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Expected EPS and EPS Growth as Determinants of Value

JAMES A. OHLSON
W. P. Carey School of Business, Arizona State University, Tempe, AZ, 85287, USA

BEATE E. JUETTNER-NAUROTH
Chair of Banking, Johannes Gutenberg-University, D-55099, Mainz, Germany

Abstract. This paper develops a parsimonious model relating a firm's price per share to, (i), next year expected earnings per share (or 12 months forward eps), (ii), short-term growth (FY-2 versus FY-1) in eps, (iii), long-term (asymptotic) growth in eps, and, (iv), cost-of-equity capital. The model assumes that the present value of dividends per share (dps) determines price, but it does not restrict how the dps-sequence is expected to evolve. All of these aspects of the model contrast sharply with the standard (Gordon/Williams) text-book approach, which equates the growth rates of expected eps and dps and fixes the growth rate and the payout rate. Though the constant growth model arises as a peculiar special case, the analysis in this paper rests on more general principles, including dividend policy irrelevancy. A second key result inverts the valuation formula to show how one expresses cost-of-capital as a function of the forward eps to price ratio and the two measures of growth in expected eps. This expression generalizes the text-book equation in which cost-of-capital equals the dps-yield plus the growth in expected eps.

Keywords: Equity valuation, EPS, EPS growth, Dividend policy

JEL Classfication: M41, G12, G14

Introduction

A central organizing principle in practical equity-valuation focuses on firms' near term expected eps and its subsequent growth. Thus making money in the stock market reduces to the idea that investors want to buy future earnings "as cheaply as possible" for a given risk-level. As an equilibrium consequence, firms with a relatively large price to next-year eps ratio (P_0/eps_1) ought to have a relatively large growth in expected eps. Such a relation would seem to be beyond dispute, at least as a first-order approximation. Much more ambiguous is how one deals with the (P_0/eps_1) ratio and expected growth in eps relation as a theoretical matter. To deal with this issue introduces a difficult and subtle conceptual problem: A formal model must find a place for expected dividends per share (dps). Dps cannot be dismissed for two reasons. First, the expected dps-sequence serves as the ultimate source of value. Second, the expected eps- and dps-sequences must be related to each other to make economic sense. The traditional, textbook, approach deals with these problems by assuming that eps and dps have a fixed relation and they grow at the same rate.[1] These assumptions suffice to derive a P_0/eps_1 ratio which increases as growth increases, but the approach barely distinguishes between eps and dps. Putting the growth assumption aside, it goes almost without saying that fixing the payout ratio confines the analysis on empirical and conceptual grounds.

This paper reconsiders how next-period eps and eps growth relate to a firm's current price per share. We impose no contrived restrictions on the dividend policy. With respect to eps growth, the model includes a short-term measure as well as a long-term measure. Consistent with conventional wisdom, the model will indeed show that the P_0/eps_1 ratio increases if one increases either of the two growth measures. A more subtle issue relates to an attribute of the two growth measures of eps. These are both independent of the firm's (future) dividend policy, In fact, the model is based on an assumption that the current price does not depend on the dividend policy in a typical MM-type of framework. In a very real sense, the core of the model shows how the current price depends on forward eps and their subsequent growth as captured by two dividend-policy independent measures of eps growth.

Development of the model starts from the standard precept that the present value of expected dps determines price. We then put into place an elementary algebraic machinery identical to the one which can also be used to derive residual income valuation (RIV).[2] The approach used here is very similar in that one simply replaces the date t book values in RIV with the period $t+1$ capitalized eps. Hence, the model rests on capitalized next-period eps as the first value component, and it then adds the present value of capitalized expected changes in earnings, adjusted for dividends, as a second component. (This analytical scheme has previously been developed in Ohlson (1998, 2000)).

We further parameterize the behavior of the expected future evolution of eps changes over time (adjusted for dps). The idea behind the parameterization is to model a smooth decay in the growth of the (adjusted for dps) expected eps-sequence as a function of the future date. Though the scheme involves some subtleties, it makes economic sense and it results in an intuitively appealing valuation formula. As indicated, this formula determines value as a function of next-period eps, near-term growth in eps, long-term growth in eps, and cost-of-capital. In some ways the model can be viewed as a generalization of the constant growth model. The constant growth model does indeed arise as a special case, but growth here has two rather than one degree of freedom. And, to be sure, there is no need here to fix the payout ratio across dates.

Text-books often show how one can exploit the constant growth model to derive the cost-of-capital as the sum of dps_1/P_0 and eps (or dps) growth. We generalize this result and derive a square-root formula which expresses cost-of-capital as a function of eps_1/P_0 and the two dividend policy irrelevant eps growth measures. This result, we believe, should be useful in research focusing on individual firms' expected returns and risks.[3]

1. Basic Model Ingredients

We consider a valuation setting from the perspective of date 0. Accordingly, the following variables/parameter will be central:

P_0 = current, date t = 0, price per share

dps_t = expected dps, date t ($t \geq 1$)

eps_t = expected eps, date t ($t \geq 1$)

$r \equiv R-1$ = Cost-of-capital, or discount factor

Naturally, the model's starting point postulates that the present value of the expected dividends per share sequence determines price per share.[4]

Assumption 1

$$P_0 = \sum_{t=1}^{\infty} R^{-t}\text{dps}_t \tag{1}$$

where $R > 1$ is a fixed constant.[5]

Applying a scheme developed in Ohlson (1998, 2000), we next introduce an expected eps-sequence in a fashion not all that different from the way RIV introduces book-values and residual earnings as a representation of a firm's value. As a first step, the following algebraic equation (or identity) is central:

$$0 = y_0 + R^{-1}(y_1 - Ry_0) + R^{-2}(y_2 - Ry_1) + \cdots \tag{2}$$

where $\{y_t\}_{t=0}^{\infty}$ can be *any* sequence of numbers that satisfy the mild transversality condition $R^{-T}y_T \to 0$ as $T \to \infty$. The equation holds simply because $y_0 + \sum_{t=1}^{T} R^{-t}(y_t - Ry_{t-1}) = R^{-T}y_T$. Adding equations (1) and (2) yields

$$P_0 = y_0 + \sum_{t=1}^{\infty} R^{-t}(y_t + \text{dps}_t - Ry_{t-1}). \tag{3}$$

One can derive the RIV model on the basis of equation (3); put y_t = expected book value per share, and assume clean surplus accounting so that $\text{eps}_t = \Delta\ \text{bvps}_t + \text{dps}_t$. RIV now obviously follows.[6] However, the scheme leading to (3) can be exploited for any specification of the $\{y_t\}_{t=0}^{\infty}$ sequence. Consider a capitalized expected eps specification:[7]

$$y_0 = \text{eps}_1/r$$

and, more generally,

$$y_t = \text{eps}_{t+1}/r \quad t = 0, 1, 2 \ldots$$

The idea of putting y_0 equal to eps_1/r would seem to be intuitively appealing. It simply means that valuation starts from next-period (FY1) expected eps capitalized. With this approach in place the issue arises as to how one should think about the premium, defined by P_0 minus eps_1/r. Conventional wisdom holds, of course, that this premium should relate to notions of expected eps growth beyond the upcoming year (FY1). The question thus reduces to how one formalizes growth in the model.

To tackle the problem, we write (3) as

$$P_0 = \frac{\text{eps}_1}{r} + \sum_{t=1}^{\infty} R^{-t} z_t \tag{4}$$

where

$$z_t \equiv \frac{1}{r}[\text{eps}_{t+1} + r\text{dps}_t - R\text{eps}_t] \quad t = 1, 2, \ldots$$

Expression (4) shows that as a matter of mechanics the present value of the z_t-sequence explains the valuation premium, P_0 minus eps_1/r. The structure of the problem accordingly requires some consideration as to how one interprets the z_t, or rz_t, variable. A good understanding of z_t will help before stating further assumptions.

In broad terms, one can think of rz_t as a measure of the expected performance for the period $(t,\ t+1)$. Since $rz_t = [\text{eps}_{t+1} - (R\ \text{eps}_t - r\ \text{dps}_t)]$, $(R\ \text{eps}_t - r\ \text{dps}_t)$ serves as a benchmark for the period $(t,\ t+1)$ expected eps. In the benchmark, the term $r\ \text{dps}_t$ is essential because eps_{t+1} depends on the expected earnings retention at date t. Such reasoning is familiar from the accounting/economics of a savings account. Thus, one naturally defines normal earnings performance as the case when $z_t = 0$. It follows that an expectation of normal earnings performance for all future periods leads to setting with zero premium. Formally,

$$z_t = 0, t \geq 1, \text{ implies } P_0 = \frac{\text{eps}_1}{r}$$

The result makes much sense; it reinforces that $z_t > 0$ reflects the expectation of superior eps performance as measured by changes in eps adjusted for earnings retention.

The metaphor of a savings account brings out that $z_t = 0$ regardless of the dividend policy. More generally, the date t eps-growth as defined by $\Delta\text{eps}_{t+1}/\text{eps}_t$ naturally depends on the date t payout, and hence the dividend-adjustment in the benchmark. To appreciate this point, consider the two extremes of full payout and zero payout.

(i) With full payout $rz_t = \Delta\text{eps}_{t+1}$ so that $z_t = 0$ if and only if there is no growth in expected eps, i.e., $\text{eps}_{t+1} = \text{eps}_t$.

(ii) With zero payout $rz_t = \text{eps}_{t+1} - R\text{eps}_t$ so that $z_t = 0$ if and only if the growth in expected eps equals r, i.e., $\Delta\text{eps}_{t+1}/\text{eps}_t = r$.

It makes no difference whether any expected superior earnings performance is due to the accounting rules, or the possibility of positive NPV projects, or some mixture of these two possibilities. The basic valuation formula still applies, without any requirement that the accounting for (expected) eps represents "truthful" accounting. In these regards, the formula (4) admits the same versatility as the RIV formula. Just as RIV holds for all accounting principles, so does expression (4) and the measurement of the "primitive" variable labeled "eps". But expression (4) does not depend on clean surplus accounting, unlike RIV.

A final point concerning z_t pertains to how one formalizes its independence of the dividend policy. This important property will be satisfied under mild conditions on the accounting and the next-period effect of on eps due to dividends. To be specific, if we fix the dps_t sequence for the dates $t+1,\ldots,\ t+\tau$ then:

$$\partial z_{t+\tau}/\partial \text{dps}_t = 0 \text{ for all } \tau \geq 0$$

if, and only if,

$$\partial \text{eps}_{t+\tau}/\partial \text{dps}_t = -rR^{\tau-1}.$$

The last condition makes sense if one, again, keeps in mind the economics of a savings account. An incremental dollar withdrawn today results in earnings being \$$r$ less in the subsequent year, \$$rR$ less for the year after the next, \$$rR^2$ less for the year after the next two, etc.

In what follows we will make an assumption on the evolution of the $\{z_t\}$-sequence which effectively subsumes dividend policy irrelevancy. There is accordingly no need for an explicit assumption $\frac{\partial \text{eps}_{t+\tau}}{\partial \text{dps}_t} = -rR^{\tau-1}$, but the relation will be essential to appreciate the workings of the model.

2. Assumptions on the z_t—Sequence and Valuation Formulae

Valuation equation (4) by itself provides guidance when one conceptualizes the P_0/eps_1 ratio. As noted, the core of this issue depends on the rate at which the eps-sequence increases, after one adjusts for implications of paying dividends. But one can reasonably hypothesize that assumptions on the (anticipated) evolution of z_t leads to sharper, and more parsimonious, insights. The following assumption virtually suggests itself.[8]

Assumption 2

The sequence $\{z_t\}_{t=1}^{\infty}$ satisfies

$$z_{t+1} = \gamma z_t, \quad t = 1, 2\ldots \tag{5}$$

where $1 \leq \gamma < R$ and $z_1 > 0$. A2 allows two degrees of freedom, z_1 and γ, but the assumption also places certain natural restrictions on the set of admissible settings. The initialization condition $z_1 > 0$ is appropriate because $z_1 = 0$ is trite, and $z_1 < 0$ leads to an exceptional scenario in which the expected earnings performance always is inferior. With respect to γ, the condition $\gamma < R$ is necessary and sufficient for convergence of $\sum_{t=1}^{\infty} R^{-t} z_t$. Settings with $\gamma < 1$ are peculiar: The condition forces the expected eps performances to become normal in the sense that $z_t \to 0$. This characteristic violates settings with conservative accounting and an expectation of positive long run growth in the free cash flows Zhang (2000), or operating assets Feltham and Ohlson (1995).

Though we argue that the restrictions $z_1 > 0$ and $\gamma \geq 1$ make intuitive sense, note that neither is required from a narrow mathematical point of view. Subsequent proposition will still hold. More importantly, given that A2 results in an ever non-decreasing sequence $z_1 \leq z_2 \leq \ldots$, the issue arises as to how one generalizes A2 to allow for broader classes of sequences. Section 6 develops such modeling.

It should be emphasized that the constant γ is viewed as being independent of the dividend policy. Subsequent results would otherwise be difficult if not impossible to interpret. But it does indeed make sense to fix γ if the condition $\frac{\partial \text{eps}_{t+\tau}}{\partial \text{dps}_t} = -rR^{\tau-1}$ holds. As noted, in that case the z_t -sequence does not depend on the dividend policy, and thus γ cannot reasonably depend on value-neutral changes in the {dps}-sequence.

Though the model Al and A2—and these are indeed the only assumptions—may seem mechanistic and unlikely to embed to interesting consequences, we will argue that they do. That is, appropriately measured growth rates will explain the P_0/eps_1-ratio in a simple reduced form valuation formula with intuitively appealing empirical content.

Proposition 1
Assumptions A1 and A2 imply

$$P_0 = \frac{\text{eps}_1}{r} + \frac{z_1}{R - \gamma} \tag{6}$$

where $z_1 \equiv r^{-1}[\text{eps}_2 + r\text{dps}_1 - R\text{eps}_1]$.

To appreciate the valuation formula (6), it helps to introduce measures of eps-growth. Assume that $\text{eps}_1 > 0$ and consider the following near-term—FY2 versus FY1—growth measure:

$$\begin{aligned}\hat{g}_2 &\equiv [(\text{eps}_2 + r\text{dps}_1)/\text{eps}_1 - 1] - r \\ &\equiv [\%\Delta\text{eps}_2 + r(\text{dps}_1/\text{eps}_1)] - r \\ &\equiv g_2 - r\end{aligned}$$

The term inside the [ ], or g_2, captures the usual measure of growth, $\%\Delta$ eps $\equiv (\text{eps}_2 - \text{eps}_1)/\text{eps}_1$, but the term also adds an adjustment for foregone earnings due to expected dividends paid at the end of FY1. Given dividend policy irrelevancy, g_2 measures the growth in eps under a pro forma assumption of any payout, including 0% or 100%. In analytical terms

$$\partial \hat{g}_2 / \partial \text{dps}_1 = 0$$

since $\partial \text{eps}_1 / \partial \text{dps}_1 = 0$ and MM presumes $\partial \text{eps}_2 / \partial \text{dps}_1 = -r$. Further, note that the definition of $\hat{g}_2$ requires that one deducts r from g_2, so that earnings are normal if and only if $\hat{g}_2 = 0$; that is, $\hat{g}_2 = 0$ corresponds to the boundary condition $z_1 = 0$.

Given the definition of $\hat{g}_2$, one readily verifies that

$$z_1 = \text{eps}_1 \hat{g}_2 / r.$$

A more intuitive version of (6) follows:

Corollary

$$\frac{P_0}{\text{eps}_1} = \frac{1}{r}\left[1 + \frac{\hat{g}_2}{R-\gamma}\right] = \frac{1}{r} \times \frac{g_2 - (\gamma - 1)}{r - (\gamma - 1)} \tag{7}$$

where $\hat{g}_2 \equiv \left[\%\Delta\text{eps}_2 + r\frac{\text{dps}_1}{\text{eps}_1}\right] - r = g_2 - r$

As should be the case, the P_0/eps_1 ratio increases in $\hat{g}_2$ (or g_2) and γ. One further sees that $g_2-(\gamma-1)$ and $r-(\gamma-1)$ provide all the necessary information to determine the correction-scalar attached to the starting point $\frac{1}{r}$. (To be sure, recall that $z_1 > 0$ implies $g_2 > r$, and $r > \gamma-1$ by the assumption necessary for $\sum R^{-t}z_t$ to converge).

To illustrate the formula (7),consider the constant growth model. In this specialized setting eps_t and dps_t grow at the same rate, and this rate is the same for all periods. Formally, assume that, for all t,

$$\text{eps}_{t+1} = G \cdot \text{eps}_t$$
$$\text{dps}_{t+1} = G \cdot \text{dps}_t$$

where $1 \leq G < R$ defines one plus the growth in eps and dpsThe rigid assumptions imply that there exists some constant payout ratio, k, such that:

$$\text{dps}_{t+1} = k.\text{eps}_{t+1}.$$

Though not necessary from a technical point of view, to make economic sense the model restricts k to satisfy $0 < k \leq 1$.

These assumptions satisfy A2; every component of z_t grows at the rate $G-1$, so that $z_t = Gz_{t\text{-}1}$, and $G = \gamma$. With respect to $\hat{g}_2$, since $rz_t = \text{eps}_{t+1} - R \cdot \text{eps}_t + r\text{dps}_t = (G-R+rk)\text{eps}_t$ it follows that

$$\hat{g}_2 = G - R + rk.$$

Applying the corollary, one obtains:

$$\frac{P_0}{\text{eps}_1} = \frac{1}{r}\left[1 + \frac{G-R+rk}{R-G}\right] = \frac{k}{R-G}$$

which is the well-known solution to the constant growth model. It is therefore clear that the model in this paper subsumes the constant growth model.

The constant growth model builds in peculiar characteristics. It satisfies superior expected eps performance when $G-R+rk > 0$. Only if this inequality holds will $\hat{g}_2 > 0$ and $P_0/\text{eps}_1 > 1/r$. The way the model is developed, one sees that the anticipated eps performance therefore depends on both growth *and* the payout parameter, k. To elaborate on this apparent MM violation, consider normal eps performance, $G + rk = R$. This case works exactly like a savings account$\left(\frac{P_0}{\text{eps}_1} = \frac{1}{r}\right)$, and the equation $G + rk = R$ suggests a direct trade-off between eps growth and the choice of a dividend policy parameter k. In other words, if one views k as a choice variable, then eps growth is a consequence specified by $G = R-rk$ (or $G-1 = r[1-k]$). Absent this knife-edge setting, however, the model is awkward because it depends on a fixed payout ratio without articulating how growth changes if the dividend policy changes. Common sense economics suggests that one cannot *a priori* postulate that eps growth is totally independent of a firm's dividend policy. The constant growth model by itself is therefore not particularly helpful as an example illustrating the valuation model's core economics. The issue remains how one can characterize the role and implications of the parameter γ when it is dividend policy irrelevant.

3. The Parameter γ

Fixing the three quantities eps_1, R and $\hat{g}_2$, it follows that P_0 increases as γ increases.This relation is intuitively obvious since $z_1 > 0$, and γ determines the extent to which future expected eps grow relative to prior-period eps. The case $\gamma = 1$ emerges as a benchmark, $z_1 = z_2 = \ldots$, and the next period's expected eps performance provides an unbiased estimate of all subsequent periods' as well. This observation yields:

$$P_0 - \frac{\text{eps}_1}{r} = E_0[P_T] - \frac{\text{eps}_{T+1}}{r}. \tag{8}$$

In other words, the premium is expected to remain the same for all future dates. To be sure, given dividend policy irrelevancy, neither the RHS nor the LHS will be affected by changes in the dividend policy, though each of the two components on the RHS of (8) depend on the dividend policy. Hence, $\gamma = 1$ captures the case of "no growth" in eps performance where the growth construct is dividend policy irrelevant.

For $\gamma = 1$, one further obtains

$$P_0 = \frac{\text{eps}_1}{r} \times \frac{g_2}{r}$$

where g_2 is the short-term growth prior to the adjustment for the norm r, so that $g_2 > r$ and $\frac{g_2}{r} > 1$. The formula has a straightforward message: It shows how one must correct for eps growth in excess of the benchmark r via the ratio-scalar $\frac{g_2}{r}$ applied to the starting point in valuation, $\frac{\text{eps}_1}{r}$. In this context it is also worthwhile to recall that $g_2 = \%\Delta\text{eps}_2$ under an as-if assumption of $\text{dps}_1 = 0$ (though we reiterate that g_2 is independent of the dividend policy given $\frac{\partial \text{eps}_2}{\partial \text{dps}_1} = -r$).

One can relate the above formula to the popular PEG-ratio, that is the P/E ratio relative to the growth of expected eps. Here we define the P/E-ratio in terms of P_0/eps_1 and the eps-growth in terms of g_2. It turns out that the so defined PEG-ratio relates directly to r. Solving for r, the cost-of capital (or expected return), one obtains the following square root formula:

$$r = \sqrt{\frac{\text{eps}_1}{P_0} \times g_2}$$

where the expression inside the $\sqrt{}$ the inverse of the PEG-ratio. The formula, in its simplicity, would seen to have obvious "first-cut" appeal if one wants to infer the cost-of-capital from analysts' expectations of eps and eps-growth.

The above observations set the stage for settings when $\gamma > 1$. Generalizing the expression (8) results in

$$\left(E_0[P_T] - \frac{\text{eps}_{T+1}}{r}\right) - \left(P_0 - \frac{\text{eps}_1}{r}\right) = z_1 - \gamma(\gamma^T - 1).$$

In words, the expected date T premium grows in the order of γ^T. This makes sense as long as one keeps in mind that the quantities are all in dollar amounts. To appreciate this issue, note that the constant growth model satisfies the property.

The constant growth model suggests loosely that one might be able to characterize γ in terms of eps-growth. One can reasonably ask whether a general proposition lurks in the background. The difficulty, of course, pertains to the dividend policy; any dividends paid affect the subsequent period's growth of eps. The constant growth model eschews this difficulty by fixing the payout ratio, and without relating the growth $(G-1)$ to the payout. Nevertheless, and this result seems less than obvious, we will show in the next proposition that A2 implies

$$\frac{\text{eps}_t}{\text{eps}_{t-1}} \rightarrow \gamma \text{ as } t \rightarrow \infty$$

provided that the asymptotic dividend policy is "sufficiently generous". Sufficiently generous in strict analytical terms means that $\frac{\text{dps}_t}{\text{eps}_t}$ must exceed $\frac{(R-\gamma)}{r}$.

One gets an intuitive feel for a sufficiently generous policy if one considers implications of a fixed but very small payout (beyond some arbitrary future date). Given that earnings retained on the margin earn a normal return, it follows that $\text{eps}_t/\text{eps}_{t-1} > \gamma$ as $t \rightarrow \infty$. This asymptotic outcome is inevitable since $r > \gamma - 1$, and one can always make earnings grow at a rate $r - \varepsilon$, where $\varepsilon > 0$ and $r - \varepsilon > \gamma - 1$, by picking a sufficiently small payout ratio (k). One only needs to recall that a savings account's earnings growth equals $r[1-k]$. It therefore seems reasonable that the equation $\gamma - 1 = r[1-k]$, or $k = (R-\gamma)/r$, should identify the minimum payout in order to be sufficiently generous.

Invoking a more elaborate perspective, suppose that we simply *define* earnings due to operations (opeps) when the payout is 100%. This payout is clearly sufficiently generous. Any policy less than 100% would thus result in additional earnings, which we define as financial earnings (fineps). Given a sufficiently generous policy (which may be less than 100%), one now obtains the convergent relations

$$\frac{\text{eps}_{t+1}}{\text{eps}_t} \rightarrow \frac{\text{opeps}_{t+1}}{\text{opeps}_t} \rightarrow \frac{\text{fineps}_{t+1}}{\text{fineps}_t} \rightarrow \gamma \text{ as } t \rightarrow \infty$$

where eps $\equiv$ opeps + fineps. The above relations reflect that the *mix* of earnings sources, $\text{opeps}_t/\text{fineps}_t$, converges to a positive constant as $t \rightarrow \infty$ (as do $\text{opeps}_t/\text{eps}_t$, and $\text{fineps}_s/\text{eps}_t$). This constant latter mix depends on the dividend policy, of course. But the possibility of having differing mixes of sources of earnings does not mean that the growth in the individual earnings sources have to depend on the mix. Hence we can deduce the conclusion $\text{eps}_{t+1}/\text{eps}_t \rightarrow \gamma$ as $t \rightarrow \infty$.

A sufficiently low payout, on the other hand, results in,

$$\frac{\text{eps}_{t+1}}{\text{eps}_t} \rightarrow \frac{\text{fineps}_{t+1}}{\text{fineps}_t} \rightarrow R - rk \text{ as } t \rightarrow \infty$$

where $k > 0$ but also small. In this case $\text{opeps}_t/\text{fineps}_t \rightarrow 0$; i.e., with a small payout the cumulative effect of normal return activities will over time overwhelm those activities that have an above-normal return. One further sees that $\lim_{t\to\infty} \frac{\text{eps}_{t+1}}{\text{eps}_t}$ has two distinct solutions depending on the sign of $k - \frac{R-\gamma}{r}$.

To formally summarize the idea of a sufficiently generous dividend policy and its consequence, the following proposition applies.

Proposition 2 Consider Assumption A2. Further, let $k = \frac{\text{dps}_t}{\text{eps}_t}$ for all $t \geq T$ where T is any arbitrary future date, and where k satisfies $k \geq \frac{(R-\gamma)}{r}$. Then

$$\lim_{t\to\infty} \frac{\text{eps}_{t+1}}{\text{eps}_t} = \gamma.$$

Proof Appendix I

To get a feel for the workings of the model—both of the propositions and the corollary—Appendix II provides a simple detailed "spread-sheet" type of example to illustrate how various aspects play out. The example is based on splitting the firm into operating and financial activities. Expected free cash flows from operating activities depend on expected operating earnings. Financial activities can be thought of as either borrowing or lending at the rate r. If free cash flows differ from dividends, which will generally be the case, then the firm must engage in borrowing or lending with related income statement consequences. If one then evaluates that model numerically it will become clear that all dividend policies result in the same P_0. Further, one can validate Propositions 1 and 2 numerically. And the example shows that one need *not* know the sequence of free cash flows to evaluate P_0 though P_0 of course equals the present value of free cash flows plus the net investment in financial assets at date zero. In other words, the example illustrates how free cash flows irrelevance can apply no less than dividend policy irrelevance.

4. Some further Comments on the Valuation Formula (7)

A practical, applications-oriented, perspective on the valuation formula (7) raises issues related to how one quantifies its ingredients. With respect to FY1 eps and FY2 versus FY1 growth in eps, these concepts are concrete enough. Analysts do, of course, provide such forecasts in abundance (and correcting for dps_1 in g_2 should generally be of only modest significance).[9]

More of a problem poses the issue of how one can think of γ in concrete terms. To visualize the empirical meaning of a quantity at some arbitrarily distant date is always difficult. But this is what the relation $\frac{\text{eps}_{t+1}}{\text{eps}_t} \to \gamma$ as $t \to \infty$ requires if one takes it literally. Perhaps the most logical interpretation is that the limit growth should correspond to the very long run steady state in which a firm's growth in expected earnings equals the growth in expected GNP. It follows that one can argue that γ should be the same for all firms in the range of 3–5%, say. Another way of thinking practically about γ focuses on the firm's industry, That is, long-run industry growth in expected eps determines $\frac{\text{eps}_{t+1}}{\text{eps}_t}$ as $t \to \infty$. In this case "long-run" may refer to only 10–15 years. After all, no industry is likely to remain in existence in the "very long-run".

Though one can thus reasonably argue that both growth measures are empirically quantifiable, the basic valuation formula (7) must also deal with the ever-present problem of measuring the cost-of-capital parameter r. Issues related to equity risk are always ambiguous, as are problems due to non-flat and stochastic term-structures of interest rates. For these reasons, it is often deemed interesting and useful to think of r as a market-determined required (internal) rate-of-return measure. In such case one infers r from current price and expectations about the future, rather than regard r as a "known" quantity necessary to calculate "intrinsic" value. This shift in emphasis can be applied to (7) without difficulty. Inverting expression (7) results in:

$$r = \frac{\gamma - 1}{2} + \sqrt{\left(\frac{\gamma - 1}{2}\right)^2 + \frac{\text{eps}_1}{P_0} \times (g_2 - (\gamma - 1))} \tag{9}$$

One naturally thinks of r as being a function of the three arguments $(\text{eps}_1/P_0, g_2, \gamma)$. As makes obvious sense, r increases if either eps_1 / P_0 or g_2 increases. Less obvious, one can also show that r increases as γ increases, provided that $z_1 > 0$ or, equivalently, $P_0/\text{eps}_1 > 1/r$.

One can argue that (9) does not really solve for r because the RHS of (9) includes the r-dependent term $g_2 \equiv \left(\frac{\Delta\text{eps}_2}{\text{eps}_1} + r\frac{\text{dps}_1}{\text{eps}_1}\right)$. While technically true, (9) reflects the usefulness of having dividend policy independent measures on the RHS. But one can, of course, also split g_2 into its two components and derive r as a function of the four arguments $(\text{eps}_1/P_0, \text{dps}_1/\text{eps}_1, \Delta\,\text{eps}_2/\text{eps}_1, \gamma)$ instead of the three arguments in (9). This alternative approach yields

$$r = A + \sqrt{A^2 + \frac{\text{eps}_1}{P_0} \times \left(\frac{\Delta\text{eps}_2}{\text{eps}_1} - (\gamma - 1)\right)}$$

where

$$A \equiv \frac{1}{2}\left(\gamma - 1 + \frac{\text{dps}_1}{P_0}\right)$$

The term dps_1/P_0 shows up because $\text{dps}_1/P_0 = (\text{dps}_1 / \text{eps}_1) / (P_0 / \text{eps}_1)$.

As a special case, if $\Delta\,\text{eps}_2 / \text{eps}_1$ happens to equal $\gamma - 1$, then one obtains the familiar formula r = growth in eps + anticipated dividend yield. The word "happens" is appropriate because the growth equivalence is entirely due to a precise calibration of the date 1 dividends policy.

5. Generalizations

Any model's degrees of freedom place a cap on its empirical validity and practical usefulness. This simple dictum applies no less to our valuation model. The $\frac{P_0}{\text{eps}_1}$ formula derived has only two degrees of freedom: Short-term growth as measured by g_2 and long-term (asymptotic) growth as measured by $\lim_{t\to\infty} \frac{\text{eps}_t}{\text{eps}_{t-1}} = \gamma$, and where $g_2 > r > \gamma - 1$. This scheme will obviously be inadequate if one visualizes more complex scenarios as to the evolution of expected eps. For example, a firm may be expected to incur increasing losses for a few years before achieving above normal eps. The so-called internet firms illustrate the case. As another scenario, for cyclical stocks one might have $z_1 > 0$, but $z_2 < 0$; that is, the upcoming "boom" trajectory will be followed by more depressed ones. These circumstances suggest that $P_0/\text{eps}_1 < 1/r$, which violates implications of Assumption A2. Though A2 may make sense "most of the time", it is nevertheless sometimes problematic even if one relaxes the restrictions on γ and z_1 . The question arises as to how one can develop a more general analysis than the one that depends on Assumption A2.

One approach dealing with the degrees of freedom problem extends A2 via an intercept. Consider the anticipation dynamic

$$z_{t+1} = \alpha + \gamma z_t \quad t = 1, 2, \ldots$$

With this assumption, combined with Al, routine calculations will show that

$$P_0 = \frac{\text{eps}_1}{r} + \frac{z_1}{R-\gamma} + \frac{\alpha}{r(R-\gamma)}$$

The expressions show that the constant α in the dynamic results in a valuation function intercept $\frac{\alpha}{r(R-\gamma)}$. Though the scheme is straightforward, it is less than apparent as to how one interprets the new term.

Another way of dealing with the degrees-of-freedom problem relies on the valuation (4) as an estimate of value at the horizon date, rather than as a current date valuation. With this approach Assumption A2 pertains to the post horizon date eps-performance dynamic. To be precise, note first that:

$$P_0 = \frac{\text{eps}_1}{r} + \sum_{t=1}^{T} R^{-t} z_t + R^{-t}\pi_T \tag{10}$$

where

$$\pi_T \equiv E_0[P_T] - \frac{\text{eps}_{T+1}}{r}.$$

Second, with an assumption that

$$z_{t+1} = \gamma z_t, \quad t \geq T$$

one obtains:

$$\pi_T = \frac{z_{T+1}}{R-\gamma} = \frac{\text{eps}_{T+1}\hat{g}_{T+2}}{r(R-\gamma)} \tag{11}$$

The use of a horizon date accordingly results in T additional degrees of freedom. This approach has practical appeal due to its flexibility, but at the same time it allows at least some parsimony. The only disadvantage- -which would seem to be relatively minor- - relates to the absence of a simple closed form solution showing the cost-of-equity capital. The disadvantage is minor in the sense that one easily derives r in equation (10) (combined with (11)) via numerical methods (given any P_0, the expectations $z_1, \ldots, z_T, z_{T+1}$ and the parameter γ).

6. Concluding Remarks

It is interesting to compare the two measures of eps growth that show up in the various formulae. Near-term (FY2 versus FY1) growth is identified by

$$g_2 \equiv \frac{\text{eps}_2 - \text{eps}_1}{\text{eps}_1} + r\frac{\text{dps}_1}{\text{eps}_1}$$

Long-term (asymptotic) growth is identified by

$$\frac{\text{eps}_{t+1} - \text{eps}_t}{\text{eps}_t}$$

or, equivalently,

$$g_{t+1} - r\frac{\text{dps}_t}{\text{eps}_t}$$

These observations precipitate an obvious question: "Why do the two growth measures differ by a payout term?" The answer is subtle; it involves a consideration of how dividend policy irrelevancy works in the model. With respect to g_2, recall that $\frac{\partial g_2}{\partial \text{dps}_1} = 0$ because $\frac{\partial \text{eps}_2}{\partial \text{dps}_1} = -r$ and (!) $\frac{\partial \text{eps}_1}{\partial \text{dps}_1} = 0$. With respect to g_t, $t > 2$, the story is quite different. One *cannot* infer that $\frac{\partial g_t}{\partial \text{dps}_\tau} = 0$ when $1 \leq \tau \leq t-2$, because the dividend policy will influence not only the numerator in g_t but *also* the denominator. Hence, only if $t = 2$ can this denominator effect be neglected. Indeed, Proposition 2 implies that a marginal increase in the dividend payout increases g_t by approximately r (for large t). To elaborate, suppose one can split *eps*$_t$ into two parts so that $\text{eps}_t = \text{opeps}_t + \text{fineps}_t$ as delineated in Section 5. An increase in the (sufficiently generous) policy will then increase the mix $\text{opeps}_t/\text{fineps}_t$; but since the mix ratio does not depend on t asymptotically, the same t-independence holds for $\Delta\,\text{eps}_{t+1}/\text{eps}_t$. One sees that g_t is dividend policy *de*pendent, in sharp contrast to $\frac{\Delta \text{eps}_{t+1}}{\text{eps}_t}$, $t \to \infty$. In sum, the "right" eps growth measures are both dividend policy irrelevant. But given their conceptual difference, care is required to appreciate why this is so.

A final comment pertains to the lack of specificity as to how eps is determined. It is noteworthy that the analysis has been silent on matters related to accounting principles. No such consideration is necessary due to the assumption which specifies the evolution of the eps-performance sequence (A2). Perhaps somewhat surprisingly, the formulae derived rest only on this assumption, aside from the PVED starting point (Al). Hence the usefulness of the formulae hinges crucially on the empirical validity of A2. It's hard to assess such validity since A2 deals with expectations. Nevertheless, one can argue that A2 is the only natural way to proceed if one aspires to derive a $\frac{P_0}{\text{eps}_1}$-ratio that depends only on two growth measures and cost-of-equity-capital.

Appendix I

First consider $T = 1$. Given the definitions

$$G_{t+1} \equiv \frac{\text{eps}_{t+1}}{eps_t} \text{ and } \beta \equiv R - rk,$$

where $k = \frac{\text{dps}_t}{\text{eps}_t}$, $t = 1, 2, \ldots$, one obtains directly from equation (5):

$$G_{t+1} = (\gamma + \beta) - \gamma \cdot \beta \cdot \frac{1}{G_t}, \quad t = 2, 3, \ldots$$

Rewrite the last equation as:

$$G_{t+1} - \gamma = \frac{\beta}{G_t}(G_t - \gamma) \tag{A1}$$

It is now readily seen that the conclusion $G_t \to \gamma$ as $t \to \infty$ follows if $G_t >> \beta$. Note next, that $G_2 > \beta$, and the dividend policy satisfies $\gamma > \beta$.

Consider the following two cases:

(a) $G_2 > \gamma > \beta$ The case is obvious since (A1) implies $G_t - \gamma > 0$ for all t.

(b) $\gamma > G_2 > \beta$ This case follows because $G_2 < \gamma$ implies that G_t increases with t.

Consider next the case when $T > 1$. Under these circumstances $\beta > G_T$ is possible. However, one readily shows that $G_{T+1} > \beta$. The result then follows.

Appendix II

6.1. A Detailed Model of Operating and Financial Activities Consistent with Assumption A2

This appendix articulates a model of a firm's operating and financial activities which jointly satisfy A2. Operating activities are in no sense linear. There are sufficient details so that the model can be evaluated numerically for various specifications. In particular, the reader can verify that (6) in fact equals PVED regardless of the dividend payout policy; further, given a sufficiently generous policy, lim $\text{eps}_{t+1}/\text{eps}_t \to \gamma$.

There are three blocks of input: Basic parameters, operating activities dynamics and financial activities dynamics.

I. Parameters.
R = 1.1 (r = .1)γ = 1.045

II. Operating Dynamics.

(a) Initialization:
operating $eps_1 \equiv ox_1 = 1$
free cash flows on a per share basis $\equiv fcf_1 = .30$
$z_1 = .25$

(b) Recursive Equations:
$z_{t+1} = \gamma\, z_t$
$ox_{t+1} = R\cdot ox_t - r\cdot fcf_t + r\cdot z_t \quad t = 1,2,\ldots$
$fcf_{t+1} = f(ox_{t+1}, t) \quad t = 1,2,\ldots$
where f is any "reasonable" function, e.g.,

$$fcf_{t+1} \equiv \left[q_1 - q_2 \cdot (4 + \sqrt{t})^{-1}\right] \cdot ox_{t+1}$$

and $q_1 = .6$, $q_2 = .3$. The function is "reasonable" in the sense that $fcf_t < ox_t$ and $fcf_t/ox_t \to q_1 = .6$ as $t \to \infty$. We introduce this peculiar but "reasonable" function to show that the model does not depend on q_1 and q_2. That is, the model builds in free cash flows irrelevancy no less than dividend policy irrelevancy.

III. Financial Dynamics.

(a) Initialization:Date $t = 0$ lending/borrowing $\equiv b_0 = 0$

(b) Recursive Equations: $\text{dps}_{t+1} = k\cdot(ox_{t+1} + r\cdot b_t) \quad t = 0,1,2,\ldots b_{t+1} = R\cdot b_t + fcf_{t+1} - \text{dps}_{t+1} \quad t = 0,1,2,\ldots$ where k is the payout, $k = .6$ (Note that t starts with $t = 0$, not $t = 1$ as in the operating dynamics)

The reader can verify the following:

(i) An evaluation of $\sum_{t=1}^{T} R^{-t}\text{dps}_t$, T large (200 is enough) approximates formula (6), $P_0 = \frac{\text{eps}_1}{r} + \frac{z_1}{R-\gamma} = (1/.1) + (25/(1.1 - 1.045)) = 14.545.$

(ii) The payout is sufficiently generous, and thus $(ox_T + r\cdot\ b_{T\text{-}1}/(ox_{T\text{-}1} + r\cdot b_{T\_2})$ approximates 1.045.

(iii) A change of q_2 does not influence conclusions (i) and (ii).

(iv) A small payout ratio, $k = .1$ say, does not change (i) but (ii) now approximates 1.09.

(v) An increase in the payout ratio will have no effect on (i) and (ii).

(vi) Assuming a zero payout for the first 10 years, and putting in $k > .6$ thereafter has no effect on (i) and (ii).

(vii) A sufficiently large (Small) k results in borrowing (lending). The precise cut-off point is not easily identified because the operating dynamic is non-linear. (Hence, the cut-off point for borrowing/lending hinges on fcf_1, q_1, and q_2).

(ix) Because $b_0 = 0$, the present value of expected free cash flows also equals P_0.

(x) If one changes b_0 from 0 to 1, then $\text{eps}_1 = \text{opeps}_1 + \text{fineps}_1 = 1 + .11 = 1.1$ and P_0 now equals 15.545 (i.e., P_0 when $b_0 = 0$ plus 1).

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Notes

1. Finance text-books usually describe the constant earnings/dividends growth model. See, for example, Brealy and Myers (1991), Damodaran (1997), and Elton and Gruber (1995).
2. To be sure, the current model does not in any sense depend on the clean surplus relation.
3. Recent examples of empirical research focusing on the inference of discount-factors (cost-of-capital) include Fama and French (2000), Botosan (1997), Easton et al. (2000), Gebhardt et al. (1999), and Claus and Thomas (1998).
4. The reader should keep in mind firmly that the notation eps_t and dps_t *never* refers to random variables. For more elaborate notation, one could use $\overline{\text{eps}_t} = E_t[\tilde{\text{eps}}_{t+1}]$, but we find it simpler to write eps_t in lieu of $\overline{\text{eps}_t}$. Phrases such as "expected eps" therefore involve a redundancy, and the word "expected" is used only to underscore its importance, or to remind the reader that we are indeed dealing with expectations.
5. As discussed in Ohlson (2000), discounting dps rather than total dividends net of capital contributions makes more economic sense. Under suitable conditions the two approaches will be equivalent, of course. But equivalence cannot be presumed in general.
6. Ohlson (2000) notes that the relation $\text{eps}_t = \Delta\ \text{bvps}_t + \text{dps}_t$ will not generally hold if shares outstanding changes. Of course, one can always *define* eps_t to satisfy the equation, a highly unorthodox approach to eps accounting. Nevertheless, if the equation is met for whatever reasons, then the PV of dps now also equals bvps_0 plus the PV of expected residual eps.
7. The model refers to eps rather than total earnings because A1 refers to dps rather than dividends net of capital contributions. We do not deal with issues related to the accounting changes in shares out-

standing per se. Hence, the reader will not miss any substantive economic/accounting insights by presuming that changes in shares outstanding do not occur (beyond date 0). But it is also well to note that the reference to eps makes practical sense in light of analysts' work-products.

8. Referring back to equations (2) and (3), it is understood that A2 also satisfies the transversality condition $\mathrm{eps}_T/R^T \to 0$ as $T \to \infty$. The condition is met given that one restricts the {dps}-sequence. Intuitively, A2 is meaningless if the dps cease at some future point, and transversality will be violated. A robust, easy-to-interpret, condition which ensures $\mathrm{eps}_T/R^T \to 0$ assumes that $\lim_{T\to\infty} \inf_{t\geq T} \{\mathrm{dps}_t/\mathrm{eps}_t\} > 0$.
9. Analysts typically estimate current year eps, the subsequent year eps, and a 5 year growth in eps rate. The first two estimates do not correspond to FY1 and FY2 estimates unless the current fiscal year just got started. In such cases an informal investigation of ours suggests that the growth FY2 and FY1 eps is almost always well approximated by the 5 year growth rate. Hence, practical applications of (9) can as a reasonable approximation use 5 year growth rates for g_2

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