolios are thus finer sorts on the variables used to construct the factors. To test the robustness of the five-factor model, FF (2016) use LHS portfolios formed on anomaly variables not directly targeted by the model. Here we study international markets, specifically, the four regions – North America (NA), Europe, Japan, and Asia Pacific (AP) – examined in Fama and French (2012). The goal is out-of-sample tests of the US results in FF (2015).

Our tests use variants of the five-factor time-series regression,

$$R_{it} - R_{Ft} = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}. \quad (1)$$

We take the perspective of a US investor and measure all returns in dollars. R_{it} is the dollar return on asset i for month t, R_{Ft} is the risk-free rate (the one-month US Treasury bill rate), Mkt_t is the value-weight (VW) market portfolio return minus the risk-free rate, and e_{it} is a zero-mean residual. The remaining right-hand-side (RHS) variables are differences between the returns on diversified portfolios of small and big stocks (SMB_t), high and low *B/M* stocks (HML_t), stocks with robust and weak profitability (RMW_t), and stocks of low and high investment firms (conservative minus aggressive, CMA_t). If the true values of the factor exposures, b_i, s_i, h_i, r_i, and c_i, capture all differences in expected returns, the intercept a_i in (1) is indistinguishable from zero for all LHS assets i.

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Chan, Hamao, and Lakonishok (1991), Fama and French (1998, 2012), Griffin (2002), Hou, Karolyi, and Kho (2011), and others identify *Size* and *B/M* patterns in international stock returns. We are also not the first to study how international returns relate to profitability and investment. Titman, Wei, and Xie (2013) show that high investment is followed by low average returns in many markets. Sun, Wei, and Xie (2013) and Watanabe, Yu, Yao, and Yu (2013) confirm this result and show that higher profitability is associated with higher future returns. These papers do not study in detail how the profitability and investment patterns in average returns vary across *Size* groups, and they do not attempt to capture these patterns in average returns in an asset pricing model. We show that, as in the case of *B/M*, small stocks pose the most serious asset pricing challenges related to profitability and investment.

Our asset pricing tests ask whether the five-factor model and variants of the model explain the *Size*, *B/M*, *OP*, and *Inv* patterns in international returns. Thus, as in FF (2015), the LHS portfolios are finer sorts of the variables used to form the RHS factors. This choice of LHS portfolios stacks the deck against rejection. The tests nevertheless provide strong challenges to the models we consider.

We examine local versions of the models, in which the returns to be explained and the factors to explain them are from the same region. The relations between average returns and profitability or investment are largely missed by local versions of the three-factor model of FF (1993), which (dropping the time subscript) include only *Mkt*, *SMB*, and *HML* in Eq. (1). Though also typically rejected in formal tests, local versions of the five-factor model absorb most of the *OP* and *Inv* patterns in average returns. We also provide evidence, brief and negative, on the performance of a global version of the five-factor model.

Our LHS portfolios reveal novel results about returns in international markets. Among the most interesting are the low average returns in Europe and Asia Pacific for small stocks with factor loadings like those of unprofitable firms that invest a lot. When we sort on profitability and investment, for example, the 1990–2015 average excess return (relative to the risk-free rate) for a value-weight portfolio of small European stocks in the lowest *OP* and highest *Inv* groups is −0.65% per month and the average for AP stocks is −0.71%. The average excess return for the analogous North American portfolio is low but less extreme, 0.12% per month. In tests on US returns, FF (2015) find that the average returns on portfolios of small stocks with factor loadings like those of firms that invest a lot despite low profitability are usually much lower than predicted by the five-factor model. This result is also prominent in the anomaly sorts in FF (2016). Although the low average returns for these stocks are more extreme in Europe and AP, in some sorts the five-factor model captures them.

We start (Section 2) with descriptions of the left-hand-side portfolios and right-hand-side factors used in estimates of Eq. (1). Section 3 tests whether asset pricing in the four international regions conforms to a global version of Eq. (1). The answer is a strong no, and the rest of the paper focuses on tests in which we use regional factors to capture LHS returns for the same region. Sections 4 and 5 present summary statistics for regional RHS and LHS returns. Sections 6–8 are the main event – tests of asset pricing models. Section 6 tests whether any regional factors are redundant in the sense that their average returns are captured by their exposures to other factors. Section 7 presents summary statistics for regression intercepts that flesh out the implications of the spanning tests. Section 8 details the intercept improvements produced by adding *RMW* and *CMA* to the FF (1993) three-factor model.

Finally, the dividend discount model that motivates Eq. (1) is useful for suggesting variables related to differences in expected asset returns, but it is silent on economic or behavioral explanations of the differences. One interpretation of Eq. (1) is that it is the regression equation for a multifactor version of Merton's (1973) intertemporal capital asset pricing model (ICAPM). In this view, *SMB*, *HML*, *RMW*, and *CMA* are not themselves state variable mimicking portfolios, but their long and short ends are [in the terminology of Fama (1996)] multifactor minimum variance portfolios that together capture the effects of state variables on returns. A less ambitious interpretation of Eq. (1) is that it is the regression equation of an empirical asset pricing model designed to span the mean-variance-efficient tangency portfolio and so capture expected asset returns. We return to interpretation issues in the conclusions, Section 9.

2. Data and variables

Our international stock returns and accounting data are primarily from Bloomberg, supplemented by Datastream and Worldscope. The sample period, July 1990 to December 2015 (henceforth 1990–2015), is constrained by data availability and the desire to have broad coverage of small and big stocks in the markets we examine. To increase the power of the tests, we use diversified LHS portfolios in the regressions. Diversification improves regression fits, which increases the precision of the intercepts that are the focus of the asset pricing tests. As in FF (2012), to diversify the LHS portfolios, we combine 23 developed markets into four regions: (1) North America (United States and Canada); (2) Japan; (3) Asia Pacific (Australia, New Zealand, Hong Kong, and Singapore); and (4) Europe (Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom). We also examine Global portfolios that combine the four regions.

Parsimony in the choice of regions is important for the power of the tests, but we want regions in which market integration is a plausible assumption. It is reasonable to assume that the US and Canada are close to one market for goods and securities during our sample period (Mittoo, 1992). For much of our sample, almost all the countries of Europe are members of the European Union (EU), and those that are not (e.g., Switzerland) participate in most of the EU's open market provisions. Our tests suggest that market integration is most questionable in the Asia Pacific region, which on average accounts for only about 4% of the market cap of the four regions, versus about 48% for NA, 30% for Europe, and 18% for Japan.

In each region, we sort stocks on *Size* and combinations of *B/M* (ratio of book equity to market equity), *OP* (ratio of operating profits – sales minus the cost of goods sold, selling, general, and administrative expenses, and interest expense – to book equity), and *Inv* (annual growth rate of assets). In our work on US stocks (e.g., FF, 1993) we use NYSE breakpoints for *Size* and other variables to avoid sorts that are dominated by tiny but plentiful AMEX and Nasdaq stocks. For the same reason, the *B/M*, *OP*, and *Inv* breakpoints here are based on large stocks, and the *Size* breakpoints are percentiles of aggregate market cap chosen to avoid undue weight on tiny stocks.

2.1. RHS factors

The RHS explanatory returns in our tests are for portfolios constructed from 2×3 sorts on *Size* and *B/M, OP*, or *Inv*. At the end of June each year t, we sort the stocks in a region on market cap. Big stocks are those in the top 90% of market cap for the region, and small stocks are in the bottom 10%. For North America, 90% of market cap corresponds roughly to the NYSE median used to define small and big stocks in FF (1993). The *B/M*, *OP*, and *Inv* breakpoints in the 2×3 sorts for a region are the 30th and 70th percentiles of the variable for the big stocks of the region. As in FF (1993), the accounting variables are for the fiscal year ending in calendar year $t-1$ and market cap in *B/M* is for the end of December of calendar year $t-1$. Global portfolios use global *Size* breaks, but to mitigate any effects of differences in accounting rules, we use each region's breakpoints for *B/M*, *OP*, and *Inv* to allocate its stocks to the Global portfolios.

For each region, the intersection of the independent 2×3 sorts on *Size* and *B/M* produces six portfolios – *SG*, *SN*, *SV*, *BG*, *BN*, and *BV*, where *S* and *B* indicate small or big and *G*, *N*, and *V* indicate growth, neutral, and value (bottom 30%, middle 40%, and top 30% of *B/M*), respectively. We compute monthly VW returns for each portfolio from July of year t to June of $t+1$. The *Size* factor, $SMB_{B/M}$, for a region is the equal-weight (EW) average of the returns on the three small stock portfolios from the 2×3 *Size-B/M* sorts for the region minus the average of the returns on the three big stock portfolios. For each region, we construct value minus growth returns for small and big stocks, $HML_S = SV - SG$ and $HML_B = BV - BG$, and *HML* is the average of HML_S and HML_B. The profitability and investment factors, *RMW* and *CMA*, are constructed in the same way as *HML* except the second sort is on either profitability (robust minus weak) or investment (conservative minus aggressive).

The 2×3 sorts used to construct *RMW* and *CMA* produce two additional *Size* factors, SMB_{OP} and SMB_{Inv}. The overall *Size* factor *SMB* is the average of $SMB_{B/M}$, SMB_{OP}, and SMB_{Inv}. Equivalently, *SMB* is the average of the returns on the nine small stock portfolios of the three 2×3 sorts minus the average of the returns on the nine big stock portfolios.

The variables used to construct *HML*, *RMW*, and *CMA* are correlated. High *B/M* value stocks, for example, tend to have low profitability and investment, and low *B/M* growth stocks, especially large low *B/M* stocks, tend to be profitable and invest aggressively (Fama and French, 1995). The correlations imply that the value, profitability, and investment factors, *HML*, *RMW*, and *CMA*, are different mixes of value, profitability, and investment effects in returns.

The breakpoints used in the factors in Eq. (1) mimic those of our previous work (e.g., FF, 1993) but are nevertheless arbitrary. The experiments in FF (2012) suggest that the performance of international models is not sensitive to choice of factor breakpoints. Since all models are imperfect, however, future work will likely refine the definitions of the factors in models like Eq. (1). For example, Ball, Gerakos, Linnainmaa, and Nikolaev (2015) find that a profitability factor based on cash profitability captures average returns associated with accruals better than the *RMW* factor of Eq. (1), which uses operating profitability. We also expect that future work will introduce additional factors. For example, a momentum factor is a common addition to the FF (1993) three-factor model. In the interests of parsimony, we are also hopeful that future work will identify redundant factors. The tests here suggest, for example, that the investment factor is on shaky ground.

2.2. LHS portfolios

At the end of June each year, we construct 25 *Size-B/M*, 25 *Size-OP*, and 25 *Size-Inv* portfolios for each region, to use as LHS assets in asset pricing regressions. The *Size* breaks for a region are the 3rd, 7th, 13th, and 25th percentiles of the region's aggregate market cap. These correspond roughly to the average market caps for NYSE quintile breakpoints for *Size* used in FF (1993, 2015). The *B/M*, *OP*, and *Inv* breakpoints in the 5×5 sorts follow the same rules as the 2×3 sorts, except we use quintile breakpoints (instead of 30–40–30 splits) for big stocks (top 90% of market cap) in each region to allocate the region's big and small stocks. The 25 VW *Size-B/M*, *Size-OP*, and *Size-Inv* portfolios for a region are the intersections of the independent 5×5 *Size* and *B/M*, *Size* and *OP*, and *Size* and *Inv* sorts.

Since *B/M*, *OP*, and *Inv* are correlated, *Size-B/M*, *Size-OP*, and *Size-Inv* portfolios do not isolate value, profitability, and investment effects in average returns. To disentangle the dimensions of average returns, we would like to sort jointly on *Size*, *B/M*, *OP*, and *Inv*, but even a $3 \times 3 \times 3 \times 3$ sort produces 81 portfolios, many poorly diversified. We compromise with $2 \times 4 \times 4$ sorts on *Size* and pairs of *B/M*, *OP*, and *Inv*. We form two *Size* groups, big and small, again defined as the top 90% and bottom 10% of the market cap of a region, and we use quartiles to form four groups for each of the other two sort variables. This process yields 32 *Size-B/M-OP*, 32 *Size-B/M-Inv*, and 32 *Size-OP-Inv* portfolios to use as LHS portfolios in asset pricing tests. The sorts on *B/M*, *OP*, and *Inv* are independent, but to spread stocks more evenly in the $2 \times 4 \times 4$ sorts, we use separate breakpoints for *B/M*, *OP*, and *Inv* for small and big stocks.

3. Tests of a global model

Fama and French (2012) find that a Global version of the FF (1993) three-factor model does not explain regional expected returns. A simple spanning test produces the same conclusion for the five-factor model. We summarize the results but do not present a table.

We estimate 20 regressions in which the LHS returns are for regional factors (five local factors for each of four regions) and the RHS returns are the five Global factors. If the Global model describes expected returns, the 20 regression intercepts are indistinguishable from zero: the Global factors span the regional factors. In fact, the intercept for the NA *Mkt* regression, 0.43%, is more than four standard errors from zero ($t = 4.10$), five intercepts are more than three standard errors from zero, and seven are more than two. The *GRS* test (Gibbons, Ross, and Shanken, 1989) says the probability the true intercepts are zero is zero to at least five decimal places. Thus, adding the regional factors to the Global factors produces a reliably large increase in the maximum Sharpe ratio from the Global factors alone. In short, the Global factors do not span the regional factors.

Tests on other sets of LHS portfolios also confirm that the Global five-factor model performs poorly when LHS portfolios are regional. For example the average return for the NA market factor, 0.62% per month, is greater than the mean for Global *Mkt*, 0.43%, and when Global factors are used to explain a wide range of NA portfolio returns, most intercepts are strongly positive. The average *Mkt* return for Japan is close to zero, 0.01%, and strong negative intercepts are the rule when Global factors are used to explain Japanese portfolio returns. We can infer that the Global five-factor model is a poor choice in applications in which the LHS portfolios are regional.

The Global five-factor model may fail because markets are not globally integrated or because we have the wrong Global model. Our goal, however, is to follow the trail of the five-factor model as far as it will go. Since the Global model fails badly when the LHS assets are regional portfolio returns, we close that branch of the trail. The asset pricing tests to come focus on five-factor models that use regional factors to explain LHS portfolio returns for the same region. To set the stage, we start with summary statistics for returns to the regional RHS factors (Section 4) and LHS portfolios (Section 5).

4. Summary statistics for RHS factor returns

Panel A of Table 1 shows that the equity premium (average *Mkt* return) for Japan for 1990–2015 is near zero (0.01% per month, $t = 0.04$). Equity premiums for the other regions are large (0.62%, $t = 2.53$ for NA; 0.47%, $t = 1.64$ for Europe; and 0.71%, $t = 2.05$ for AP). The 1990–2015 *Size* premium (average *SMB* return) is close to zero in all regions. The largest *Size* premium is 0.17% per month ($t = 1.05$) for NA.

Large value premiums (average *HML* returns) are the rule for 1990–2015. Only the value premium for North America, 0.20% ($t = 1.08$), is less than 2.19 standard errors from zero. The value premium for Japan, 0.36% per month ($t = 2.19$), is the only Japanese premium not close to zero–an important result for the asset pricing tests to come. Profitability premiums (average *RMW* returns) for NA and Europe are substantial (0.34%, $t = 2.46$ for NA; 0.41%, $t = 4.76$ for Europe). The average *RMW* return for Asia Pacific is lower, 0.21% per month ($t = 1.25$). Investment premiums (average *CMA* returns) for NA, Europe, and AP are 0.20% to 0.39% per month and 1.86–2.60 standard errors from zero.

Panel B of Table 1 confirms the evidence in FF (2012) that, for NA, Europe, and AP, the value premium is larger for small stocks. For Japan, however, average HML_B, 0.41% per month ($t = 1.97$), is larger than average HML_S, 0.30% ($t = 1.67$), but the average difference is only 0.51 standard errors from zero. The new evidence is that, except for Japan, where there are no reliable profitability and investment premiums, average *RMW* and *CMA* returns are also larger for small stocks. For NA, Europe, and AP, the average values of RMW_S range from 1.80 (AP) to 7.79 (Europe) standard errors from zero. The average values of CMA_S are all more than 2.9 standard errors from zero. Average RMW_B and CMA_B returns are all positive, but none break the two standard error barrier. The average values of the difference $RMW_{S\text{-}B} = RMW_S - RMW_B$ are 0.61 (AP) to 2.09 (Europe) standard errors from zero, so the evidence that the expected profitability premium is larger for small stocks is weak. The evidence that the expected investment premium is larger for small stocks is stronger, at least for the two major regions, North America and Europe. The average values of CMA_S for NA (0.45%, $t = 3.03$) and Europe (0.31%, $t = 2.95$) are about three times those of CMA_B (0.13%, $t = 0.70$ for NA, and 0.09%, $t = 0.68$ for Europe), and the average values of the spread, $CMA_{S\text{-}B} = CMA_S - CMA_B$, are 2.59 (NA) and 2.14 (Europe) standard errors above zero. For AP, average CMA_S (0.51%) is almost twice average CMA_B (0.28%), but the average difference is only 1.04 standard errors from zero.

Panel C of Table 1 shows correlations across regions for each of the five factors. Market factors (*Mkt*) are most correlated. The correlation between the market factors of NA and Europe is 0.80, and the correlations of AP with NA and Europe are 0.72 and 0.75. The correlations of the Japanese market factor with the other three are lower, 0.42–0.51. Correlations are lower for other factors. Europe and NA on average account for almost 80% of the market cap of the sample, so their correlations are of special interest. Of the four non-market factors, the *HML* returns of Europe and NA are most correlated (0.61), next is *CMA* (0.57), then *SMB* (0.31) and *RMW* (0.21). The profitability factor *RMW* is least correlated across regions. The 0.21 correlation for Europe and NA is the largest in the matrix.

Average premiums for 1990–2015 (306 months) have large sampling errors as estimates of expected premiums. For example, the standard error of the average *Mkt* (equity premium) return for Japan is 0.34%, and the plus and minus two standard error range about the 0.01% mean is wide, from −0.67% to 0.69%.

More important, average factor returns are interesting, but the contribution of a factor to a model depends on its marginal information about average returns (the intercept in the spanning regression of the factor's return on the returns of the model's other factors), not on its average return. We examine factor spanning regressions in Section 6, after discussing the LHS average returns of portfolios formed on *Size*, *B/M*, profitability, and investment that variants of the five-factor model attempt to explain.

Table 1

Summary statistics for factor returns: July 1990–December 2015, 306 months.

We construct regional factors for North America (NA), Europe, Japan, and Asia Pacific (AP) (excluding Japan). To construct *HML*, we form portfolios at the end of June of each year *t* by sorting stocks in a region into two market cap and three book-to-market equity (*B/M*) groups. Big stocks are those in the top 90% of June market cap for the region, and small stocks are those in the bottom 10%. The *B/M* breakpoints for the four regions are the 30th and 70th percentiles of lagged (fiscal year $t-1$) *B/M* for the big stocks of a region. The independent 2×3 sorts on *Size* and *B/M* produce six value-weight portfolios, *SG*, *SN*, *SV*, *BG*, *BN*, and *BV*, where S and B indicate small or big and G, N, and V indicate growth, neutral, and value (bottom 30%, middle 40%, and top 30% of *B/M*). $SMB_{B/M}$ is the equal-weight average of the returns on the three small stock portfolios for the region minus the average of the returns on the three big stock portfolios. We construct value – growth returns for small and big stocks, $HML_S = SV - SG$ and $HML_B = BV - BG$, and *HML* is the equal-weight average of HML_S and HML_B. The profitability and investment factors, *RMW* and *CMA*, and their small and big components are constructed in the same way as *HML* except the second sort is either on *OP* (the ratio of operating profits to book equity, sorted from robust to weak) or investment (the rate of growth of total assets, sorted from conservative to aggressive). The 2×3 sorts used to construct *RMW* and *CMA* produce two additional *Size* factors, SMB_{OP} and SMB_{Inv}. The *Size* factor *SMB* is the average of the returns on the nine small stock portfolios of the three 2×3 sorts minus the average of the returns on the nine big stock portfolios. All returns are in US dollars. *Mkt* is the return on a region's value-weight market portfolio minus the US one-month T-bill rate. The average T-bill rate is 0.28%. *Mean* and *Std Dev* are the mean and standard deviation of the return, and *t*-Mean is the ratio of *Mean* to its standard error. The table shows *Mean*, *Std Dev*, and *t-Mean* for factor returns (Panel A), the small and big components of the factors and the difference between them (Panel B), e.g., HML_S, HML_B, and $HML_{SB} = HML_S - HML_B$, and the correlations of each factor across regions (Panel C).

Panel A: Means and standard deviations of factor returns

	Mkt	*SMB*	*HML*	*RMW*	*CMA*	*Mkt*	*SMB*	*HML*	*RMW*	*CMA*
	North America					Europe				
Mean	0.62	0.17	0.20	0.34	0.29	0.47	0.05	0.32	0.41	0.20
Std Dev	4.30	2.81	3.25	2.45	2.70	5.01	2.23	2.42	1.50	1.87
t-Mean	2.53	1.05	1.08	2.46	1.86	1.64	0.39	2.30	4.76	1.89
	Japan					Asia Pacific				
Mean	0.01	0.09	0.36	0.13	0.08	0.71	−0.08	0.59	0.21	0.39
Std Dev	5.94	3.35	2.84	2.19	2.46	6.06	3.01	3.05	2.90	2.66
t-Mean	0.04	0.46	2.19	1.03	0.59	2.05	−0.44	3.38	1.25	2.60

Panel B: Small and Big components of factor returns

	HML_S	HML_B	$HML_{S\text{-}B}$	RMW_S	RMW_B	$RMW_{S\text{-}B}$	CMA_S	CMA_B	$CMA_{S\text{-}B}$
North America									
Mean	0.41	−0.01	0.43	0.43	0.26	0.18	0.45	0.13	0.32
Std Dev	4.06	3.05	3.05	2.86	2.66	2.53	2.57	3.21	2.15
t-Mean	1.78	−0.07	2.44	2.65	1.68	1.22	3.03	0.70	2.59
Europe									
Mean	0.45	0.19	0.26	0.55	0.27	0.28	0.31	0.09	0.23
Std Dev	2.62	2.99	2.87	1.23	2.39	2.35	1.87	2.28	1.84
t-Mean	2.99	1.09	1.60	7.79	1.95	2.09	2.95	0.68	2.14
Japan									
Mean	0.30	0.41	−0.11	0.15	0.11	0.03	0.07	0.10	−0.03
Std Dev	3.15	3.65	3.76	2.33	3.33	3.71	2.29	3.37	3.02
t-Mean	1.67	1.97	−0.51	1.09	0.59	0.16	0.53	0.50	−0.16
Asia Pacific									
Mean	0.86	0.32	0.54	0.28	0.13	0.15	0.51	0.28	0.23
Std Dev	3.15	4.09	4.00	2.72	4.27	4.19	2.23	4.09	3.88
t-Mean	4.78	1.36	2.36	1.80	0.55	0.61	4.01	1.19	1.04

Panel C: Correlation matrices for the same factor in different regions

	NA	Europe	Japan	AP	NA	Europe	Japan	AP
	Mkt				*SMB*			
NA	1.00	0.80	0.42	0.72	1.00	0.31	0.08	0.24
Europe	0.80	1.00	0.51	0.75	0.31	1.00	0.33	0.47
Japan	0.42	0.51	1.00	0.47	0.08	0.33	1.00	0.14
AP	0.72	0.75	0.47	1.00	0.24	0.47	0.14	1.00
	HML				*RMW*			
NA	1.00	0.61	0.44	0.24	1.00	0.21	−0.02	0.12
Europe	0.61	1.00	0.39	0.23	0.21	1.00	0.09	0.08
Japan	0.44	0.39	1.00	0.12	−0.02	0.09	1.00	0.06
AP	0.24	0.23	0.12	1.00	0.12	0.08	0.06	1.00
	CMA							
NA	1.00	0.57	0.22	0.31				
Europe	0.57	1.00	0.36	0.28				
Japan	0.22	0.36	1.00	0.18				
AP	0.31	0.28	0.18	1.00				

5. Average excess returns for LHS portfolios

Average returns for the 5 × 5 *Size-B/M*, *Size-OP*, and *Size-Inv* sorts in Panel A of Table 2 provide detail on how average *B/M*, *OP*, and *Inv* return spreads vary with *Size*. The *Size-B/M* sorts show a positive relation between average returns and *B/M*, but except for Japan, it is progressively weaker in bigger *Size* quintiles. During 1990–2015 there is no relation between average return and *B/M* in the Big (megacap) row of the 5 × 5 *Size-B/M* matrix for North America, but this result is special to NA. For Europe and Asia Pacific, average returns increase with *B/M*, even for megacaps. For Japan, there is a value effect in all *Size* quintiles, and the spread in average returns is larger for megacaps (0.55% per month) than for the Small quintile (microcaps, 0.21% per month).

A note of caution is in order. The *Size* breakpoints in the LHS sorts are set so that a *Size* group contains the same percent of market cap in each region, but the average *Size* of the firms in a *Size* group (which we do not report) declines from NA to Europe, Japan, and then AP. For example, on average the megacaps of Europe are only a bit smaller than NA megacaps, but the megacaps of Japan average less than half the *Size* of NA megacaps, and AP megacaps are about one-third the *Size* of NA megacaps. Thus, putting aside the important role of chance, it is meaningful to compare patterns in average returns across *Size* groups of a region, but comparisons across regions can be misleading.

In the *Size-OP* sorts for NA, Europe, and AP, average returns increase with *OP* in all *Size* quintiles, and decay in the return spread in larger *Size* quintiles is less severe than in the *Size-B/M* sorts. For Japan, average returns in the highest *OP* quintile are larger than in the lowest quintile, but the spreads are small, and the relation between profitability and average returns is weak.

Fama and French (2015) find that in US data for 1963–2013, average returns for the 5 × 5 *Size-Inv* portfolios decline with *Inv* for megacaps, but the striking feature of average returns for the other four *Size* quintiles is a sharp drop from the fourth to the highest investment quintile. Table 2 confirms this pattern for NA and, more interesting, shows there is a similar pattern in *Size-Inv* average returns for Europe. For AP, the drop in average returns in the highest *Inv* quintile extends to AP's much smaller megacaps. There is no clear relation between *Inv* and average returns for Japan. We see later that, Japan aside, the low returns of smaller high investment firms pose a challenge to the five-factor model that is not fully met, despite the presence of the investment factor, *CMA*.

Average 5 × 5 *Size-B/M*, *Size-OP*, and *Size-Inv* in Table 2 also reinforce the Table 1 evidence that the *Size* effect is weak (NA) to nonexistent (Europe, Japan, and AP) during 1990–2015.

Panel B of Table 2 shows average excess returns from the 2 × 4 × 4 sorts. Although they condition on an additional variable, these sorts largely just reinforce the 5 × 5 sorts. In brief, for the small stocks of NA, Europe, and AP, average returns increase rather smoothly with *B/M* and profitability, but the relation between average returns and investment is dominated by a drop in average returns in the highest *Inv* quartile. There are similar *B/M*, *OP*, and *Inv* patterns in average returns for the big stocks of NA, Europe, and AP, but they tend to be weaker and less consistent. For Japan, there is a strong positive relation between average returns and *B/M* for small and big stocks, but profitability is weakly related to average returns, and there is no systematic relation between average returns and *Inv*.

In the Asia Pacific *Size-B/M-OP* sorts, we do not show an average return for the big stock portfolio in the highest *OP* and *B/M* quartiles. Profitability and *B/M* are negatively correlated; high *B/M* (value) stocks tend to be less profitable than low *B/M* (growth) stocks. As a result, especially for big stocks, the portfolio of stocks in the highest *B/M* and *OP* quartiles is less diversified. In the AP region, this portfolio is sometimes empty and we exclude it from all tests.

Finally, in the US 2 × 4 × 4 *Size-OP-Inv* sorts in FF (2015), the low 1963–2013 average return of small stocks in the lowest *OP* and highest *Inv* quartiles (unprofitable firms that invest a lot) is deadly for the five-factor model. Small low profitability firms that invest a lot are also problematic for the five-factor model in many anomaly sorts in FF (2016). Panel B of Table 2 confirms that the NA small stock portfolio in the lowest *OP* and highest *Inv* quartiles has a low average excess return (0.14% per month), as does the big stock portfolio in the lowest *OP* and highest *Inv* quartiles (0.11%). A new result in Table 2 is that these two portfolios have lower average excess returns in the *Size-OP-Inv* sorts for Europe, −0.66% for small stocks and −0.11% for big stocks, and the average excess return for the small, low *OP*, high *Inv* portfolio in AP is even lower, −0.72% per month. It will be interesting to see if, as in the US results of FF (2015), small stock portfolios that combine low profitability and high investment are lethal for the five-factor model in NA, Europe, and AP. (They are not an issue for Japan.)

6. Factor spanning tests

In US data for 1963–2013, FF (2015) find that the intercept in the regression of *HML* on the other four factors of the five-factor model is close to zero. This implies that *HML* is redundant for describing US average returns during this period. Its average return is spanned (fully explained) by the average returns on the other four factors. It is interesting to examine whether this result holds for the shorter period used here, for other markets, or other factors.

Table 3 shows regressions (five for each region) in which four factors explain returns on the fifth. For North America, Europe, and Asia Pacific, the market factor *Mkt* is not redundant. The intercepts in the *Mkt* regressions are strongly positive for NA (0.99% per month, $t = 4.70$), Europe (0.74%, $t = 2.82$), and AP (1.28%, $t = 4.37$). These intercepts exceed average *Mkt* returns (Table 1), largely due to negative *RMW* and *CMA* slopes. The intercept in the *Mkt* regression for Japan is smaller, 0.36% ($t = 1.12$), but above the average *Mkt* return, 0.01%, largely due to a strong negative *HML* slope.

The value factor, *HML*, is important for describing 1990–2015 average returns in all regions. The intercepts in the regressions of *HML* on other factors are positive and more than 2.25 standard errors from zero for Europe, Japan, and

Table 2
Average monthly percent excess returns on the portfolios from regional 5×5 and $2 \times 4 \times 4$ sorts: July 1990–December 2015, 306 months.

Panel A shows average monthly excess returns for the portfolios of the three 5×5 sorts. At the end of June of each year, we construct 25 *Size-B/M* portfolios for each region. The *Size* breakpoints are the 3rd, 7th, 13th, and 25th percentiles of aggregate market cap for a region. The *B/M* quintile breakpoints use the big stocks (top 90% of market cap) of a region. The intersections of the independent *Size* and *B/M* sorts for a region produce 25 value-weight *Size-B/M* portfolios. The 25 *Size-OP* and *Size-Inv* portfolios are formed in the same way except that the second sort is on *OP* (profitability) or *Inv* (investment). Panel B shows average excess returns for the portfolios of the $2 \times 4 \times 4$ sorts. At the end of June each year, stocks in each region are allocated to two *Size* groups (Small and Big) using the 90th percentile of market cap for the region as breakpoint. Small and Big stocks are allocated independently to four *OP* quartiles (Low *OP* to High *OP*) and four *Inv* quartiles (Low *Inv* to High *Inv*), using *OP* and *Inv* breakpoints for the small or big *Size* group of the region. The intersections of the three sorts produce 32 *Size-OP-Inv* portfolios for each region. The 32 *Size-B/M-OP* and the 32 *Size-B/M-Inv* portfolios are formed in the same way, except the second sort is on *B/M* and the third sort is on *OP* or *Inv*.

Panel A: Size-B/M, Size-OP, and Size-Inv portfolios

	Low *B/M*	2	3	4	High *B/M*	Low *B/M*	2	3	4	High *B/M*
	North America					Europe				
Small	0.43	0.57	0.89	0.83	1.11	−0.10	0.28	0.42	0.49	0.68
2	0.27	0.59	0.78	0.74	0.80	0.23	0.39	0.48	0.64	0.72
3	0.76	0.63	0.74	0.71	0.87	0.30	0.53	0.50	0.51	0.66
4	0.79	0.64	0.79	0.72	0.81	0.44	0.47	0.48	0.56	0.62
Big	0.60	0.57	0.59	0.63	0.49	0.34	0.48	0.52	0.61	0.49
	Japan					Asia Pacific				
Small	0.16	0.14	0.23	0.24	0.37	0.41	0.32	0.75	1.04	1.49
2	0.20	−0.21	−0.02	0.17	0.16	−0.06	0.18	0.39	0.60	0.92
3	−0.27	−0.07	−0.05	−0.00	0.23	0.17	0.32	0.67	0.70	0.70
4	−0.26	−0.02	−0.02	0.15	0.20	0.56	0.79	0.59	0.81	1.03
Big	−0.14	0.01	0.02	0.21	0.41	0.64	0.74	0.72	0.74	0.92

	Low *OP*	2	3	4	High *OP*	Low *OP*	2	3	4	High *OP*
	North America					Europe				
Small	0.79	1.00	1.04	1.03	1.06	0.08	0.60	0.66	0.85	0.71
2	0.38	0.71	0.91	0.98	1.07	0.21	0.50	0.62	0.71	0.89
3	0.56	0.72	0.76	0.84	1.06	0.18	0.52	0.67	0.59	0.78
4	0.53	0.72	0.90	0.75	0.89	0.12	0.50	0.68	0.65	0.65
Big	0.21	0.50	0.57	0.67	0.67	0.15	0.49	0.50	0.42	0.59
	Japan					Asia Pacific				
Small	0.21	0.28	0.23	0.27	0.48	0.89	1.25	1.14	1.31	1.05
2	−0.01	0.06	0.02	0.26	0.19	0.27	0.59	0.66	0.79	0.59
3	−0.01	−0.02	0.00	0.03	0.12	0.10	0.67	0.74	0.86	0.82
4	−0.17	0.03	0.08	0.18	−0.05	0.57	0.81	0.57	0.80	0.96
Big	−0.07	−0.04	0.14	0.09	0.03	0.48	0.77	0.82	0.94	0.63

	Low *Inv*	2	3	4	High *Inv*	Low *Inv*	2	3	4	High *Inv*
	North America					Europe				
Small	1.17	1.09	0.96	0.94	0.47	0.51	0.62	0.64	0.56	0.10
2	0.85	0.85	0.81	0.78	0.31	0.54	0.73	0.70	0.57	0.29
3	0.90	0.85	0.85	0.72	0.50	0.61	0.59	0.69	0.45	0.21
4	0.89	0.92	0.80	0.81	0.47	0.58	0.58	0.58	0.58	0.32
Big	0.71	0.60	0.59	0.56	0.50	0.50	0.57	0.40	0.46	0.42
	Japan					Asia Pacific				
Small	0.23	0.24	0.36	0.24	0.31	1.15	1.29	1.23	1.20	0.50
2	0.11	0.12	0.12	0.15	−0.02	0.51	0.88	0.65	0.72	0.02
3	0.08	0.10	0.01	−0.04	−0.09	0.60	0.93	0.64	0.71	0.25
4	0.06	0.02	0.13	−0.04	−0.03	0.61	0.74	1.00	0.85	0.51
Big	0.04	−0.04	−0.09	0.06	−0.09	0.78	0.77	0.78	0.76	0.35

(*continued on next page*)

Table 2 (*Continued*)

Panel B: Portfolios formed on Size, B/M, and OP; Size, B/M, and Inv; and Size, OP, and Inv

	Small				Big			
	Low *B/M*	2	3	High *B/M*	Low *B/M*	2	3	High *B/M*
North America								
Low *OP*	0.25	0.54	0.67	0.87	0.68	0.36	0.42	0.36
2	0.41	0.65	0.81	0.91	0.35	0.61	0.68	0.73
3	0.72	0.75	0.94	1.00	0.66	0.64	0.71	0.64
High *OP*	0.88	0.98	1.02	1.20	0.65	0.61	0.69	0.96
Europe								
Low *OP*	−0.63	−0.32	0.16	0.27	−0.06	−0.07	0.36	0.40
2	−0.21	0.25	0.55	0.75	0.05	0.59	0.60	0.65
3	0.23	0.52	0.83	0.96	0.47	0.48	0.60	0.73
High *OP*	0.58	0.84	1.11	0.98	0.46	0.61	0.70	0.74
Japan								
Low *OP*	−0.22	−0.19	0.17	0.35	−0.21	−0.16	−0.14	0.13
2	−0.48	0.06	0.10	0.32	−0.15	0.04	0.05	0.25
3	−0.26	0.17	0.28	0.44	−0.06	0.09	0.20	0.47
High *OP*	0.08	0.27	0.47	0.57	−0.15	0.20	0.37	0.55
Asia Pacific								
Low *OP*	−0.40	−0.23	0.62	0.73	−0.55	0.21	0.48	0.82
2	0.18	−0.04	0.45	1.14	0.54	0.83	0.78	1.25
3	0.10	0.56	0.96	1.21	0.97	0.76	0.92	1.17
High *OP*	0.52	1.14	1.30	1.86	0.75	0.84	0.97	–

	Small				Big			
	Low *B/M*	2	3	High *B/M*	Low *B/M*	2	3	High *B/M*
North America								
Low *Inv*	0.82	0.97	0.94	1.17	0.65	0.64	0.85	0.74
2	0.87	0.83	0.89	1.06	0.63	0.60	0.67	0.70
3	0.72	0.82	1.11	0.76	0.56	0.59	0.61	0.67
High *Inv*	0.35	0.60	0.63	0.66	0.80	0.55	0.51	0.18
Europe								
Low *Inv*	0.20	0.55	0.71	0.66	0.34	0.60	0.57	0.55
2	0.49	0.58	0.80	0.86	0.37	0.62	0.65	0.74
3	0.48	0.61	0.71	0.77	0.38	0.47	0.57	0.66
High *Inv*	0.04	0.24	0.40	0.46	0.43	0.34	0.55	0.30
Japan								
Low *Inv*	0.01	0.11	0.25	0.43	−0.18	0.07	0.00	0.36
2	−0.12	0.11	0.29	0.36	−0.28	0.02	0.08	0.26
3	−0.25	0.13	0.18	0.40	−0.29	0.09	0.16	0.22
High *Inv*	−0.00	0.12	0.21	0.50	−0.15	0.00	0.14	0.37
Asia Pacific								
Low *Inv*	0.02	0.34	0.97	1.05	0.74	0.59	0.77	1.05
2	0.67	0.95	1.16	1.52	1.27	0.60	0.90	1.08
3	0.49	0.62	0.98	1.09	0.51	0.85	0.56	0.98
High *Inv*	−0.08	0.41	0.39	0.85	0.26	0.64	0.42	0.94

	Small				Big			
	Low *OP*	2	3	High *OP*	Low *OP*	2	3	High *OP*
North America								
Low *Inv*	0.85	0.89	0.88	1.27	0.67	0.87	0.72	0.64
2	0.89	0.85	0.89	0.92	0.56	0.61	0.67	0.65
3	0.22	0.88	0.83	0.96	0.51	0.60	0.58	0.63
High *Inv*	0.14	0.23	0.58	0.88	0.11	0.43	0.84	1.00
Europe								
Low *Inv*	0.07	0.59	0.64	0.80	0.32	0.51	0.68	0.57
2	0.15	0.62	0.72	0.92	0.38	0.50	0.61	0.65
3	−0.04	0.43	0.68	0.83	0.32	0.47	0.42	0.55
High *Inv*	−0.66	0.14	0.27	0.54	−0.11	0.61	0.45	0.58
Japan								
Low *Inv*	0.12	0.16	0.22	0.25	−0.17	0.22	0.18	0.16
2	0.13	0.11	0.19	0.18	−0.07	0.01	0.10	−0.02
3	0.09	0.07	0.15	0.12	−0.21	0.07	0.09	−0.02
High *Inv*	0.08	−0.11	−0.00	0.27	0.11	−0.19	0.07	−0.09
Asia Pacific								
Low *Inv*	0.10	0.59	0.92	0.92	0.61	1.04	0.59	0.94
2	0.67	0.99	1.12	1.27	0.80	0.93	0.74	1.25
3	0.15	0.37	0.75	0.90	0.40	0.74	1.13	0.43
High *Inv*	−0.72	0.22	0.22	0.64	0.49	0.63	0.60	0.26

Table 3
Using four factors in regressions to explain average returns on the fifth: July 1990–December 2015, 306 months.

Mkt is the value-weight return on the market portfolio of the stocks of a region, minus the one-month Treasury bill rate; *SMB* (small minus big) is the size factor; *HML* (high minus low *B/M*) is the value factor; *RMW* (robust minus weak *OP*) is the profitability factor; and *CMA* (conservative minus aggressive *Inv*) is the investment factor. The factors are constructed using separate sorts of stocks into two *Size* groups and three *B/M* groups (*HML*), three *OP* groups (*RMW*), or three *Inv* groups (*CMA*).

	Coefficient						*t*-statistic						
	Int	*Mkt*	*SMB*	*HML*	*RMW*	*CMA*	*Int*	*Mkt*	*SMB*	*HML*	*RMW*	*CMA*	R^2
North America													
Mkt	0.99		0.01	0.52	−0.53	−1.02	4.70		0.14	5.02	−5.20	−8.30	0.30
SMB	0.34	0.01		0.13	−0.49	−0.12	2.23	0.14		1.72	−7.06	−1.27	0.17
HML	−0.28	0.15	0.07		0.30	0.95	−2.44	5.02	1.72		5.54	20.31	0.65
RMW	0.46	−0.16	−0.29	0.31		−0.11	3.97	−5.20	−7.06	5.54		−1.46	0.36
CMA	0.31	−0.18	−0.04	0.61	−0.07		3.41	−8.30	−1.27	20.31	−1.46		0.68
Europe													
Mkt	0.74		−0.42	0.82	−0.54	−1.42	2.82		−3.85	5.95	−2.83	−9.19	0.30
SMB	0.16	−0.11		0.05	−0.14	−0.11	1.20	−3.85		0.66	−1.39	−1.17	0.04
HML	0.34	0.13	0.03		−0.56	0.72	3.31	5.95	0.66		−8.16	13.07	0.53
RMW	0.52	−0.05	−0.05	−0.32		0.05	7.20	−2.83	−1.39	−8.16		0.91	0.30
CMA	0.09	−0.15	−0.04	0.50	0.06		1.08	−9.19	−1.17	13.07	0.91		0.46
Japan													
Mkt	0.36		0.14	−0.56	−1.06	−0.27	1.12		1.43	−4.13	−5.31	−1.33	0.13
SMB	0.05	0.05		0.08	−0.09	0.20	0.29	1.43		1.01	−0.77	1.66	0.05
HML	0.30	−0.10	0.04		−0.02	0.63	2.29	−4.13	1.01		−0.23	8.36	0.35
RMW	0.18	−0.08	−0.02	−0.01		−0.59	2.09	−5.31	−0.77	−0.23		−13.46	0.51
CMA	0.05	−0.02	0.05	0.30	−0.63		0.60	−1.33	1.66	8.36	−13.46		0.59
Asia Pacific													
Mkt	1.28		−0.21	−0.07	−0.68	−1.02	4.37		−2.23	−0.60	−4.98	−8.76	0.35
SMB	0.14	−0.08		−0.10	−0.33	−0.09	0.77	−2.23		−1.36	−3.95	−1.09	0.05
HML	0.62	−0.02	−0.06		−0.74	0.32	4.60	−0.60	−1.36		−14.77	5.57	0.45
RMW	0.53	−0.11	−0.15	−0.57		0.20	4.46	−4.98	−3.95	−14.77		3.89	0.53
CMA	0.31	−0.20	−0.05	0.29	0.24		2.36	−8.76	−1.09	5.57	3.89		0.34

AP. The intercept in the *HML* regression for NA is −0.28% per month ($t = -2.44$). The negative intercept is due to positive *Mkt*, *SMB*, *RMW*, and *CMA* slopes that more than suffice to absorb the positive average *HML* return for NA (0.20% per month). We can also confirm that the tiny intercept in the US regression for 1963–2013 in FF (2015) is due to a positive intercept for 1963–1989 (0.23, $t = 2.34$) that offsets the negative intercept for the subsequent period used here.

The factor spanning tests suggest that the profitability factor, *RMW*, is important for describing NA, Europe, and AP average returns. The intercepts in the *RMW* regressions for these regions are positive and more than 3.95 standard errors from zero. Even for Japan, *RMW* may have a marginal role in describing average returns. The intercept in the *RMW* regression for Japan is 0.18% per month ($t = 2.09$).

The evidence that the investment factor, *CMA*, helps describe average returns is mixed. The intercept in the *CMA* regression is strong for NA (0.31% per month, $t = 3.41$) and AP (0.31%, $t = 2.36$), but trivial for Europe (0.09%, $t = 1.08$) and Japan (0.05%, $t = 0.60$). Finally, at least for 1990–2015 the *Size* factor *SMB* seems redundant everywhere except North America. The intercept in the *SMB* regressions for NA is 0.34% per month ($t = 2.23$). The intercepts for the other three regions are 0.16 or less and at most 1.20 standard errors from zero.

In short, all five factors are important for describing NA average returns for 1990–2015. The heavy lifting in Europe and AP is left to *Mkt*, *HML*, and *RMW*, with an assist from *CMA* in Asia Pacific. Despite its large average return (Table 1), the spanning tests in Table 3 say that *CMA* adds little to the description of 1990–2015 European average returns. In Japan, the big player is *HML* with perhaps an assist from *RMW*.

We caution that factor spanning inferences can be sample specific. Given the different results for *HML* here and in FF (2015), we would not be surprised to find that factors that are redundant for describing average returns in one period are important in another. Evidence of redundancy from factor spanning tests is nevertheless definitive within a sample. If a factor's average return for a period is captured by its exposures to the other factors in a model, that factor has no role in describing average returns in that model for that period. And no set of LHS portfolios can overturn this conclusion (Fama, 1998; Barillas and Shanken, 2015).

Evidence that a factor's average return is not captured by its exposures to the other factors in a model does not, however, imply that the factor helps describe average returns for all LHS assets that have nontrivial loadings on the factor. There may be assets that violate the model's predictions about how factor loadings relate to expected returns. For example, in the sorts on *Size* and accruals in FF (2016), microcaps do not have the lower average returns predicted by their negative *RMW* slopes. In the LHS sorts examined here, however, factors that produce strong intercepts in the spanning tests of Table 3 are important for describing average returns for LHS assets that have nontrivial loadings on the factors. As a result, much of what we learn from the tests that follow just fleshes out the factor spanning tests.

7. Summary asset pricing tests

If an asset pricing model explains expected returns, the intercept is indistinguishable from zero in the time series regression of any asset's excess returns on the model's factors. We use the *GRS* statistic of Gibbons, Ross, and Shanken (1989) and other summary metrics to test this hypothesis for variants of regression (1). The variants we focus on are the three-factor model of FF (1993) in which the explanatory returns are *Mkt*, *SMB*, and *HML*, the four-factor model that adds *RMW*, and the five-factor model (1). Our main goal is to compare the performance of the three-factor and five-factor models, but we also show results for the four-factor model that drops *CMA* to check on the factor spanning evidence that the investment factor is redundant for describing European average returns. Skipping the details, we can report that for NA, Europe, and AP, the five-factor model almost always outperforms four-factor models that drop *HML* or *RMW*. The exceptions are LHS sorts in which five-factor exposures to the factor omitted from a four-factor model tend to be trivial.

Asset pricing models are simplified propositions about expected returns that are rejected in tests with power. In our tests, *GRS* typically rejects with high confidence. We are, however, more interested in the relative performance of competing models, which we judge using *GRS* and other statistics.

Using *A* to indicate an average value, the other statistics we use to evaluate competing models include the average absolute intercept, $A|a_i|$, and a ratio that measures the dispersion of the intercepts produced by a model (the unexplained part of LHS average returns) relative to the dispersion of LHS average returns. We require reference points to compute dispersion. The asset pricing hypothesis is that the true intercepts are zero, so the reference point for the intercepts is zero. FF (2016) argue that from an asset pricing perspective, the VW market average excess return is the natural reference point for measuring the dispersion of LHS average excess returns.

Define $\bar{r}_i$ as the difference between the average return on LHS portfolio *i* and the average return on the VW market. The ratio of unexplained dispersion of LHS average returns relative to the total dispersion of LHS average returns is $Aa_i^2/A\bar{r}_i^2$, the average squared intercept over the average squared value of $\bar{r}_i$. We also show estimates of the proportion of unexplained dispersion in average returns attributable to sampling error, $As^2(a_i)/Aa_i^2$, where $As^2(a_i)$ is the average of the squared sample standard errors of the a_i and Aa_i^2 is the average squared intercept (the numerator of $Aa_i^2/A\bar{r}_i^2$). A low value of $Aa_i^2/A\bar{r}_i^2$ is good news for an asset pricing model: it says intercept dispersion is low relative to the dispersion of LHS average returns. In contrast, a low value of $As^2(a_i)/Aa_i^2$ is bad news: it says little of the dispersion of the intercepts is sampling error rather than dispersion of the true intercepts.

Table 4 shows that Japanese average returns pose no problems for the models we consider. In all sorts of Japanese stocks, all models pass the *GRS* test, often with *F*-statistics close to 1.0, the median under the hypothesis that a model describes expected returns. There is a strong positive relation between *B/M* and average returns in Japan, but average returns are at best weakly related to profitability (*OP*) or investment (*Inv*) (Table 2). Given the *GRS* results and the weak *OP* and *Inv* patterns in Japanese returns, what we expect from the intercept ratios depends on whether a sort involves *B/M*.

For the Japanese *Size-B/M* portfolios, for example, the estimate of $A(a_i^2)/A(\bar{r}_i^2)$ for the FF three-factor model is 0.48, so in units of return squared, the model fails to explain almost half the dispersion of average returns. $As^2(a_i)/Aa_i^2 = 0.69$ then estimates that more than two-thirds of the unexplained dispersion of average returns is sampling error. In contrast, in the tests on Japanese *Size-OP-Inv* portfolios (which do not involve *B/M*), the *GRS* statistic for the three-factor model, 1.02 (*p*-value = 0.45) is close to the median under the null, 1.00, and $A(a_i^2)/A(\bar{r}_i^2)$ and $As^2(a_i)/Aa_i^2$ are near 1.0. These results suggest that the meager dispersion of Japanese average returns in the *Size-OP-Inv* sorts is sampling error.

We conclude that, aside from a strong value effect, there is not enough variation in 1990–2015 average returns for Japan to challenge the three-factor model or any other model we consider. We save space by dropping Japan from the results and discussion to come.

The spanning tests in Table 3 say that all five factors are important in describing NA average returns. Table 4 confirms that for all LHS sorts, the five-factor model outperforms the FF three-factor model on all summary metrics. In the *Size-OP* and *Size-B/M-OP* sorts, however, the NA five-factor model does not outperform the four-factor model that drops the investment factor, *CMA*. This happens because these two LHS sorts produce little variation in *CMA* slopes.

For Europe, the factor spanning tests in Table 3 say that *MKT*, *HML*, and *RMW* are important for describing average returns, but the average *CMA* return (0.20, $t = 1.89$ in Table 1) is captured by the exposures of *CMA* to the other factors of the five-factor model. The asset pricing tests in Table 4 confirm that for Europe, the performance of the four-factor model that drops *CMA* is always close to that of the five-factor model. Both models typically outperform the FF three-factor model by large margins. Table 4 also confirms the factor spanning evidence that for AP, *CMA* contributes to the five-factor model's description of average returns, but *RMW* is responsible for most of the improvement over the FF three-factor model.

The five-factor model beats the three-factor model's description of average returns for NA, Europe, and AP, but only the NA *Size-B/M-OP* and the European *Size-Inv* portfolios produce *GRS* statistics that do not reject the five-factor model. Confirming *GRS*, $As^2(a_i)/Aa_i^2$ close to 1.0 suggest that chance is the reason the five-factor intercepts for these portfolios differ from zero.

More generally, the *GRS* tests imply that the five-factor model misses variation in expected returns. For example, $A(a_i^2)/A(\bar{r}_i^2)$ for the European *Size-OP-Inv* portfolios estimates that only 32% of the dispersion of average returns is unexplained by the five-factor model, and $As^2(a_i)/Aa_i^2$ estimates that 36% of the unexplained dispersion in average returns, Aa_i^2, is noise. The five-factor model nevertheless

Table 4

Summary asset pricing tests for portfolios from 5 × 5 and 2 × 4 × 4 sorts: July 1990 – December 2015, 306 months.

The table shows summary tests of asset pricing models for the 5 × 5 and 2 × 4 × 4 portfolios of each region. Results are shown for the three-factor model of Fama and French (1993) in which the explanatory returns are *Mkt*, *SMB*, and *HML*, a four-factor model that adds *RMW*, and the five-factor model (1). The *GRS* statistic and its *p*-value, *p*(*GRS*), test whether the expected values of all 25 or 32 intercept estimates in the regressions for a region are zero. Also shown are (1) the average absolute value of the intercepts, $A|a_i|$, (2) $Aa_i^2/A\bar{r}_i^2$, the average squared intercept over the average squared value of $\bar{r}_i$, which is the average return on portfolio *i* minus the average VW market portfolio return, (3) $As^2(a_i)/Aa_i^2$, the average of the estimates of the variances of the sampling errors of the estimated intercepts over Aa_i^2, and (4) AR^2, the average of the regression R^2, adjusted for degrees of freedom.

Model factors	*GRS*	*p*(*GRS*)	$A\|a_i\|$	$\frac{Aa_i^2}{A\bar{r}_i^2}$	$\frac{As^2(a_i)}{Aa_i^2}$	AR^2	*GRS*	*p*(*GRS*)	$A\|a_i\|$	$\frac{Aa_i^2}{A\bar{r}_i^2}$	$\frac{As^2(a_i)}{Aa_i^2}$	AR^2
Panel A: 25 Size-B/M portfolios	North America						Europe					
Mkt SMB HML	2.85	0.00	0.12	0.86	0.27	0.93	2.53	0.00	0.09	0.59	0.34	0.94
Mkt SMB HML RMW	2.48	0.00	0.12	0.74	0.31	0.93	2.05	0.00	0.09	0.41	0.55	0.94
Mkt SMB HML RMW CMA	2.34	0.00	0.11	0.64	0.37	0.93	2.08	0.00	0.09	0.45	0.49	0.95
	Japan						Asia Pacific					
Mkt SMB HML	1.40	0.10	0.11	0.48	0.69	0.93	2.96	0.00	0.22	0.74	0.27	0.88
Mkt SMB HML RMW	1.35	0.13	0.10	0.41	0.81	0.93	2.48	0.00	0.20	0.63	0.33	0.88
Mkt SMB HML RMW CMA	1.37	0.11	0.11	0.43	0.78	0.93	2.27	0.00	0.18	0.52	0.40	0.88
Panel B: 25 Size-OP portfolios	North America						Europe					
Mkt SMB HML	2.53	0.00	0.17	0.60	0.20	0.91	5.19	0.00	0.20	1.15	0.09	0.94
Mkt SMB HML RMW	1.94	0.01	0.09	0.17	0.54	0.93	2.74	0.00	0.10	0.33	0.32	0.95
Mkt SMB HML RMW CMA	2.06	0.00	0.09	0.18	0.52	0.93	2.80	0.00	0.11	0.34	0.31	0.95
	Japan						Asia Pacific					
Mkt SMB HML	1.23	0.21	0.13	0.89	0.49	0.93	4.47	0.00	0.32	1.99	0.13	0.88
Mkt SMB HML RMW	1.11	0.33	0.10	0.52	0.76	0.94	3.17	0.00	0.22	0.98	0.25	0.89
Mkt SMB HML RMW CMA	1.20	0.24	0.10	0.55	0.70	0.94	2.94	0.00	0.20	0.83	0.30	0.90
Panel C: 25 Size-Inv portfolios	North America						Europe					
Mkt SMB HML	2.69	0.00	0.16	0.66	0.19	0.92	1.91	0.01	0.09	0.65	0.35	0.94
Mkt SMB HML RMW	2.61	0.00	0.14	0.62	0.20	0.92	1.03	0.42	0.07	0.27	0.99	0.94
Mkt SMB HML RMW CMA	2.10	0.00	0.10	0.35	0.32	0.94	1.02	0.45	0.06	0.22	1.04	0.95
	Japan						Asia Pacific					
Mkt SMB HML	0.76	0.79	0.10	0.67	0.86	0.93	3.71	0.00	0.25	1.17	0.20	0.87
Mkt SMB HML RMW	0.82	0.72	0.08	0.55	0.98	0.93	3.75	0.00	0.25	1.21	0.21	0.87
Mkt SMB HML RMW CMA	0.82	0.72	0.08	0.56	0.87	0.94	3.56	0.00	0.23	0.97	0.24	0.89
Panel D: 32 Size-B/M-OP portfolios	North America						Europe					
Mkt SMB HML	1.53	0.04	0.16	0.77	0.46	0.85	3.72	0.00	0.27	0.82	0.12	0.87
Mkt SMB HML RMW	1.08	0.36	0.09	0.25	1.30	0.87	1.89	0.00	0.15	0.23	0.49	0.88
Mkt SMB HML RMW CMA	1.09	0.34	0.10	0.29	1.12	0.87	1.85	0.01	0.15	0.21	0.53	0.88
	Japan						Asia Pacific					
Mkt SMB HML	1.23	0.19	0.15	0.46	0.59	0.88	2.60	0.00	0.42	0.73	0.23	0.77
Mkt SMB HML RMW	1.08	0.36	0.11	0.31	0.80	0.89	1.75	0.01	0.26	0.31	0.55	0.78
Mkt SMB HML RMW CMA	1.10	0.34	0.11	0.32	0.78	0.89	1.95	0.00	0.27	0.32	0.54	0.79
Panel E: 32 Size-B/M-Inv portfolios	North America						Europe					
Mkt SMB HML	2.58	0.00	0.16	0.79	0.30	0.87	2.62	0.00	0.12	0.70	0.42	0.90
Mkt SMB HML RMW	2.17	0.00	0.16	0.74	0.33	0.87	1.66	0.02	0.09	0.39	0.88	0.90
Mkt SMB HML RMW CMA	1.79	0.01	0.11	0.36	0.62	0.89	1.66	0.02	0.09	0.30	1.02	0.91
	Japan						Asia Pacific					
Mkt SMB HML	0.68	0.90	0.09	0.28	1.16	0.89	2.78	0.00	0.28	0.97	0.29	0.82
Mkt SMB HML RMW	0.77	0.81	0.07	0.18	1.73	0.90	2.22	0.00	0.24	0.79	0.38	0.82
Mkt SMB HML RMW CMA	0.76	0.82	0.07	0.20	1.42	0.91	2.03	0.00	0.23	0.69	0.40	0.83
Panel F: 32 Size-OP-Inv portfolios	North America						Europe					
Mkt SMB HML	2.81	0.00	0.21	1.09	0.21	0.86	4.33	0.00	0.25	1.18	0.10	0.89
Mkt SMB HML RMW	2.31	0.00	0.16	0.63	0.33	0.87	2.49	0.00	0.13	0.35	0.36	0.90
Mkt SMB HML RMW CMA	1.90	0.00	0.13	0.40	0.50	0.88	2.45	0.00	0.13	0.32	0.36	0.91
	Japan						Asia Pacific					
Mkt SMB HML	1.02	0.45	0.11	1.09	0.92	0.88	4.09	0.00	0.36	1.26	0.21	0.79
Mkt SMB HML RMW	0.94	0.56	0.09	0.79	1.15	0.89	3.06	0.00	0.24	0.60	0.45	0.81
Mkt SMB HML RMW CMA	0.94	0.57	0.09	0.86	0.97	0.90	2.85	0.00	0.21	0.56	0.45	0.82

leaves enough unexplained dispersion for the *GRS* statistic, 2.45 (p-value = 0.00), to reject the model.

The five-factor model fails badly in the tests on the AP *Size-OP* and *Size-Inv* portfolios. For these two sets of portfolios, $A(a_i^2)/A(\bar{r}_i^2)$ is near 1.0. This means the dispersion of five-factor intercepts is similar to the dispersion of average returns, a result never observed in the five-factor tests for NA and Europe. The five-factor model also performs poorly in other LHS sorts for AP. Like FF (2012), we suggest that asset pricing in the four AP markets (Australia, New Zealand, Hong Kong, and Singapore) is probably not well integrated.

The factor spanning tests in Table 3 suggest that the *Size* factor, *SMB*, is important for describing NA average returns for 1990–2015, but for Europe and AP its role is minor. Skipping the details, we can confirm that dropping *SMB* from the NA five-factor model produces a large drop in performance on all metrics in Table 4. For Europe and AP, model performance is similar with and without *SMB*. Even for Europe and AP, *SMB* improves the precision of five-factor intercepts by absorbing LHS return variance. Thus, dropping *SMB* produces large declines in average R^2, between 0.08 and 0.12 for the 5×5 portfolios and between 0.06 and 0.09 for the $2 \times 4 \times 4$ portfolios. The resulting decline in the precision of regression intercepts is a drawback in applications, such as evaluating the performance of managed portfolios.

In contrast, Table 4 shows that adding *RMW* and *CMA* to the FF three-factor model produces small increases in average R^2 (0.01 and 0.02 are typical), but together they have a big impact on the regression intercepts in NA, Europe, and AP sorts that involve *OP*, *Inv*, or both. The improvements relative to the three-factor model are large on all metrics in Table 4. For example, in the $2 \times 4 \times 4$ *Size-OP-Inv* sorts for NA the average absolute intercept, $A|a_i|$, falls from 0.21 for three-factor model to 0.13 for the five-factor model.

8. Asset pricing details

The intercepts from the asset pricing regressions provide perspective on the summary statistics in Table 4. Since we are particularly interested in how the description of average returns changes when we go from the FF (1993) three-factor model to the five-factor model, we save space by showing results for only these two models. We also save space by showing results for only North America and Europe. (AP results are roughly like those for Europe.) The factor spanning tests (Table 3) suggest that all five factors contribute to the description of NA average returns, but for Europe *CMA* is redundant during 1990–2015. The summary asset pricing tests (Table 4) confirm these inferences. Since redundant factors cause no harm, we restrict attention to the five-factor model.

Tables 5–10 show three- and five-factor intercepts for the six sets of LHS portfolios. *Mkt* slopes in the three- and five-factor regressions are close to 1.0 and so cannot help much to describe differences in average returns. *SMB* slopes are similar for the two models and so are also not much help for discriminating between them. Thus, our explanations of intercept improvements produced by the five-factor model focus on five-factor *HML*, *RMW*, and *CMA* slopes, shown in Appendix Tables A1 and A2.

8.1. Size-B/M portfolios

For NA and Europe, the low average returns of small stocks in the lowest *B/M* quintile (Table 2) are deadly for the three-factor model. The three-factor intercepts (Table 5) for NA portfolios in the lowest *B/M* and two lowest *Size* quintiles are −0.34% per month (t=−2.30) and −0.45% (t=−4.32). The three-factor intercepts for the matching European portfolios are −0.47% (t=−4.62) and −0.15% (t=−1.80). The five-factor intercepts for these portfolios are closer to zero, and only the intercept for the NA portfolio in the lowest *B/M* and second *Size* quintile is more than 2.0 standard errors from zero.

The NA and European small stock portfolios in the lowest *B/M* quintile that cause problems for the three-factor model have negative five-factor slopes on *HML*, *RMW*, and *CMA* (Table A1). In other words, these are small growth stocks with returns that behave like those of firms that invest a lot despite low profitability. In the US results of FF (2015), this combination is deadly for the three-factor model, and despite noticeable improvements, it is lethal for the five-factor model in tests on the US 5×5 *Size-B/M* portfolios. The five-factor model more effectively captures the average returns on these portfolios in the NA and European tests for the shorter sample period examined here.

The *GRS* tests (Table 4) show, however, that the five-factor model has other problems describing average returns on the *Size-B/M* portfolios of NA and Europe. For example, the high average return of the NA microcap portfolio in the highest *B/M* quintile and the low average return of the NA megacap portfolio in the highest *B/M* quintile are also deadly for the three-factor model, producing three-factor intercepts 3.80 and −3.89 standard errors from zero. The five-factor model moves the intercept for the megacap value portfolio toward zero (t=−2.09) but, using the Bonferonni inequality, the intercept for the NA microcap value portfolio (0.33% per month, t = 4.41) by itself rejects the five-factor model as a description of expected returns on the 25 portfolios.

8.2. Size-OP portfolios

The three-factor model misses the profitability pattern in average returns for NA and Europe observed in Table 2. The three-factor intercepts (Table 6) increase from low to high profitability, and the pattern is stronger for Europe. The three-factor intercepts for Europe are more than 3.7 standard errors below zero for the five portfolios in the lowest profitability quintile, and they rise to more than 2.1 standard errors above zero in the highest *OP* quintile. The difference between the intercepts for the highest and lowest European *OP* quintiles is at least 0.60% per month in each size quintile.

The five-factor model largely absorbs the profitability pattern in the three-factor intercepts for NA and Europe. Not surprisingly, the heavy lifting is done by the slopes for the profitability factor, *RMW*, which are negative for low *OP* portfolios and positive for high *OP* portfolios (Table A1).

Table 5
Three-factor and five-factor intercepts for 25 *Size-B/M* portfolios of North America and Europe: July 1990–December 2015, 306 months.

We construct 25 *Size-B/M* portfolios for each region at the end of June of each year. The *Size* breakpoints are the 3rd, 7th, 13th, and 25th percentiles of aggregate market cap for a region. The *B/M* (book-to-market equity) quintile breakpoints use the big stocks (top 90% of market cap) of a region. The intersections of the 5×5 independent *Size* and *B/M* sorts for a region produce 25 value-weight *Size-B/M* portfolios. The table shows three-factor and five-factor intercepts and their *t*-statistics. The five-factor model is
$R_{it} - R_{Ft} = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$.

	a					*t*(*a*)				
	Low *B/M*	2	3	4	High *B/M*	Low *B/M*	2	3	4	High *B/M*
Three-factor intercepts										
North America										
Small	−0.34	−0.19	0.12	0.09	0.28	−2.30	−1.70	1.22	1.19	3.80
2	−0.45	−0.15	0.01	−0.05	−0.09	−4.32	−1.77	0.19	−0.75	−1.51
3	0.05	−0.12	−0.03	−0.06	0.04	0.54	−1.30	−0.34	−0.72	0.51
4	0.16	−0.06	0.07	−0.04	−0.01	1.45	−0.70	0.84	−0.39	−0.08
Big	0.15	0.02	−0.05	−0.03	−0.36	2.35	0.27	−0.74	−0.46	−3.89
Europe										
Small	−0.47	−0.13	−0.03	−0.02	0.10	−4.62	−1.63	−0.42	−0.38	1.73
2	−0.15	−0.08	−0.04	0.05	0.05	−1.80	−1.27	−0.76	0.77	0.85
3	−0.05	0.05	−0.05	−0.10	−0.04	−0.60	0.69	−0.65	−1.37	−0.58
4	0.09	−0.01	−0.06	−0.06	−0.10	0.97	−0.19	−0.87	−0.69	−1.27
Big	0.11	0.11	0.04	0.04	−0.24	1.54	1.52	0.58	0.50	−2.47
Five-factor intercepts										
North America										
Small	−0.07	−0.04	0.24	0.17	0.33	−0.47	−0.37	2.60	2.23	4.41
2	−0.27	−0.03	−0.01	−0.07	−0.15	−2.68	−0.35	−0.12	−1.04	−2.55
3	0.15	−0.08	−0.08	−0.15	−0.01	1.47	−0.82	−0.93	−1.83	−0.13
4	0.30	0.00	0.01	−0.15	−0.05	2.68	0.05	0.08	−1.61	−0.67
Big	0.05	−0.04	−0.03	0.03	−0.19	0.79	−0.63	−0.40	0.33	−2.09
Europe										
Small	−0.20	0.03	0.12	−0.04	0.07	−1.93	0.38	1.66	−0.69	1.22
2	0.02	0.03	−0.06	−0.01	0.04	0.29	0.42	−1.01	−0.19	0.74
3	0.17	0.05	−0.15	−0.20	−0.10	1.93	0.61	−1.79	−2.52	−1.27
4	0.27	−0.03	−0.15	−0.12	−0.10	3.00	−0.32	−1.94	−1.33	−1.11
Big	0.11	−0.08	0.04	0.08	0.03	1.40	−1.16	0.59	0.88	0.33

Despite this success, *GRS* (Table 4) rejects the five-factor model for the NA and European *Size-OP* portfolios. Table 6 suggests that the major problem is large positive intercepts for many microcap portfolios.

8.3. Size-Inv portfolios

Strong negative intercepts in Table 7 show that for NA and Europe, the three-factor model misses the drop in average returns in the highest *Inv* quintile (Table 2). The five-factor model largely cures this problem and, confirming the *GRS* test (Table 4), all the five-factor intercepts for Europe are close to zero. For NA, the three-factor model produces positive intercepts for microcap portfolios in the four lower *Inv* quintiles that show up about as strongly in the five-factor model. Only one NA five-factor intercept in the other four *Size* quintiles is more than two standard errors from zero.

The five-factor improvements in the intercepts for European portfolios in the highest *Inv* quintile trace to negative *RMW* and *CMA* slopes (Table A1) that lower five-factor predictions of average returns. Since these portfolios also have negative *HML* slopes, we can conclude that their returns behave like the stock returns of growth firms that invest a lot despite low profitability. The improvements for NA are also primarily due to negative *RMW* and *CMA* slopes, but *HML* slopes are less consistently negative. The five-factor model absorbs the low average returns of NA portfolios in the highest *Inv* quintile more effectively for 1990–2015 than in US tests for the longer 1963–2013 period in FF (2015).

8.4. Size-B/M-OP portfolios

Table 8 shows intercepts for the portfolios from the $2 \times 4 \times 4$ *Size-B/M-OP* sorts for NA and Europe. As in the US results of FF (2015), the portfolio of small NA stocks in the lowest *B/M* and *OP* quartiles has a low average excess return, 0.25% per month (Table 2), and the three-factor intercept for this portfolio is strongly negative, −0.52% per month ($t = -2.90$, Table 8). There is also a profitability pattern in the three-factor intercepts for NA small stocks: the intercepts increase from low to high *OP*. There is a similar but weaker pattern in the three-factor intercepts for NA big stocks. The serious three-factor problem for NA big stocks is the negative intercept, −0.46% per month ($t = -4.53$), for the portfolio in the lowest *OP* and highest *B/M* quartiles (large unprofitable value stocks).

The extreme North America intercepts of the three-factor model are attenuated in the five-factor model. The intercept for the small stock portfolio in the lowest *B/M* and *OP* quartiles rises from −0.52% ($t = -2.90$) to −0.20% ($t = -1.28$). The intercept for the big stock portfolio in the highest *B/M* and lowest *OP* quartiles rises from −0.46% ($t = -4.53$) to −0.23% ($t = -2.44$). This is the only five-factor NA intercept more than two standard errors from zero, and the profitability pattern in the three-factor intercepts disappears in the five-factor intercepts. The *GRS* test rejects

Table 6
Three-factor and five-factor intercepts for 25 *Size-OP* portfolios of North America and Europe: July 1990–December 2015, 306 months

We construct 25 *Size-OP* (profitability) portfolios for each region at the end of June of each year. The *Size* breakpoints are the 3rd, 7th, 13th, and 25th percentiles of aggregate market cap for a region. The *OP* quintile breakpoints use the big stocks (top 90% of market cap) of a region. The intersections of the 5×5 independent *Size* and *OP* sorts for a region produce 25 value-weight *Size-OP* portfolios. The table shows three-factor and five-factor intercepts and their *t*-statistics. The five-factor model is
$R_{it} - R_{Ft} = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$.

	a					$t(a)$				
	Low *OP*	2	3	4	High *OP*	Low *OP*	2	3	4	High *OP*
Three-factor intercepts										
North America										
Small	−0.01	0.25	0.27	0.21	0.22	−0.10	3.16	3.41	2.04	2.09
2	−0.43	−0.07	0.14	0.16	0.22	−4.62	−1.11	1.79	1.77	1.95
3	−0.24	−0.03	0.03	0.06	0.22	−2.10	−0.35	0.38	0.59	1.97
4	−0.24	−0.02	0.20	0.04	0.16	−1.78	−0.23	2.44	0.48	1.54
Big	−0.51	−0.18	−0.06	0.13	0.18	−4.11	−2.00	−0.95	2.09	2.37
Europe										
Small	−0.41	0.09	0.17	0.37	0.25	−5.50	1.51	2.94	6.23	3.19
2	−0.33	−0.08	0.08	0.17	0.37	−4.85	−1.15	1.30	2.70	5.33
3	−0.39	−0.03	0.13	0.06	0.26	−4.96	−0.51	1.82	0.78	3.55
4	−0.46	−0.08	0.12	0.12	0.17	−4.62	−1.05	1.68	1.55	2.13
Big	−0.47	−0.07	0.04	−0.01	0.28	−3.80	−0.95	0.58	−0.14	3.84
Five-factor intercepts										
North America										
Small	0.24	0.23	0.18	0.13	0.17	2.42	2.83	2.29	1.34	1.77
2	−0.14	−0.12	0.01	0.01	−0.03	−2.50	−1.82	0.17	0.13	−0.37
3	0.10	−0.07	−0.08	−0.13	0.02	1.15	−0.80	−0.95	−1.57	0.21
4	0.07	0.00	0.11	−0.12	−0.01	0.61	0.02	1.32	−1.59	−0.08
Big	−0.08	0.03	−0.06	0.15	−0.04	−0.90	0.35	−0.85	2.37	−0.66
Europe										
Small	−0.15	0.10	0.17	0.28	0.16	−2.12	1.53	2.71	4.42	1.99
2	−0.08	−0.00	−0.03	0.07	0.26	−1.19	−0.06	−0.42	1.00	3.52
3	−0.18	−0.01	0.02	−0.07	0.11	−2.30	−0.12	0.31	−0.81	1.51
4	−0.09	−0.09	0.05	−0.01	0.07	−0.94	−1.09	0.63	−0.17	0.79
Big	0.19	0.19	0.03	−0.24	0.00	2.03	2.61	0.38	−3.56	0.06

the five-factor model in the *Size-B/M-OP* sorts for a longer US sample (Fama and French, 2015), but the *GRS* test here (p-value = 0.34, Table 4) suggests that the five-factor NA intercepts for 1990–2015 are chance deviations from zero.

The average excess return of the NA small stock portfolio in the lowest *B/M* and *OP* quartiles is low but positive. In the *Size-B/M-OP* sorts for Europe, this portfolio has a negative average excess return and adjacent portfolios have low returns (Table 2). European big stocks in the lowest *B/M* and *OP* quartiles also have a negative average excess return. The combination of low *B/M* and low *OP* (low profitability growth stocks) thus poses a more serious asset pricing challenge for Europe than for NA.

The three-factor model is not up to the challenge. The low average return of European small stocks in the lowest *B/M* and *OP* quartiles shows up as a strong negative three-factor intercept, −0.99% per month ($t = -6.01$). The three-factor model also fails to capture the profitability pattern in *Size-B/M-OP* average returns for Europe. The three-factor intercepts for small stocks are 3.08–6.01 standard errors below zero for portfolios in the lowest *OP* quartile, and 1.76–5.03 standard errors above zero for the portfolios in the highest *OP* quartile. Though less extreme, three-factor intercepts for European big stock portfolios are negative in the lowest *OP* quartile and positive in the highest.

As in the NA results, the five-factor model attenuates the three-factor intercepts in the *Size-B/M-OP* sorts for Europe. The profitability pattern in the three-factor intercepts is weaker in the five-factor intercepts, and only one of the five-factor intercepts for big stocks is more than 1.7 standard errors from zero. The intercept for the small stock portfolio in the lowest *B/M* and *OP* quintiles rises from −0.99% ($t = -6.01$) to −0.37% ($t = -2.40$). Most intercepts for small stock portfolios are closer to zero in the five-factor model, but some are large relative to their standard errors and *GRS* rejects the five-factor model in the tests on the European *Size-B/M-OP* portfolios (p-value = 0.01, Table 4).

In the US tests of FF (2015), the portfolios in the lower *B/M* and *OP* quartiles have high average *Inv*. Skipping the details, we can report that NA and European portfolios that combine low *B/M* and low *OP* also show high average *Inv*. In other words, these are firms that invest a lot despite low profitability. At least for the 1990–2015 period examined here, the low average returns of these portfolios are in large part captured by the five-factor model.

8.5. *Size-B/M-Inv* portfolios

The summary results in Table 4 show that the five-factor model describes average returns for the *Size-B/M-Inv* portfolios of North America and Europe more accurately than the three-factor model. The intercepts in Table 9 show that the most notable three-factor problems for NA and European big stocks are the low average returns (Table 2) of portfolios in the highest *B/M* and *Inv*

Table 7
Three-factor and five-factor intercepts for 25 *Size-Inv* portfolios of North America and Europe: July 1990–December 2015, 306 months.

We construct 25 *Size-Inv* (investment) portfolios for each region at the end of June of each year. The *Size* breakpoints are the 3rd, 7th, 13th, and 25th percentiles of aggregate market cap for a region. The *Inv* quintile breakpoints use the big stocks (top 90% of market cap) of a region. The intersections of the 5 × 5 independent *Size* and *Inv* sorts for a region produce 25 value-weight *Size-Inv* portfolios. The table shows three-factor and five-factor intercepts and their *t*-statistics. The five-factor model is
$R_{it} - R_{Ft} = a_i + b_iMkt_t + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$.

	a					$t(a)$				
	Low *Inv*	2	3	4	High *Inv*	Low *Inv*	2	3	4	High *Inv*
Three-factor intercepts										
North America										
Small	0.31	0.35	0.24	0.17	−0.34	2.58	4.38	3.23	2.30	−3.00
2	−0.02	0.09	0.07	−0.03	−0.50	−0.32	1.27	0.94	−0.39	−5.52
3	0.08	0.13	0.11	−0.06	−0.29	0.94	1.67	1.39	−0.78	−2.45
4	0.12	0.23	0.09	0.10	−0.28	1.37	2.90	1.13	1.23	−2.26
Big	0.09	0.05	0.02	−0.03	−0.09	1.25	0.69	0.35	−0.37	−0.80
Europe										
Small	−0.03	0.11	0.15	0.08	−0.34	−0.49	1.83	2.44	1.28	−3.79
2	−0.06	0.18	0.17	0.03	−0.19	−0.92	2.85	2.73	0.42	−2.56
3	−0.00	0.00	0.13	−0.09	−0.26	−0.04	0.05	1.86	−1.20	−3.03
4	0.00	0.01	0.05	0.04	−0.15	0.05	0.09	0.65	0.51	−1.51
Big	0.02	0.10	−0.06	0.05	−0.01	0.24	1.41	−0.94	0.62	−0.16
Five-factor intercepts										
North America										
Small	0.39	0.33	0.22	0.26	−0.06	3.55	4.12	2.97	3.46	−0.59
2	−0.08	0.00	−0.02	−0.03	−0.22	−1.21	0.03	−0.21	−0.39	−3.15
3	−0.03	−0.02	0.02	−0.06	0.06	−0.29	−0.29	0.25	−0.75	0.64
4	0.03	0.07	0.02	0.12	0.00	0.32	0.93	0.28	1.37	0.04
Big	−0.01	−0.10	−0.04	0.09	0.16	−0.14	−1.69	−0.62	1.14	1.75
Europe										
Small	0.05	0.07	0.10	0.09	−0.12	0.75	1.06	1.44	1.38	−1.43
2	−0.06	0.11	0.13	0.01	0.01	−0.91	1.68	1.94	0.20	0.08
3	−0.11	−0.05	0.07	−0.07	−0.09	−1.34	−0.67	0.91	−0.90	−1.20
4	−0.05	−0.05	0.03	0.06	0.01	−0.53	−0.68	0.43	0.71	0.16
Big	0.00	−0.02	−0.08	0.10	0.01	0.01	−0.31	−1.09	1.42	0.15

quartiles–stocks of big value firms that invest a lot. The three-factor intercepts for these portfolios are −0.62% per month (t = −4.13) for NA and −0.41% (t = −2.60) for Europe. The five-factor intercepts are closer to zero, −0.23% (t = −1.81) for NA and −0.13% (t = −0.83) for Europe, largely due to strong negative *RMW* and *CMA* slopes (Table A2). The sorts guarantee that these are high investment firms, and portfolio averages of *OP* (not shown) confirm that they have low profitability.

For small stocks, the main three-factor problem for the *Size-B/M-Inv* portfolios for NA and Europe is low average returns in the highest investment and lower *B/M* quartiles, observed in Table 2. Table 9 shows that the strong negative three-factor intercepts of small, high investment, European growth (lower *B/M*) stocks are absorbed by the five-factor model. For example, the intercept for the portfolio in the lowest *B/M* and highest *Inv* quartiles rises from −0.36% per month (t = −4.23) for the three-factor model to −0.10% (t = −1.27) for the five-factor model. For the corresponding NA portfolio of small growth stocks that invest a lot, adding *RMW* and *CMA* to the model shrinks the intercept from −0.42% (t = −4.99) to −0.21%, but it is still 2.82 standard errors below zero.

The *Size-B/M-Inv* sorts guarantee that firms in the highest *Inv* portfolios invest aggressively. These portfolios have strong negative *CMA* slopes (Table A2), which improve the predictions of average returns provided by the five-factor model. The small stock NA and European portfolios in the lowest *B/M* and highest *Inv* quartiles also have negative *RMW* slopes, so these portfolios contain small stocks with returns that behave like those of firms that invest a lot despite low profitability. Negative *RMW* slopes also help capture the extreme low average returns of these portfolios (Table 2).

8.6. Size-OP-Inv portfolios

GRS rejects the three- and five-factor models in the tests on the *Size-Op-Inv* portfolios of North America and Europe (Table 4), but the five-factor model performs better on all metrics. The intercepts in Table 10 detail the problems of the three-factor model. The model fails to capture the profitability pattern in average returns. For small and big NA and European stocks, three-factor intercepts increase with *OP* in every *Inv* quartile. The model also fails to capture the low average returns of NA and European small and big stocks of firms that invest a lot despite low profitability. The large negative intercepts in Table 10 for the small and big stock portfolios in the lowest *OP* and highest *Inv* quartiles are among the three-factor model's biggest failures in the sorts we consider.

For big stocks, the five-factor model largely cures the problems of the three-factor model in the tests on the *Size-OP-Inv* portfolios. The profitability pattern in the three-factor intercepts for big stocks disappears in the five-factor results. Big stocks that invest a lot despite low profitability are also not a problem for the five-factor model. The

Table 8
Three-factor and five-factor intercepts for 32 *Size-B/M-OP* portfolios of North America and Europe: July 1990–October 2015.

At the end of June each year, stocks in each region are allocated to two *Size* groups (Small and Big) using the 90th percentile of market cap for the region as breakpoint. Small and big stocks are allocated independently to four *B/M* (book-to-market equity) quartiles (Low *B/M* to High *B/M*) and four *OP* (profitability) quartiles (Low *OP* to High *OP*), using *B/M* and *OP* breakpoints for the small or big *Size* group of the region. The intersections of the three sorts produce 32 *Size-B/M-OP* portfolios for each region. The table shows three-factor and five-factor intercepts and their *t*-statistics.

	Small								Big							
	a				$t(a)$				a				$t(a)$			
B/M →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
Three-factor intercepts: $R_{it} - R_{Ft} = a_i + b_iMkt_t + s_iSMB_t + h_iHML_t + e_{it}$																
North America																
Low *OP*	−0.52	−0.34	−0.27	−0.17	−2.90	−1.48	−1.45	−0.85	0.01	−0.30	−0.23	−0.46	0.03	−1.81	−1.89	−4.53
2	−0.37	−0.20	−0.01	0.01	−2.36	−1.87	−0.09	0.14	−0.18	−0.04	−0.06	−0.07	−0.86	−0.34	−0.63	−0.75
3	−0.03	0.01	0.16	0.11	−0.29	0.14	2.38	1.10	0.19	0.03	0.08	−0.13	1.59	0.35	0.84	−1.01
High *OP*	0.12	0.16	0.16	0.21	1.56	2.12	1.70	1.22	0.17	0.05	0.06	0.05	2.25	0.39	0.36	0.21
Europe																
Low *OP*	−0.99	−0.83	−0.46	−0.43	−6.01	−5.99	−4.06	−3.08	−0.29	−0.64	−0.24	−0.29	−1.31	−3.61	−2.06	−2.87
2	−0.60	−0.25	−0.03	0.08	−4.34	−2.93	−0.49	1.11	−0.30	0.17	0.06	−0.10	−1.79	1.69	0.69	−0.97
3	−0.20	−0.01	0.22	0.28	−2.53	−0.24	3.78	3.07	0.20	0.05	0.04	0.02	1.77	0.50	0.42	0.15
High *OP*	0.14	0.26	0.44	0.31	3.06	4.20	5.03	1.76	0.19	0.22	0.18	0.08	2.48	1.88	1.05	0.34
Five-factor intercepts: $R_{it} - R_{Ft} = a_i + b_iMkt_t + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$																
North America																
Low *OP*	−0.20	0.10	0.08	0.10	−1.28	0.51	0.48	0.52	0.43	0.06	−0.07	−0.23	1.94	0.38	−0.58	−2.44
2	−0.13	−0.14	−0.02	0.03	−0.87	−1.24	−0.33	0.33	0.08	−0.01	−0.03	−0.03	0.38	−0.10	−0.26	−0.33
3	−0.04	−0.09	0.08	0.06	−0.35	−1.31	1.19	0.62	0.14	−0.06	0.08	−0.15	1.16	−0.64	0.85	−1.08
High *OP*	−0.06	0.04	0.05	0.21	−1.02	0.71	0.55	1.19	−0.03	−0.13	−0.04	−0.04	−0.43	−1.09	−0.24	−0.20
Europe																
Low *OP*	−0.37	−0.50	−0.26	−0.18	−2.40	−3.57	−2.15	−1.23	0.26	−0.16	0.14	0.12	1.11	−0.92	1.27	1.32
2	−0.16	−0.06	−0.03	0.09	−1.19	−0.65	−0.45	1.18	−0.12	0.30	0.15	−0.08	−0.64	2.81	1.70	−0.76
3	−0.03	−0.08	0.10	0.14	−0.38	−1.38	1.59	1.43	0.11	−0.09	−0.14	−0.04	0.91	−0.97	−1.30	−0.25
High *OP*	0.10	0.08	0.23	0.15	2.13	1.31	2.56	0.81	0.02	−0.14	−0.15	−0.10	0.31	−1.18	−0.84	−0.39

Table 9
Three-factor and five-factor intercepts for *Size-B/M-Inv* portfolios of North America and Europe: July 1990–October 2015.

At the end of June each year, stocks in each region are allocated to two *Size* groups (Small and Big) using the 90th percentile of market cap for the region as breakpoint. Small and big stocks are allocated independently to four *B/M* (book-to-market equity) quartiles (Low *B/M* to High *B/M*) and four *Inv* (investment) quartiles (Low *Inv* to High *Inv*), using *B/M* and *OP* breakpoints for the small or big *Size* group of the region. The intersections of the three sorts produce 32 *Size-B/M-Inv* portfolios for each region. The table shows three-factor and five-factor intercepts and their *t*-statistics.

	Small								Big							
	a				$t(a)$				a				$t(a)$			
B/M →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
Three-factor intercepts: $R_{it} - R_{Ft} = a_i + b_iMkt_t + s_iSMB_t + h_iHML_t + e_{it}$																
North America																
Low *Inv*	0.02	0.14	0.06	0.15	0.17	1.18	0.57	1.23	0.14	0.07	0.19	−0.05	1.15	0.63	1.81	−0.53
2	0.14	0.08	0.14	0.16	1.64	1.07	2.02	1.69	0.16	0.01	0.02	−0.09	1.35	0.08	0.23	−0.89
3	−0.01	0.06	0.31	−0.15	−0.16	0.92	4.93	−1.35	0.08	−0.03	−0.06	−0.24	0.71	−0.25	−0.53	−1.69
High *Inv*	−0.42	−0.28	−0.26	−0.24	−4.99	−2.78	−2.60	−1.48	0.28	−0.12	−0.21	−0.62	2.03	−0.84	−1.55	−4.13
Europe																
Low *Inv*	−0.25	−0.03	0.08	−0.08	−2.40	−0.31	0.88	−0.67	0.05	0.13	0.01	−0.16	0.35	1.05	0.11	−1.38
2	0.04	0.03	0.19	0.20	0.56	0.52	2.84	2.10	0.03	0.22	0.08	0.03	0.28	2.38	0.83	0.26
3	0.04	0.07	0.11	0.10	0.58	1.31	1.48	1.26	0.14	0.03	0.06	−0.09	1.25	0.23	0.61	−0.77
High *Inv*	−0.36	−0.27	−0.22	−0.22	−4.23	−3.17	−2.66	−1.71	0.14	−0.07	−0.03	−0.41	1.15	−0.69	−0.22	−2.60
Five-factor intercepts: $R_{it} - R_{Ft} = a_i + b_iMkt_t + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$																
North America																
Low *Inv*	0.02	0.13	−0.02	0.16	0.19	1.26	−0.27	1.25	−0.08	−0.10	0.09	−0.07	−0.68	−0.94	0.89	−0.71
2	0.01	−0.03	0.03	0.15	0.14	−0.48	0.46	1.53	−0.14	−0.09	0.06	−0.12	−1.34	−0.72	0.63	−1.15
3	−0.06	−0.04	0.26	−0.10	−0.83	−0.57	4.09	−0.92	0.10	−0.06	−0.00	−0.07	0.88	−0.57	−0.01	−0.53
High *Inv*	−0.21	−0.15	−0.15	−0.07	−2.82	−1.70	−1.66	−0.45	0.40	0.08	−0.16	−0.23	3.27	0.58	−1.22	−1.81
Europe																
Low *Inv*	−0.11	−0.04	0.05	−0.04	−0.97	−0.41	0.50	−0.31	−0.08	−0.05	0.14	−0.09	−0.62	−0.40	1.24	−0.68
2	0.08	−0.07	0.17	0.18	1.07	−1.04	2.51	1.86	−0.00	0.06	0.08	0.11	−0.02	0.62	0.76	0.93
3	0.06	0.02	0.01	0.08	0.95	0.39	0.16	0.87	0.25	0.04	0.04	−0.01	2.08	0.34	0.35	−0.07
High *Inv*	−0.10	−0.15	−0.20	−0.12	−1.27	−1.85	−2.34	−0.87	0.14	−0.01	−0.05	−0.13	1.29	−0.10	−0.38	−0.83

Table 10
Three-factor and five-factor intercepts for 32 *Size-OP-Inv* portfolios of North America and Europe: July 1990–October 2015.

At the end of June each year, stocks in each of the four regions are allocated to two *Size* groups (Small and Big) using the 90th percentile of market cap for the region as breakpoint. Small and big stocks are allocated independently to four *OP* (profitability) quartiles (Low *OP* to High *OP*) and four *Inv* (investment) quartiles (Low *Inv* to High *Inv*), using *OP* and *Inv* breakpoints for the small or big *Size* group of the region. The intersections of the three sorts produce 32 *Size-OP-Inv* portfolios for each region. The table shows three-factor and five-factor intercepts and their *t*-statistics.

	Small								Big							
	a				$t(a)$				a				$t(a)$			
OP →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
Three-factor intercepts: $R_{it} - R_{Ft} = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + e_{it}$																
North America																
Low *Inv*	−0.08	−0.04	0.07	0.43	−0.43	−0.39	0.70	4.45	−0.09	0.20	0.16	0.11	−0.70	1.73	1.48	1.05
2	0.00	0.03	0.17	0.15	0.00	0.43	2.33	2.01	−0.16	−0.08	0.07	0.13	−1.30	−0.64	0.83	1.28
3	−0.69	0.13	0.08	0.16	−3.11	1.49	1.21	2.07	−0.29	−0.07	−0.01	0.13	−1.83	−0.58	−0.10	1.14
High *Inv*	−0.63	−0.63	−0.26	0.04	−3.00	−5.49	−2.87	0.38	−0.52	−0.32	0.25	0.44	−3.61	−2.04	1.59	2.54
Europe																
Low *Inv*	−0.48	−0.03	0.04	0.21	−3.33	−0.29	0.40	2.23	−0.33	−0.04	0.16	0.26	−2.82	−0.34	1.34	2.19
2	−0.45	0.06	0.15	0.36	−3.64	0.80	2.46	5.42	−0.24	−0.03	0.11	0.26	−1.94	−0.33	1.08	2.73
3	−0.63	−0.16	0.15	0.32	−4.45	−2.27	2.89	6.37	−0.29	−0.05	0.01	0.17	−2.14	−0.52	0.07	1.58
High *Inv*	−1.12	−0.37	−0.26	0.09	−7.65	−3.74	−3.38	1.14	−0.69	0.16	−0.04	0.23	−4.61	1.23	−0.31	1.81
Five-factor intercepts: $R_{it} - R_{Ft} = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$																
North America																
Low *Inv*	0.19	−0.04	−0.04	0.25	1.26	−0.40	−0.36	2.67	0.09	0.12	0.05	−0.17	0.78	1.14	0.44	−1.97
2	0.29	0.03	0.05	−0.04	1.59	0.45	0.77	−0.63	−0.14	0.03	−0.03	−0.11	−1.11	0.25	−0.28	−1.16
3	−0.34	0.22	0.05	0.03	−1.61	2.58	0.68	0.43	0.05	0.06	−0.00	0.05	0.36	0.52	−0.01	0.43
High *Inv*	−0.13	−0.36	−0.19	−0.01	−0.68	−3.37	−2.18	−0.07	−0.05	−0.11	0.34	0.39	−0.39	−0.74	2.24	2.51
Europe																
Low *Inv*	−0.05	0.05	−0.05	−0.00	−0.38	0.58	−0.53	−0.04	0.01	0.03	−0.01	−0.05	0.14	0.28	−0.13	−0.46
2	−0.29	0.10	0.08	0.19	−2.22	1.37	1.36	2.92	0.22	0.02	−0.07	−0.07	1.93	0.18	−0.72	−0.70
3	−0.32	−0.08	0.05	0.18	−2.21	−1.13	0.88	3.62	0.21	0.13	−0.08	−0.02	1.66	1.24	−0.64	−0.16
High *Inv*	−0.62	−0.09	−0.17	0.11	−4.50	−1.00	−2.25	1.47	−0.10	0.29	−0.17	0.09	−0.79	2.31	−1.48	0.84

intercept for the big NA portfolio in the lowest *OP* and highest *Inv* quartiles rises from −0.52% per month ($t = -3.61$) in the three-factor model to −0.05% ($t = -0.39$) in the five-factor model. The matching intercept for Europe rises from −0.69% ($t = -4.61$) to −0.10% ($t = -0.79$).

The five-factor model also improves the intercepts for small NA and European stocks, but problems remain. There is no profitability pattern in the five-factor intercepts for North America, and the pattern is muted for the small stocks of Europe. For each region, however, there is a portfolio in the highest *Inv* and lower *OP* quartiles that has an extreme negative five-factor intercept. The intercept for the NA small stock portfolio in the lowest *OP* and highest *Inv* quartiles rises from −0.63% per month ($t = -3.00$) in the three-factor model to −0.13% ($t = -0.68$) in the five-factor model, which is good news. But the five-factor intercept for the portfolio in the second *OP* and highest *Inv* quartiles, −0.36%, is 3.37 standard errors below zero. The extreme negative three-factor intercept for the European small stock portfolio in the lowest *OP* and highest *Inv* quartiles, −1.12 ($t = -7.65$), rises by almost half to −0.62% in the five-factor model, but the five-factor intercept is still 4.50 standard errors from zero.

In short, the five-factor model captures much of the variation in average returns related to profitability and investment. As in the US results of FF (2015), however, the low returns of small firms that invest a lot despite low profitability are a problem, indeed the only systematic problem for the five-factor model in the tests on the *Size-OP-Inv* portfolios of NA and Europe.

9. Conclusions

For the small stocks of North America, Europe, and Asia Pacific, average returns increase with *B/M* and *OP*, but these value and profitability patterns in average returns are weaker for big stocks. In sorts on investment, average returns for small stocks of the three regions are substantially lower for the highest *Inv* portfolios. This drop in average returns for high *Inv* portfolios is less consistent for big stocks and, except for NA, there is no other systematic *Inv* pattern in average returns for small or big stocks. In short, the role of the investment factor of the five-factor model may largely be to absorb the low average returns of high investment small stocks.

Global three-factor and five-factor models perform poorly in tests on regional portfolios, so we concentrate on local models in which the factors and returns to be explained are from the same region. A strong positive relation between *B/M* and average returns is the only clear pattern in Japanese average returns, and it is captured well by the local version of the FF (1993) three-factor model. For NA, Europe, and AP, the profitability and investment patterns in average returns are left unexplained by local three-factor models. Though also typically rejected in formal tests, local versions of the five-factor model absorb most of the value, profitability, and investment patterns in average returns.

Factor spanning tests say that all five factors have unique information about NA average returns for 1990–2015, but the investment factor, *CMA*, is redundant for Europe and Japan. Detailed asset pricing tests confirm these inferences, as they must. Thus, for two of four regions, dropping *CMA* from the five-factor model has little effect on the description of average returns, at least for 1990–2015.

One result, common to all our recent work on factor models, merits attention. In tests on US returns like those examined here, FF (2015) find that portfolios of small stocks whose returns behave like those of firms that invest a lot despite low profitability have low average returns that cause problems for the three-factor and five-factor models in many different sorts. Here we find similar evidence for markets outside the US. Indeed, the low average returns and resulting three-factor problems posed by such stocks are more extreme in Europe and Asia Pacific. The five-factor model captures the troublesome average returns in some sorts, but not in the *Size-OP-Inv* sorts that best isolate stocks of firms that invest a lot despite low profitability. The low average returns on these stocks and the problems they create for asset pricing models are a challenge for future research.

The dividend discount model provides motivation for the five-factor model, but it leaves important stones unturned. For example, the dividend discount model does not rule out CAPM pricing, which means it offers no help in explaining why the premiums in average returns related to *B/M*, profitability, and investment are not explained by the CAPM. The dividend discount model is also silent on whether the *B/M*, profitability, and investment patterns in average returns it predicts are the result of rational or irrational pricing. The rationality of the value (*B/M*) premium is an old and contentious issue (Fama and French, 1993, 1995; Lakonishok, Shleifer, and Vishny, 1994; Daniel, Hirshleifer, and Subramanyam, 1998). Titman, Wei, and Xie (2013), Watanabe, Yu, Yao, and Yu (2013), and Sun, Wei, and Xie (2013) test whether the investment and profitability patterns in average returns are due to rational or irrational pricing. They come down on the side of rational pricing, but the conclusion hinges on the interpretation of their instruments, and there is room for disagreement. More generally, models like Eq. (1) are silent on whether asset pricing is rational (Fama and French, 2007; Lewellen, Nagel, and Shanken 2010; Kozak, Nagel, and Santosh, 2015). Even if a model works perfectly, for example, its factor premiums may be due to mispricing.

In the end, the five-factor model, like the FF (1993) three-factor model, is an empirical asset pricing model, designed to capture prominent patterns in average returns. Given their flimsy theoretical underpinnings, empirical asset pricing models can be judged only on empirical robustness. This is the rationale for our papers on the five-factor model. FF (2015) test the model on US data for portfolios designed to produce large spreads in average returns along the *Size*, *B/M*, profitability, and investment dimensions targeted by the model. FF (2016) move on to sorts on other so-called anomaly variables. Here we extend the tests to international data.

For those inclined to dismiss empirical asset pricing models, we offer a reality check. There are three theory-based rational asset pricing models: (1) the CAPM of Sharpe (1964), Lintner (1965), and Black (1972), (2) the consumption based CAPM of Lucas (1978) and Breeden (1979), and (3) Merton's (1973) ICAPM. Though often used in applications, the CAPM is a widely acknowledged empirical failure. Despite the efforts of many talented empiricists, the consumption based CAPM rests in empirical limbo, and we are unaware of anyone who suggests it is useful in applications. The ICAPM provides a theoretical framework that can accommodate factor-based empirical asset pricing models. But until researchers identify and empirically validate the model's state variables, other stories for the success of empirical asset pricing models are viable competitors.

We would prefer to have a theoretical model that captures the salient features of expected returns. The experience of the last 50 years says, however, that the task is difficult and the wait for a successful model is likely to be long. In the meantime, in the spirit of Huberman and Kandel (1987) and Gibbons, Ross, and Shanken (1989), there is value in searching for a small set of RHS portfolios that span the Markowitz (1952) mean-variance-efficient set and so capture expected returns on all assets. Complete success is almost certainly impossible, but less-than-perfect models can provide useful descriptions of expected returns.

Would we recommend the five-factor model in applications? Factor models are often used to evaluate portfolio performance. FF (2010) find that inferences about the performance of actively managed US equity mutual funds are different for the CAPM and the FF three-factor model but similar for the three-factor model and a four-factor model that adds a momentum factor. It would be interesting to extend the tests to the five-factor model and a six-factor model that adds a momentum factor. On the other side of the coin, the experience of the FF three-factor model suggests that an empirical asset pricing model that stands up to robustness checks can be a tool for portfolio design.

We are less confident that asset pricing models like the five-factor model can provide meaningful estimates of the cost of equity capital. Suppose, for example, the CAPM is the right model and we are given returns on the true market portfolio. To estimate the cost of equity for a firm, we need estimates of its market beta and the expected market premium. The standard errors of the estimates are large, even if we ignore the evidence that the expected market premium and market betas vary through time (Fama and French, 1997; Pastor and Stambaugh, 1999). In short, estimates of the cost of equity capital from asset pricing models are quite imprecise, and so arguably useless.

In contrast, time variation in the slopes is a potential problem, but using a factor model like Eq. (1) to evaluate the performance of a managed portfolio does not require estimates of expected factor premiums, variation in expected premiums is not a problem since it is embedded in the RHS returns, and the standard error of the regression intercept (alpha) incorporates the imprecision of the estimated factor slopes.

Appendix

Table A1 reports the *HML*, *RMW*, and *CMA* slopes (h, r, and c) and their t-statistics from five-factor regressions

Table A1

Five-factor *HML*, *RMW*, and *CMA* slopes and *t*-statistics for the 5 × 5 *Size-B/M*, 5 × 5 *Size-OP*, and 5 × 5 *Size-Inv* portfolios of North America and Europe: July 1990–December 2015, 306 months

The five-factor model is

$R_{it} - R_{Ft} = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$.

Panel A: 25 Size-B/M portfolios	Low *B/M*	2	3	4	High *B/M*	Low *B/M*	2	3	4	High *B/M*
North America										
			h					*t(h)*		
Small	−0.54	−0.34	−0.10	0.08	0.40	−7.82	−6.05	−2.06	2.08	10.71
2	−0.59	−0.36	−0.08	0.28	0.50	−11.68	−8.30	−2.12	8.93	17.23
3	−0.64	−0.07	0.14	0.32	0.46	−12.30	−1.46	3.46	7.78	12.22
4	−0.63	0.00	0.15	0.36	0.54	−11.45	0.09	3.46	7.85	14.45
Big	−0.49	−0.12	0.24	0.47	0.80	−15.96	−3.54	6.64	12.14	17.71
			r					*t(r)*		
Small	−0.60	−0.31	−0.28	−0.20	−0.12	−8.92	−5.63	−6.19	−5.52	−3.20
2	−0.22	−0.24	−0.01	0.06	0.07	−4.47	−5.83	−0.37	1.99	2.57
3	−0.18	0.06	0.18	0.18	0.07	−3.54	1.29	4.50	4.46	1.86
4	−0.27	−0.03	0.11	0.25	0.08	−4.97	−0.68	2.70	5.73	2.30
Big	0.21	0.06	−0.04	−0.09	−0.22	7.09	1.69	−1.19	−2.27	−5.09
			c					*t(c)*		
Small	−0.07	−0.07	−0.03	0.03	−0.01	−0.84	−0.96	−0.44	0.67	−0.29
2	−0.27	−0.07	0.10	−0.02	0.11	−4.26	−1.29	2.18	−0.54	3.05
3	−0.07	−0.25	−0.10	0.05	0.06	−1.15	−4.20	−1.97	1.08	1.34
4	−0.08	−0.19	0.05	0.01	0.03	−1.17	−3.37	1.00	0.12	0.65
Big	0.03	0.13	−0.02	−0.08	−0.27	0.88	3.07	−0.37	−1.68	−4.81
Europe										
			h					*t(h)*		
Small	−0.57	−0.38	−0.20	0.06	0.26	−9.99	−8.07	−5.04	1.72	8.02
2	−0.55	−0.25	0.02	0.33	0.45	−12.05	−6.43	0.57	9.35	14.00
3	−0.64	−0.14	0.19	0.38	0.56	−13.04	−3.12	4.22	8.81	12.75
4	−0.48	−0.04	0.17	0.39	0.64	−9.73	−0.87	4.00	7.79	13.07
Big	−0.63	−0.22	0.01	0.31	0.81	−14.43	−5.87	0.20	6.35	15.80
			r					*t(r)*		
Small	−0.45	−0.27	−0.25	0.02	0.00	−6.06	−4.45	−4.76	0.48	0.01
2	−0.26	−0.20	0.02	0.11	−0.04	−4.23	−3.93	0.42	2.29	−0.92
3	−0.35	0.01	0.17	0.18	0.09	−5.38	0.16	2.88	3.14	1.51
4	−0.28	−0.01	0.14	0.11	−0.01	−4.19	−0.15	2.40	1.70	−0.20
Big	−0.01	0.27	−0.02	−0.06	−0.41	−0.17	5.58	−0.45	−0.96	−5.98
			c					*t(c)*		
Small	−0.28	−0.15	−0.12	0.08	0.20	−4.14	−2.64	−2.62	1.92	5.18
2	−0.29	−0.05	0.07	0.01	0.20	−5.31	−1.15	1.77	0.16	5.18
3	−0.32	−0.03	0.04	0.01	0.10	−5.46	−0.53	0.80	0.22	1.88
4	−0.30	0.14	0.13	0.03	0.01	−5.03	2.45	2.40	0.51	0.20
Big	0.05	0.31	0.06	−0.04	−0.47	1.01	7.10	1.44	−0.62	−7.63
Panel B: 25 Size-OP portfolios	Low *OP*	2	3	4	High *OP*	Low *OP*	2	3	4	High *OP*
North America										
			h					*t(h)*		
Small	−0.13	0.32	0.24	0.33	0.32	−2.72	7.82	6.14	6.79	6.58
2	−0.21	0.21	0.14	0.24	0.24	−7.28	6.37	3.85	6.12	5.49
3	−0.19	0.07	0.24	0.24	0.22	−4.52	1.73	5.54	5.66	4.77
4	−0.14	0.19	0.17	0.13	0.11	−2.43	4.78	4.09	3.44	2.50
Big	0.26	0.17	0.14	−0.01	−0.27	5.85	4.30	3.88	−0.26	−8.92
			r					*t(r)*		
Small	−0.61	0.06	0.22	0.28	0.31	−12.64	1.64	5.89	5.79	6.38
2	−0.66	0.08	0.27	0.42	0.68	−23.76	2.46	7.36	10.87	16.28
3	−0.69	0.03	0.33	0.53	0.58	−16.49	0.68	7.80	12.82	12.98
4	−0.68	−0.03	0.19	0.31	0.43	−12.32	−0.71	4.75	8.35	9.74
Big	−0.76	−0.43	−0.00	0.04	0.42	−17.92	−11.14	−0.08	1.31	14.02
			c					*t(c)*		
Small	0.03	−0.04	−0.02	−0.16	−0.31	0.49	−0.76	−0.39	−2.58	−5.00
2	−0.01	0.06	0.04	−0.11	−0.13	−0.34	1.32	0.94	−2.18	−2.41
3	−0.15	0.09	−0.09	−0.12	−0.17	−2.79	1.84	−1.58	−2.22	−2.95
4	−0.05	−0.03	0.03	0.10	−0.08	−0.73	−0.54	0.61	2.05	−1.43
Big	−0.36	−0.09	−0.01	−0.14	0.15	−6.53	−1.73	−0.24	−3.40	3.88

(*continued on next page*)

Table A1 (*continued*)

Panel B: 25 Size-OP portfolios	Low *OP*	2	3	4	High *OP*	Low *OP*	2	3	4	High *OP*
Europe										
			h					*t*(*h*)		
Small	−0.22	0.10	−0.00	0.07	−0.01	−5.50	2.72	−0.02	1.88	−0.24
2	−0.10	0.17	0.12	0.15	0.03	−2.71	4.45	3.23	3.98	0.80
3	−0.07	0.14	0.24	0.15	0.10	−1.48	3.71	5.79	3.32	2.52
4	0.01	0.28	0.22	0.14	0.01	0.18	5.96	5.13	3.09	0.26
Big	0.03	0.14	0.01	0.06	−0.21	0.60	3.35	0.20	1.55	−5.44
			r					*t*(*r*)		
Small	−0.48	−0.04	−0.01	0.16	0.18	−9.01	−0.85	−0.33	3.56	2.93
2	−0.46	−0.15	0.17	0.19	0.23	−9.74	−2.93	3.59	3.93	4.30
3	−0.39	−0.04	0.22	0.24	0.30	−6.65	−0.86	3.89	4.08	5.55
4	−0.70	0.00	0.14	0.25	0.22	−10.17	0.01	2.50	4.06	3.61
Big	−1.21	−0.49	0.01	0.42	0.53	−17.86	−9.07	0.13	8.71	10.43
			c					*t*(*c*)		
Small	−0.05	0.10	0.06	0.02	−0.10	−1.11	2.30	1.54	0.47	−1.76
2	−0.10	0.07	0.14	0.01	−0.08	−2.25	1.51	3.32	0.24	−1.64
3	0.01	−0.01	−0.06	−0.02	−0.13	0.10	−0.17	−1.14	−0.43	−2.62
4	−0.03	0.08	−0.02	0.04	−0.12	−0.41	1.52	−0.41	0.66	−2.16
Big	−0.10	−0.04	0.06	0.02	0.01	−1.69	−0.90	1.28	0.55	0.23

Panel C: 25 Size-Inv portfolios	Low *Inv*	2	3	4	High *Inv*	Low *Inv*	2	3	4	High *Inv*
North America										
			h					*t*(*h*)		
Small	−0.04	0.12	0.08	0.17	0.05	−0.67	3.08	2.26	4.59	0.91
2	0.02	0.11	0.13	0.15	−0.09	0.56	3.06	3.25	4.21	−2.46
3	0.17	0.18	0.17	0.15	−0.16	3.99	4.79	4.28	3.77	−3.37
4	0.13	0.17	0.21	0.08	−0.14	3.24	4.66	5.47	1.84	−2.46
Big	−0.06	−0.01	0.01	0.01	−0.00	−1.83	−0.48	0.22	0.22	−0.09
			r					*t*(*r*)		
Small	−0.40	−0.09	−0.06	−0.17	−0.27	−7.42	−2.35	−1.77	−4.47	−5.56
2	−0.16	−0.01	0.07	0.10	−0.23	−5.11	−0.18	1.87	2.85	−6.59
3	0.09	0.20	0.17	0.14	−0.36	2.09	5.49	4.30	3.60	−7.58
4	−0.01	0.18	0.14	0.07	−0.28	−0.31	4.84	3.70	1.66	−5.21
Big	−0.08	0.11	0.04	−0.06	−0.04	−2.48	3.73	1.48	−1.61	−0.88
			c					*t*(*c*)		
Small	0.30	0.20	0.15	−0.07	−0.57	4.39	4.12	3.20	−1.44	−9.05
2	0.43	0.33	0.21	−0.15	−0.63	10.79	7.62	4.27	−3.34	14.24
3	0.25	0.24	0.06	−0.22	−0.70	4.48	4.99	1.29	−4.46	−11.45
4	0.34	0.29	0.01	−0.16	−0.56	6.55	6.20	0.29	−2.93	−8.01
Big	0.48	0.35	0.14	−0.32	−0.85	12.19	9.56	3.74	−6.75	−14.65
Europe										
			h					*t*(*h*)		
Small	−0.06	0.11	0.15	0.01	−0.31	−1.57	3.16	4.02	0.22	−6.64
2	0.11	0.16	0.18	0.15	−0.19	2.83	4.61	4.70	4.12	−5.15
3	0.18	0.25	0.26	0.15	−0.19	3.94	6.03	6.02	3.50	−4.46
4	0.12	0.16	0.21	0.22	−0.13	2.33	3.97	4.92	4.99	−2.62
Big	−0.14	0.00	−0.08	−0.05	0.23	−3.41	0.13	−2.13	−1.24	5.51
			r					*t*(*r*)		
Small	−0.23	0.03	0.11	−0.03	−0.30	−4.62	0.55	2.20	−0.63	−4.86
2	−0.08	0.07	0.06	0.05	−0.26	−1.55	1.51	1.15	0.99	−5.33
3	0.14	0.05	0.10	0.01	−0.18	2.36	0.93	1.76	0.14	−3.19
4	0.02	0.03	0.02	0.01	−0.15	0.24	0.54	0.28	0.16	−2.35
Big	−0.12	0.13	−0.01	−0.00	0.12	−2.15	2.65	−0.15	−0.03	2.28
			c					*t*(*c*)		
Small	0.30	0.25	−0.02	0.02	−0.47	6.62	6.05	−0.42	0.55	−8.31
2	0.34	0.29	0.08	−0.10	−0.46	7.18	6.79	1.69	−2.36	−10.48
3	0.26	0.21	0.08	−0.17	−0.55	4.74	4.27	1.53	−3.26	−10.59
4	0.35	0.33	0.04	−0.19	−0.66	5.89	6.78	0.72	−3.61	−10.94
Big	0.65	0.40	0.15	−0.43	−0.73	13.27	9.21	3.20	−9.25	−14.77

Table A2
Five-factor *HML*, *RMW*, and *CMA* slopes and *t*-statistics for the 2 × 4 × 4 *Size-B/M-OP*, 2 × 4 × 4 *Size-B/M-Inv*, and 2 × 4 × 4 *Size-OP-Inv* portfolios of North America and Europe: July 1990 – December 2015.

Panel A: Size-B/M-OP portfolios

	Small								Big							
B/M →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
North America																
	h				*t*(*h*)				*h*				*t*(*h*)			
Low *OP*	−0.97	−0.44	0.08	0.58	−12.19	−4.37	0.87	5.96	−0.70	−0.18	0.11	0.61	−6.35	−2.44	1.90	13.26
2	−0.55	0.01	0.31	0.57	−7.37	0.24	9.42	12.85	−0.60	−0.02	0.41	0.68	−5.72	−0.37	8.52	14.57
3	−0.36	0.19	0.44	0.69	−6.51	5.54	12.67	13.49	−0.35	0.16	0.30	0.70	−5.87	3.62	6.07	10.38
High *OP*	−0.02	0.44	0.60	0.79	−0.82	14.49	13.73	9.12	−0.41	0.00	0.43	0.81	−12.68	0.00	5.61	7.71
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *OP*	−0.81	−1.12	−0.78	−0.54	−10.47	−11.33	−9.11	−5.73	−0.68	−0.72	−0.34	−0.44	−6.28	−9.88	−5.79	−9.71
2	−0.51	−0.21	−0.05	−0.04	−7.02	−3.88	−1.62	−0.92	−0.41	−0.08	−0.07	−0.01	−4.04	−1.43	−1.48	−0.26
3	0.04	0.19	0.14	0.18	0.69	5.49	4.25	3.56	0.21	0.18	−0.05	0.16	3.60	4.00	−0.97	2.43
High *OP*	0.44	0.39	0.35	0.16	15.90	13.10	8.22	1.95	0.33	0.33	0.41	0.49	10.72	5.78	5.54	4.79
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *OP*	0.12	0.11	−0.09	−0.12	1.17	0.85	−0.85	−1.02	−0.48	−0.17	−0.05	−0.18	−3.44	−1.79	−0.69	−3.06
2	−0.06	0.08	0.14	0.00	−0.60	1.20	3.30	0.00	−0.29	0.03	−0.01	−0.11	−2.21	0.39	−0.18	−1.96
3	−0.03	0.08	0.07	−0.10	−0.43	1.81	1.60	−1.56	−0.15	0.05	0.06	−0.20	−1.97	0.85	0.91	−2.33
High *OP*	−0.05	−0.17	−0.14	−0.25	−1.46	−4.62	−2.47	−2.34	0.18	0.12	−0.29	−0.44	4.38	1.60	−3.09	−3.34
Europe																
	h				*t*(*h*)				*h*				*t*(*h*)			
Low *OP*	−0.92	−0.30	0.28	0.24	−10.94	−3.89	4.25	3.00	−1.00	−0.21	0.09	0.47	−7.83	−2.10	1.39	9.21
2	−0.59	−0.14	0.24	0.49	−7.91	−2.79	6.31	11.55	−0.54	−0.29	0.16	0.92	−5.39	−4.98	3.38	15.01
3	−0.42	0.14	0.45	0.54	−9.20	4.37	13.30	9.92	−0.55	−0.08	0.43	0.68	−8.12	−1.43	7.39	8.22
High *OP*	−0.20	0.33	0.65	0.56	−7.56	9.75	13.22	5.26	−0.39	−0.00	0.50	0.55	−8.99	−0.06	5.00	4.03
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *OP*	−1.07	−0.52	−0.36	−0.54	−9.64	−5.10	−4.12	−5.07	−1.01	−0.96	−0.72	−0.73	−5.99	−7.39	−8.95	−10.79
2	−0.74	−0.39	−0.03	−0.07	−7.60	−6.04	−0.67	−1.23	−0.39	−0.31	−0.16	0.03	−2.91	−4.04	−2.55	0.33
3	−0.29	0.10	0.21	0.23	−4.84	2.24	4.75	3.22	0.19	0.20	0.34	0.14	2.08	2.88	4.37	1.31
High *OP*	0.08	0.33	0.40	0.24	2.38	7.29	6.18	1.71	0.30	0.69	0.65	0.37	5.12	8.09	4.91	2.05
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *OP*	−0.46	−0.38	−0.08	0.27	−4.55	−4.02	−1.03	2.77	−0.12	0.25	0.06	−0.23	−0.79	2.07	0.87	−3.73
2	−0.35	0.10	0.14	0.21	−3.88	1.66	3.02	4.09	0.15	0.34	−0.05	−0.24	1.28	4.81	−0.84	−3.27
3	−0.12	0.14	0.10	0.17	−2.14	3.46	2.48	2.59	−0.08	0.27	−0.00	−0.15	−0.95	4.10	0.00	−1.47
High *OP*	−0.08	0.03	−0.04	0.26	−2.37	0.63	−0.68	2.03	0.06	−0.05	−0.09	−0.19	1.06	−0.62	−0.77	−1.13

Panel B: Size-B/M-Inv portfolios

	Small								Big							
B/M →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
North America																
	h				*t*(*h*)				*h*				*t*(*h*)			
Low *Inv*	−0.48	−0.20	0.13	0.63	−8.59	−3.93	2.86	9.82	−0.42	−0.19	0.16	0.51	−7.32	−3.72	3.25	9.72
2	−0.36	0.20	0.30	0.58	−8.50	5.91	9.28	11.64	−0.41	0.04	0.33	0.66	−7.78	0.74	6.66	13.09
3	−0.26	0.21	0.48	0.63	−6.94	6.28	15.27	11.11	−0.40	0.04	0.41	0.79	−7.19	0.69	7.26	11.27
High *Inv*	−0.41	0.29	0.60	0.76	−10.93	6.33	12.90	9.44	−0.45	0.24	0.34	0.80	−7.46	3.49	5.06	12.53
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *Inv*	−0.29	−0.38	−0.15	−0.09	−5.25	−7.76	−3.41	−1.52	0.12	0.02	−0.06	−0.10	2.16	0.43	−1.33	−1.92
2	0.05	0.09	0.04	−0.01	1.25	2.59	1.42	−0.19	0.36	0.08	−0.08	0.07	7.00	1.32	−1.67	1.34
3	0.08	0.23	0.16	−0.07	2.20	7.13	5.09	−1.17	−0.02	0.15	0.02	−0.31	−0.38	3.00	0.37	−4.57
High *Inv*	−0.19	0.07	0.08	−0.03	−5.31	1.61	1.66	−0.41	0.19	−0.03	0.10	−0.30	3.11	−0.44	1.59	−4.73
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *Inv*	0.43	0.64	0.52	0.12	6.19	10.18	9.15	1.46	0.60	0.55	0.45	0.22	8.33	8.69	7.24	3.36
2	0.39	0.25	0.31	0.05	7.44	5.89	7.77	0.73	0.51	0.22	−0.02	0.01	7.81	3.02	−0.33	0.10
3	0.06	0.01	−0.05	−0.05	1.27	0.25	−1.28	−0.75	−0.05	−0.11	−0.24	−0.10	−0.73	−1.68	−3.31	−1.10
High *Inv*	−0.44	−0.55	−0.49	−0.54	−9.33	−9.70	−8.40	−5.41	−0.72	−0.66	−0.30	−0.94	−9.46	−7.78	−3.62	−11.69

(*continued on next page*)

Table A2 (*continued*)

Panel B: Size-B/M-Inv portfolios

	Small								Big							
B/M →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
Europe																
	h				*t*(*h*)				*h*				*t*(*h*)			
Low *Inv*	−0.41	−0.06	0.25	0.47	−6.75	−1.13	4.94	6.56	−0.56	−0.15	0.01	0.47	−7.96	−2.16	0.13	6.67
2	−0.25	0.12	0.30	0.39	−6.04	3.51	7.97	7.28	−0.46	−0.26	0.20	0.58	−6.62	−5.04	3.45	8.67
3	−0.14	0.21	0.46	0.54	−3.97	6.21	10.71	10.86	−0.71	−0.15	0.33	0.92	−10.77	−2.29	5.47	14.20
High *Inv*	−0.54	0.06	0.46	0.46	−13.17	1.25	9.45	6.14	−0.22	−0.02	0.47	0.92	−3.72	−0.39	6.97	11.03
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *Inv*	−0.32	−0.08	−0.01	−0.17	−4.03	−1.24	−0.14	−1.78	0.05	0.21	−0.36	−0.21	0.57	2.28	−4.28	−2.25
2	−0.13	0.12	−0.03	−0.06	−2.31	2.52	−0.65	−0.87	−0.02	0.21	−0.06	−0.14	−0.23	3.16	−0.84	−1.59
3	−0.00	0.10	0.17	0.04	−0.03	2.13	3.09	0.56	−0.20	−0.10	0.15	−0.05	−2.34	−1.12	1.88	−0.56
High *Inv*	−0.38	−0.11	0.04	−0.14	−6.91	−1.84	0.58	−1.38	0.20	0.01	0.14	−0.33	2.59	0.12	1.57	−2.97
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *Inv*	0.21	0.43	0.28	0.37	2.93	6.97	4.63	4.35	0.78	0.59	0.48	0.27	9.24	6.87	6.35	3.17
2	0.21	0.30	0.28	0.41	4.28	7.16	6.09	6.45	0.38	0.38	0.30	−0.07	4.48	6.21	4.28	−0.85
3	−0.19	−0.00	0.02	0.05	−4.62	−0.03	0.32	0.78	−0.02	0.30	−0.42	−0.43	−0.21	3.76	−5.82	−5.53
High *Inv*	−0.49	−0.46	−0.31	−0.19	−9.92	−8.35	−5.31	−2.12	−0.85	−0.55	−0.44	−0.90	−12.11	−7.75	−5.41	−8.94

Panel C: Size-OP-Inv portfolios

	Small								Big							
OP →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
North America																
	h				*t*(*h*)				*h*				*t*(*h*)			
Low *Inv*	−0.50	0.30	0.23	0.31	−6.55	5.95	4.67	6.58	0.09	0.02	0.13	−0.20	1.48	0.41	2.42	−4.47
2	−0.48	0.15	0.28	0.29	−5.33	4.07	8.00	9.03	0.26	0.27	0.17	−0.14	4.12	4.52	3.80	−2.98
3	−0.19	0.01	0.22	0.25	−1.76	0.18	6.39	7.48	0.35	0.10	0.18	−0.30	4.71	1.68	3.55	−5.27
High *Inv*	−0.62	0.09	0.16	0.17	−6.54	1.77	3.74	4.69	0.07	0.35	−0.19	−0.18	1.29	4.68	−2.47	−2.36
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *Inv*	−0.95	−0.19	0.10	0.26	−12.88	−3.78	2.11	5.62	−0.60	−0.14	0.02	0.33	−10.52	−2.64	0.45	7.72
2	−0.91	−0.17	0.11	0.37	−10.29	−4.67	3.14	12.03	−0.19	−0.24	0.08	0.35	−3.09	−4.15	1.92	7.83
3	−0.76	−0.29	0.09	0.40	−7.32	−7.06	2.58	12.23	−0.65	−0.24	0.12	0.20	−9.03	−4.14	2.41	3.61
High *Inv*	−0.85	−0.36	0.09	0.45	−9.14	−6.92	2.11	12.72	−0.63	−0.03	0.15	0.55	−11.24	−0.46	1.99	7.32
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *Inv*	0.48	0.28	0.22	0.24	5.08	4.50	3.47	4.14	0.25	0.47	0.35	0.49	3.45	6.91	5.40	8.87
2	0.36	0.26	0.23	0.11	3.19	5.56	5.32	2.73	0.21	−0.01	0.22	0.30	2.68	−0.09	4.01	5.22
3	−0.07	0.14	−0.01	−0.12	−0.54	2.56	−0.29	−2.93	−0.24	−0.10	−0.21	−0.03	−2.63	−1.29	−3.34	−0.36
High *Inv*	−0.48	−0.41	−0.39	−0.53	−3.99	−6.21	−7.19	−11.67	−0.73	−0.68	−0.56	−0.65	−10.08	−7.32	−5.91	−6.81
Europe																
	h				*t*(*h*)				*h*				*t*(*h*)			
Low *Inv*	−0.43	0.10	0.20	0.24	−5.55	1.92	3.74	4.63	0.14	−0.08	0.12	−0.40	2.41	−1.29	1.86	−6.40
2	0.12	0.11	0.18	0.20	1.61	2.76	5.27	5.48	−0.03	−0.01	0.08	0.02	−0.51	−0.21	1.34	0.41
3	0.17	0.29	0.23	0.15	2.16	7.03	7.84	5.39	0.29	0.16	−0.15	−0.15	4.13	2.78	−2.34	−2.46
High *Inv*	−0.50	−0.11	0.03	−0.17	−6.63	−2.13	0.76	−4.26	0.06	0.15	0.29	0.08	0.88	2.12	4.76	1.21
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *Inv*	−0.87	−0.23	0.10	0.34	−8.44	−3.56	1.38	4.85	−0.73	−0.29	0.19	0.45	−9.22	−3.59	2.22	5.50
2	−0.32	−0.17	0.05	0.26	−3.31	−3.20	1.22	5.42	−0.92	−0.18	0.25	0.58	−10.89	−2.35	3.35	8.48
3	−0.48	−0.13	0.19	0.28	−4.58	−2.44	4.94	7.82	−0.84	−0.31	0.16	0.33	−9.15	−4.00	1.85	4.09
High *Inv*	−0.82	−0.41	−0.07	0.07	−8.17	−6.01	−1.34	1.40	−0.96	−0.06	0.38	0.47	−9.91	−0.69	4.62	5.73
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *Inv*	0.26	0.37	0.29	0.27	2.75	6.23	4.50	4.27	0.37	0.68	0.58	0.57	5.16	9.17	7.62	7.59
2	0.10	0.39	0.32	0.26	1.18	8.29	7.94	6.05	0.17	0.36	0.41	0.19	2.24	5.03	5.88	3.02
3	−0.48	−0.02	−0.02	−0.08	−5.04	−0.32	−0.53	−2.47	−0.43	−0.16	−0.01	0.06	−5.14	−2.34	−0.13	0.78
High *Inv*	−0.51	−0.50	−0.39	−0.48	−5.59	−7.96	−7.85	−9.78	−0.59	−0.77	−0.57	−0.90	−6.69	−9.24	−7.70	−11.95

to explain monthly excess returns on the 5×5 *Size-B/M*, *Size-OP*, and *Size-Inv* portfolios of North America and Europe. Table A2 reports *h*, *r*, and *c* and their *t*-statistics for the $2 \times 4 \times 4$ *Size-B/M-OP*, *Size-B/M-Inv*, and *Size-OP-Inv* portfolios.

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